

## ENERGY LOSS OF BUNCHED BEAMS IN SSC RF CAVITIES†

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A bunched beam that is accelerated by the fundamental resonant mode of RF cavities necessarily loses energy by exciting the higher-order modes of these very cavities. We consider here the calculation of the power lost by gaussian bunches traversing a structure with resonator impedance of arbitrary quality factor. Known formulas are used for the infinite summations, which reduce to other approximate ones for the broad-band case. For the narrow-band case we present a simple analytic expression. We apply our results to calculate the power lost to the higher-order modes of the SSC RF cavities. Our conclusion is that if the resonant frequencies of all higher-order modes of the cavities are sufficiently far from integral multiples of the bunch frequency, the energy loss is strongly reduced over that calculated by a simple broad-band model; but if this condition is not satisfied, then the resulting resonant loss for even a weak mode may be several orders of magnitude greater than the broad-band calculation.

### Introduction

Because of a combination of several requirements on the SSC [1], including large size, high luminosity and few interactions per crossing, it must operate with a large number of bunches, on the order of 17,000. This translates into a bunch frequency  $f_b = 62.5$  MHz. On the other hand, the RF cavities considered have a fundamental mode frequency  $f_R = 375$  MHz. The condition for a resonator mode to be considered broad-banded when traversed by this beam is that its resonant frequency  $f_R$  and quality factor  $Q$  satisfy  $f_R/Q \gg f_b$ . For the fundamental mode, this would require  $Q \ll 6$ , which is patently violated by the PEP-type cavities considered [2], which have quality factors of about 40,000 to 80,000, depending on the mode. Of course higher-order modes are "de-Qed," but never to the point that the above inequality is reasonably satisfied. Therefore it is not legitimate to use broad-band resonator formulas in this case, and previous calculations [3] must be re-examined [4].

### Description of the Calculation

We consider  $M$  identical, equally spaced, well-separated gaussian bunches, each with an rms size  $\sigma$  (in time units). These bunches travel through the ring with revolution frequency  $\omega_0 = 2\pi f_0$ , and traverse a structure with impedance  $Z(\omega)$ . Then the power lost by the bunches is given by [4]

$$P = M Z_{loss} I_b^2 \quad (1)$$

where  $I_b$  is the "bunch current," related to the peak current  $\hat{I}$  by  $I_b = (\sqrt{2\pi}\sigma f_0)\hat{I}$  and to the average beam current  $\bar{I}$  by  $\bar{I} = MI_b$ . The effective "loss impedance" is defined by

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$$Z_{loss} \equiv M \sum_{p=-\infty}^{\infty} \text{Re}[Z(p\omega_b)] e^{-(p\sigma\omega_b)^2} \quad (2)$$

Here  $\omega_b \equiv M\omega_0$  is the angular bunch frequency, and the summation is over all integers (if the structure has multiple modes, there is an additional summation over modes.)

The motivation for splitting an  $M^2$  factor between Eqs. (1) and (2) is that, for short bunches ( $\sigma\omega_b \ll 1$ ) and very broad-banded impedances ( $\omega_b \ll \Delta\omega$ ),  $Z_{loss}$  is independent of  $M$ , as can be easily verified by replacing the summation by an integral in Eq. (2), and therefore the power loss is simply  $M$  times the power lost by a single bunch.

We are concerned here only with resonator impedances of the form

$$Z(\omega) = \frac{R_S}{1 + iQ(\frac{\omega_R}{\omega} - \frac{\omega}{\omega_R})} \quad (3)$$

For the SSC, some nominal values are  $M = 17,280$  (idealized),  $f_0 = 3.617$  kHz, and  $\sigma = 0.233$  nsec. Therefore the bunch frequency is  $f_b \equiv \omega_b/2\pi = 62.5$  MHz, and so the bunches can be considered "short" in the sense defined above, since the parameter  $a \equiv \sigma\omega_b$  that appears in the exponential in Eq. (2) has the value  $a = 0.092$ . In this case the summation can be done analytically with great accuracy [5] assuming only that  $Q > \frac{1}{2}$ , yielding

$$Z_{loss} = \frac{2\pi R_S}{\omega_0 \sigma S} \text{Re}\{z[w(z) - e^{-z^2}(1 - i \cot(\pi z/a))]\} \quad (4)$$

with an error of order  $\exp(-\pi^2/a^2) \simeq 10^{-400}$ . In the above  $z$  and  $S$  are defined by

$$z = \frac{\omega_R \sigma}{2Q}(S + i) \quad (5)$$

and  $S = \sqrt{4Q^2 - 1}$ , and  $w(z)$  is the complex error function [6] (the generalization for  $Q \leq \frac{1}{2}$  is discussed in Ref. [8].)

Note that the bunch frequency appears in Eq. (4) only in the cotangent term, through the parameter  $a$ . Thus when a resonator is broad-banded (i.e. when  $f_R/Qf_b \gg 1$ ) then the argument of the cotangent has a large positive imaginary part, the factor  $(1 - i \cot(\pi z/a))$  vanishes exponentially and the broad-band result [7] is recovered.

However, in the case of interest for the SSC, the 20 or so lowest cavity modes [2] have  $Q \simeq 40,000 - 80,000$  and frequencies  $f_R \simeq 375 - 2,300$  MHz, and therefore it is the very-narrow-band limit that is relevant, since the parameter

$$\Delta \equiv \frac{\pi f_R}{2Qf_b} \quad (6)$$

has values in the range  $\sim 1 \times 10^{-4} - 4 \times 10^{-4}$ . In this case,  $\pi z/a$  is approximately

$$\pi \frac{z}{a} \simeq \pi \frac{f_R}{f_b} \left( 1 + \frac{i}{2Q} \right) \quad (7)$$

Since the imaginary part is very small, the cotangent term provides large, narrow spikes whenever  $f_R/f_b = \text{integer}$ . In fact, for all modes for which  $Q \gg 1$  and  $\Delta \ll 1$ , our final result for  $Z_{loss}$  is, to a very good approximation [6],

$$Z_{loss} = 2MR_S \frac{\Delta^2 e^{-(\omega_R \sigma)^2}}{\sin^2(\pi f_R/f_b) + \Delta^2} \quad (8)$$

which explicitly exhibits the resonances mentioned above (the quantity  $\omega_R \sigma$  is in the range 0.5 – 2.5.)

If we carry out the same approximations in the (incorrect) broad-band model [7], we obtain

$$Z_{loss}^{BB} = \left( \frac{\pi R_S f_R}{Q f_0} \right) e^{-(\omega_R \sigma)^2} \quad (9)$$

If we now compare the correct power loss to the broad-band calculation, we see that on-resonance the true power loss is larger by a factor  $\Delta^{-1}$  than the broad-band model, whereas off-resonance it is suppressed by a factor  $\Delta$ .

### Conclusions

We have derived an analytical expression for the power loss due to the high-order modes of the RF cavities considered for the SSC. The power spectrum shows high, narrow resonances whenever the mode frequency is a multiple of the bunch frequency, and therefore this condition is to be avoided in the final design of the cavities. If this is achieved, the power loss is some 4 orders of magnitude smaller than that calculated from a broad-band model [3]. If this is not the case, and some higher order mode is a harmonic of the bunch frequency, the power loss could be some 4 orders of magnitude *higher* than the broad-band model prediction.

The PEP-type cavities considered for the SSC have modes with large quality factors and frequency spacing typically larger than the bunch frequency, and therefore the resonances in the power spectrum are well-separated and easy to avoid. The nominal design contemplates 40 such cavities, and the higher order modes are expected to be “de-Qed,” but not to the point that  $Q \sim 1 - 10$ . Therefore our approximations will remain valid (a numerical calculation of the power loss taking into account the 20 first modes of the 40 cavities, appropriately “de-Qed,” is presented in Ref. [4].)

Our calculation is based on the assumption that the bunches are identical, evenly spaced (no gaps) and well-separated. In reality the beam will have relatively short gaps (less than 2% of the buckets will be empty.) The spectrum will therefore contain mainly multiples of the bunch frequency (62.5 MHz), while the revolution frequency harmonics will remain at a very small level. Therefore our calculation should not be significantly affected.

During injection only part of the ring is filled with bunches, so the revolution frequency harmonics have a stronger spectrum. Because of the narrow spacing of the lines, there is a much higher probability of falling onto a resonance. The power loss could therefore be larger than when the beam is full. This is partly compensated by the reduced current, and, in any case, it is limited to the injection period of some 20 min. The evolution of the power loss during injection is discussed in the appendix of Ref. [4].

### References

1. SSC Central Design Group, “Superconducting Super Collider Conceptual Design Report,” SSC-SR-2020, March 1986.
2. Z. D. Farkas and K. Bane, SSC-N-124.
3. M. Furman, “Bunch-Length Dependence of Power Loss for the SSC,” SSC-N-143, May 1986.
4. M. Furman, H. Lee and B. Zotter, “Energy Loss of Bunched Beams in RF Cavities,” SSC-86, August 1986.
5. B. Zotter, CERN Report ISR/TH 80/03, 1980.
6. M. Abramowitz and I. A. Stegun, eds., “Handbook of Mathematical Functions,” Dover, 1965, p. 297.
7. M. Furman, “Practical Formula for Bunch Power Loss in Resonators of Almost Arbitrary Quality Factor,” SSC-N-142, May 1986.
8. H. Lee, “Generalized Formula for Power Loss of a Gaussian Bunch in Resonators,” SSC-N-179, May 1986.