EFFECT OF THE SPACE-CHARGE FORCE ON TRACKING AT LOW ENERGY[†]

Miguel A. Furman

SSC Central Design Group*

c/o Lawrence Berkeley Laboratory

Berkeley, CA   94720

We present tracking results for the SSC's Low Energy Booster at injection energy, including the effect of the space-charge force. The bunches are assumed to be gaussian with elliptical cross-section. Magnet errors and sextupoles are not included, but an RF cavity is. We compare the phase space with and without synchrotron oscillations, with and without space-charge. The effective emittance is not significantly altered. We also present results on tune shifts with amplitude.

Introduction

The Low Energy Booster (LEB) [1] is the second element of the injector complex in the SSC. Since the particles injected by the linac into the LEB are nonrelativistic ($\beta = 0.79$), it is important to study the effect of the space charge force, which is maximal at low energy. Because of its nonlinear nature, it has the potential for distorting the phase space to the point where the effective emittance is unacceptably large. In order to assess its effect, we have performed tracking simulations up to 500 turns, and looked at the phase space and tune shifts. We include one RF cavity, so particles perform synchrotron oscillations. We have left out the sextupole magnets as well as magnet errors, so that the only nonlinearities are those introduced by the space-charge force and the kinematics. We conclude that the space-charge force does not alter phase space significantly. We also obtain the tune shifts as a function of amplitude, and compare with the nominal values [1] and also with the analytic results in the linear approximation.

LEB Lattice and Parameters

The nominal [1] LEB lattice has a FODO structure with circumference $C = 249.6$ m distributed over 5 superperiods. The horizontal and vertical tunes are $\nu_x = 4.39$ and $\nu_y = 4.41$, and the average beta functions are $\bar{\beta}_x \simeq \bar{\beta}_y \simeq 9$ m. There are 52 bunches equally spaced by 4.8 m (one per RF bucket, making the harmonic number $h = 52$). The bunches have $N_B = 7.3\times10^9$ particles and bunching factor 0.25, corresponding to an rms bunch length $\sigma_z = 0.48$ m. Injection momentum is $pc = 1.22$ GeV, corresponding to $\gamma = 1.64$ and $\beta = 0.79$. The normalized vertical and horizontal emittances are equal, $\epsilon_{Nx} = \epsilon_{Ny} \equiv \epsilon_N = 0.75$ mm-mrad, and the longitudinal emittance is $\epsilon_L = 1.6$ meV-sec. The RF cavities provide a total voltage around the ring $V_0 = 350$ kV. The momentum spread is $\Delta p/p = 9.2 \times 10^{-4}$. The lattice design is such that the LEB operates always below transition.

Description of the Simulation

All results presented here are at injection energy [3]. The space-charge force is computed in the approximation that the bunch length is infinite compared with the bunch width (a very good approximation in our case, where length $\simeq 0.48$ m and width $\simeq 1.5$ mm). We assume that the bunch has gaussian charge distribution in all 3 dimensions, with (in general) unequal rms sizes, and we approximate the electric field by a simple rational function that is accurate both at short and long distance [2]. We track one single particle which feels the electromagetic force produced by all the other particles in the bunch, but its effect is not fed back into the bunch size or shape. The bunch is assumed to be always gaussian, with transverse rms sizes σ_x and σ_y determined only by the lattice functions and momentum spread at every point of the lattice. This approximation is justified a posteriori by the relative smallness of the effects.

Since we want to focus on the (nonlinear) effect of the space-charge force, we set all other nonlinearities of the lattice (magnet errors and chromatic sextupole strengths) to zero. We replace the 5 RF cavities by a single cavity with voltage $V_0 = 350$ kV, placed in the middle of one of the drift sections. This is a point where the dispersion function and its derivative are far from zero, so there is significant synchro-betatron coupling. This is, generally, an undesirable feature, and future lattice designs should correct it. For our present purposes, however, we take this into account by finding the 6-dimensional fixed point which describes the coupling, and dealing always with the normal coordinates. This is achieved with the tracking program FRANKENSPOT [4], which is a variant of the thin-element tracking program TEAPOT [5]. Once a thin-lens description of the lattice is set up, the tracking is done exactly, except, of course, for machine precision. The space-charge force is also represented by a thin elements (kicks). In practice, we have adopted the criterion that the space-charge kicks should typically be $\frac{1}{10}$ as strong as the quadrupole kicks. This implies, for our specific simulation, that there is one space-charge kick per quadrupole magnet.

From the tracking program we collect 6-dimensional tracking data at one specific observation point (located immediately before the RF cavity,) and calculate the 6-dimensional fixed point, eigenvalues (which yield the 3 tunes,) and eigenvectors (normal coordinates.) FRANKENSPOT does these computations with Lie-algebraic techniques, and has the capability of computing higher-order maps which account for higher-order nonlinearities. This part is based on the program MARYLIE [6].

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Results

Tune vs. Amplitude

To study this we use on-momentum particles only (no synchrotron oscillations,) launched with several different amplitudes, and track them for 100 turns. We inject all particles with transverse displacements $x = y$ and slopes $x' = y' = 0$ in all cases. Fig. (1) shows ν_x and ν_y plotted vs. x. As mentioned above, the linear lattice has $\nu_x = 4.39$ and $\nu_y = 4.41$, and we see that the simulation results indeed approach these values for large amplitude oscillations, as it should be the case. For small amplitudes, the tunes are shifted downwards by the amounts $\Delta\nu_x = -0.065$ and $\Delta\nu_y = -0.093$ which should be compared with the nominal CDR [1] value of -0.17, and with the linear tune shift calculation,

$$\Delta\nu_x = -\frac{r_0\, N_B\, C\, \bar{\beta}_x}{2\pi\beta^2\gamma^3\, \bar{\sigma}_x(\bar{\sigma}_x + \bar{\sigma}_y)\sqrt{2\pi}\sigma_z} \tag{1}$$

and a corresponding expression for $\Delta\nu_y$. Here $r_0 = 1.536 \times 10^{-18}$ m is the classical proton radius, and the $\bar{\sigma}$s are the average beam sizes given by $\bar{\sigma}_x = \sqrt{\bar{\beta}_x\epsilon_B + \overline{(\eta\,\Delta p/p)^2}}$ and $\bar{\sigma}_y = \sqrt{\bar{\beta}_y\epsilon_B}$. The dispersion function has a squared average $\overline{\eta^2} \simeq 25$ m^2, and the "beam emittance" is $\epsilon_B = \epsilon_N/(\beta\gamma) = 0.577$ mm-mrad. This yields $\bar{\sigma}_x \simeq 5.5$ mm and $\bar{\sigma}_y \simeq 2.5$ mm, so that $\Delta\nu_x \simeq -0.035$, and $\Delta\nu_y \simeq -0.075$.
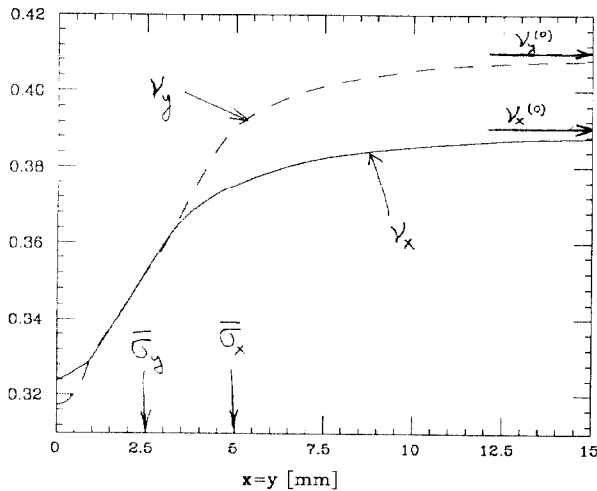

Fig. 1 : Tunes vs. Launching Amplitude

Phase Space

We track particles for 500 turns (the synchrotron period is $\simeq 29$ turns) and plot the horizontal phase space in the normal coordinates x_1, x_2. Fig. (2) shows 3 particles launched on-momentum (no synchrotron motion), with transverse amplitudes corresponding to $\frac{1}{2}$-σ, 1-σ, and 2-σ. In Fig. (3) the same particles are launched off-momentum, performing 1-σ synchrotron oscillations. The additional "smear" in Fig. (3) is due to the space-charge force. The fact that the particles in Fig. (3) appear to have smaller emittances than in Fig. (2) is not physical, and is due to a poor choice of transverse initial conditions (we launched with the same values of x and x', so that the normal coordinates were *not* the same due to the longitudinal coupling.) In Fig. (4) the vertical motion is turned off, and the particles

move with (A-curves) or without (B-curves) synchrotron oscillations (1-σ.) Note that the "smear" of the A-curves has almost completely disappeared (compare with Fig. (2)) indicating that this "smear" is due mostly to the $x - y$ coupling introduced by the space-charge force than to its nonlinear quality. In Fig. (5) the launching conditions are similar to those in Figs. (2) and (3) combined, but the space-charge force is turned off, revealing the nonlinearity due to the longitudinal coupling.

Effective Beam Emittance

In order to sumarize the effect of the space-charge force exhibited in the phase space plots, we compute an "effective emittance" of the beam as follows: we track 1600 particles with different values of the injection emittances ϵ_x, ϵ_y, and compute for each the resulting average emittances $\bar{\epsilon}_x, \bar{\epsilon}_y$. We assume that the particles are gaussian-distributed, so that the effective emittance is defined to be
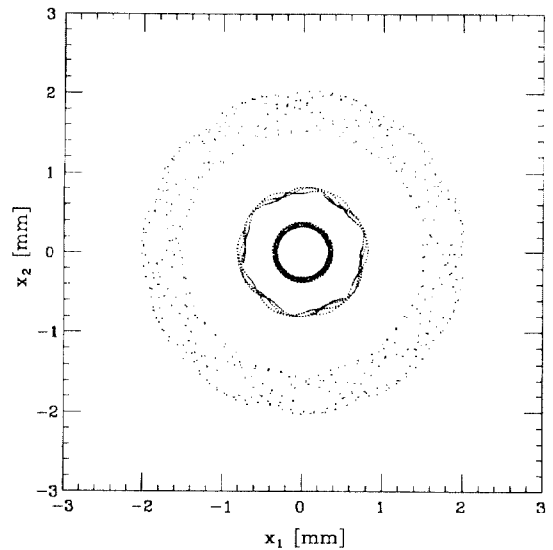

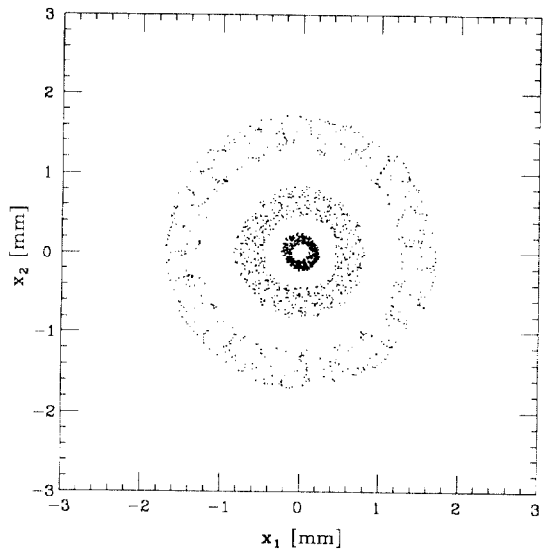Fig. 2 : With Space Charge, no Synchrotron Oscillations

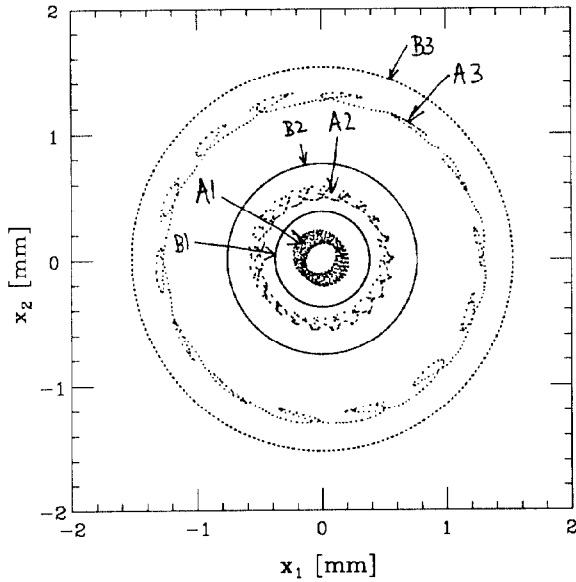Fig. 3 : With Space Charge and Synchrotron Oscillations

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Fig. 4 : With Space Charge, no Vertical Motion
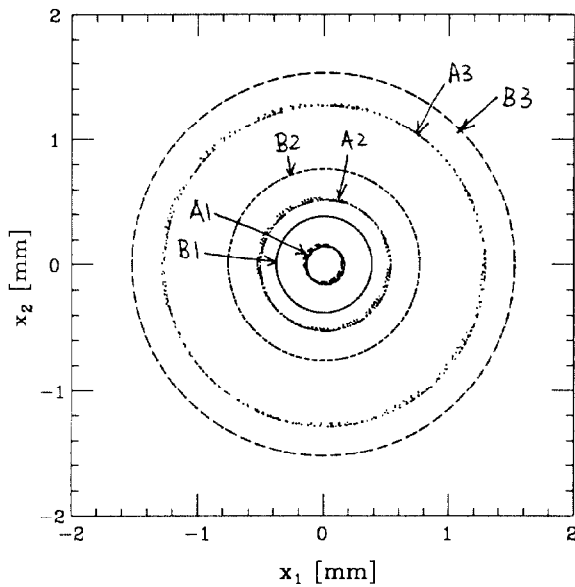A: with Synchr. Osc.; B: without Synchr. Osc.

Fig. 5 : Space Charge turned off
A: with Synchr. Osc.; B: without Synchr. Osc.

$$\epsilon_{x,eff} = \frac{1}{\epsilon_B^2} \int_0^\infty d\epsilon_x \, d\epsilon_y \, e^{-(\epsilon_x+\epsilon_y)/\epsilon_B} \, \bar{\epsilon}_x(\epsilon_x,\epsilon_y) \qquad (2)$$

and a similar expression for $\epsilon_{y,eff}$. We present the results comparing emittances with and without space charge, when the longitudinal amplitude is 1-σ,

$$\frac{\epsilon_{x,eff}(with\ SPC)}{\epsilon_{x,eff}(without\ SPC)} = 1.044 \qquad (3)$$

$$\frac{\epsilon_{y,eff}(with\ SPC)}{\epsilon_{y,eff}(without\ SPC)} = 0.980 \qquad (4)$$

Very similar results (very close to 1) are obtained when the longitudinal amplitude is 0 or $\frac{1}{2}$-σ.

Conclusions

Since the tune shifts with amplitude are smaller in absolute value than the nominal CDR value (mostly due to the effect of the dispersion,) one may consider modifying the injection system by lowering the injection energy into the LEB (lower energy linac) or increasing the beam intensity.

For the specific parameters considered here (which are nominal,) the space-charge force does not have significant consequences on phase space distortion nor emittance growth. The fractional change of the β-function is a few percent, corresponding to a few percent smear and negligible emittance growth (the smallness of the effect justifies our basic approximation.)

Extensions of this work is much needed and is in progress. We are considering repeating the simulations done here for lower energy and/or higher bunch intensity, and more spread-out, weaker space-charge kicks. The relatively large tune shift with amplitude implies that several resonances are crossed near the working point, and it would be interesting to see the corresponding effects. Finally, the lattice needs to be re-examined regarding the RF cavities location and the dispersion function.

Acknowledgments

References

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