BEAM BREAKUP WITH TRANSVERSE FOCUSSING*

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Introduction

The large transient beam displacement which occurs in a high current linear accelerator consisting of identical sections can be reduced by including transverse focussing[1]. In the work conducted at SLAC[1], the focussing was considered weak compared to the beam breakup "defocussing". More recently[2], Yokoya has shown that there is some amplification of the transverse oscillations due to beam breakup even if the external focussing is strong.

We have started with the difference equations for cumulative beam breakup[3] and have obtained expressions for the transient behavior of the transverse displacement in both limits. Simulations are then used to connect these limits through the region of intermediate transverse focussing strength.

Differential Equations

If one extracts the $\exp(iM\omega\tau)$ behavior from both the cavity excitation and the transverse displacement and angle, the resulting difference equations involving slowly varying functions of N and M can readily be approximated by difference equations[3]. Here N is cavity number, M is bunch number, $\omega/2\pi$ is the deflection mode frequency, and τ is the time interval between beam bunches. (We assume the non-resonant condition that $\omega\tau/2\pi$ is not near an integer.) If we write the displacement as

$$\xi(N,M) = \mathrm{Re}\,[w(N,M)\, e^{iM\omega\tau}], \tag{1}$$

we obtain[4]

$$\frac{\partial^2 w}{\partial N^2} + \mu^2 w = \frac{z}{i}\ , \quad \frac{\partial z}{\partial M} = \frac{r}{2}\, w. \tag{2}$$

Here μ is the phase advance of the (smooth) transverse oscillation in the absence of beam breakup, and $r = RL/\gamma$ is a parameter proportional to beam current and the transverse shunt impedance of each cavity[3]. The complex parameter $z(N,M)$ is proportional to the amplitude of excitation of the Nth cavity at the time the Mth bunch transverses the cavity.

If we assume that w and z have exponential dependence of the form

$$w(N,M) = We^{q(N,M)}\ , \quad z(N,M) = Ze^{q(N,M)} \tag{3}$$

and neglect derivatives of W and Z, as well as second derivatives of the slowly varying exponent $q(N,M)$, we obtain the equation

$$\left[\left(\frac{\partial q}{\partial N}\right)^2 + \mu^2\right] \frac{\partial q}{\partial M} \cong \frac{r}{2i}\ , \tag{4}$$

where we permit r to have dependence on N in the general case. Our task is to solve Eq. (4) for small μ (treating the transverse focussing as a small perturbation) and for small r (treating beam breakup as a small perturbation).

Focussing as a Small Pertubation

It is possible to solve Eq. (4) as a power series in μ^2. The result for the first two terms is

$$q(N,M) \cong \frac{3}{2} M^{1/3} G^{2/3} e^{-i\pi/6} - \frac{e^{i\pi/6}}{2} M^{-1/3} G^{1/3} \int_0^N \frac{\mu^2 dn}{\sqrt{r(n)}} \tag{5}$$

where $G(N) = \int_0^N dn\, \sqrt{r(n)}$. The corresponding result for the maximum displacement with a single offset pulse is

$$\frac{|\xi_{max}|}{\xi_0} = \frac{(MG^2)^{1/6}}{M\sqrt{6\pi}}\, e^{p_b}, \tag{6}$$

where the real part of the exponent for the case where beam breakup dominates is

$$p_b(N,M) = -\frac{M\omega\tau}{2Q} + \frac{3\sqrt{3}}{4} (MG^2)^{1/3} - \frac{9}{32} \rho (MG^2)^{-1/3} \frac{Q}{\omega\tau}\ . \tag{7}$$

Here

$$\rho = \frac{8}{3\sqrt{3}} \frac{\omega\tau}{Q} \frac{\int_0^N \frac{\mu^2}{\sqrt{r}} dn}{\int_0^N \sqrt{r}\, dn} = \frac{8}{3\sqrt{3}} \frac{\omega\tau}{Q} \frac{\mu^2}{r} \tag{8}$$

where the last form is valid for μ and r independent of n. The peak of $p_b(N,M)$ occurs at bunch number M_b given by

$$\frac{M_b}{M_o} = \left(\frac{\sqrt{1+\rho}+1}{2}\right)^{3/2}\ , \tag{9}$$

at which p_b has the value

$$\frac{p_b}{p_o} = (2 - \sqrt{1+\rho}) \left(\frac{\sqrt{1+\rho}+1}{2}\right)^{1/2}\ , \tag{10}$$

where M_o and p_o are the peak bunch number and exponent for $\mu = 0$ ($\rho = 0$). Thus ρ is a scale parameter for measuring the importance of focussing in suppressing beam breakup. An alternate form of ρ is easily obtained from Eq. (8) as

$$\rho = \left(\frac{\mu N}{p_o}\right)^2\ , \tag{11}$$

which is the square of the ratio of the total phase advance in N cavities to the maximum exponent.

Beam Breakup as a Small Perturbation

The solution of Eq. (4) can also be obtained as a power series in r. The result for the first two terms is

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$$q(N,M) \cong i \int_0^N \mu dn + [M \int_0^N \frac{r(n)}{\sin\mu} dn]^{1/2}. \qquad (12)$$

The corresponding result for the maximum displacement with a single offset pulse is

$$\frac{|\xi_{max}|}{\xi_o} \cong \frac{[M \int_0^N \frac{r(n)}{\sin\mu} dn]^{1/4}}{M\sqrt{8\pi}} e^{P_f}, \qquad (13)$$

where the real part of the exponent for the case where focussing dominates is

$$P_f(N,M) = -\frac{M\omega\tau}{2Q} + (MG)^{1/2} \left(\frac{8\omega\tau}{3Q\sqrt{3}}\right)^{1/4} \rho^{-1/4} . \qquad (14)$$

Here

$$\rho = \frac{8}{3\sqrt{3}} \frac{\omega\tau}{Q} \left[\frac{\int_0^N \sqrt{r}\, dn}{\int_0^N \frac{r(n)\, dn}{\sin\mu}}\right] = \frac{8}{3\sqrt{3}} \frac{\omega\tau}{Q} \frac{\sin^2\mu}{r} \qquad (15)$$

where the last form is valid for μ and r independent of n. Comparison with Eq. (8), which holds for small μ, suggests that μ should be replaced by sin μ in Eq. (8). The peak of $P_f(N,M)$ occurs at

$$\frac{M_f}{M_o} \cong \frac{8}{3\sqrt{3}} \rho^{-1/2} \qquad (16)$$

at which P_f has the value

$$\frac{P_f}{P_o} \cong \frac{4}{3\sqrt{3}} \rho^{-1/2} . \qquad (17)$$

Comparison With Simulations

Simulations have been performed for the same parameters used earlier[3], namely $\omega\tau/Q = 11.6 \times 10^{-3}$, $r = 2.88 \times 10^{-3}$, for values of sin μ from 0 to 1. In each case the value of M at which the transient peak occurs and the value of the peak exponent $[\ell n(|\xi_{max}|/\xi_o)]$ are compared with the corresponding values in the absence of focussing (μ = 0). The quantities M_{max} and $\ell n(|\xi_{max}|/\xi_o)$ are plotted in Figs. 1 and 2 against $\sin^2\mu$. The figures also contain the low and high ρ approximation in Eqs. (9), (10), (16) and (17) and confirm the validity of these approximations as well as the universal dependence of the results on

$$\rho = \frac{9}{3\sqrt{3}} \frac{\omega\tau}{Q} \frac{\sin^2\mu}{r} \qquad (18)$$

Discussion

We have successfully derived the transient beam breakup behavior in the presence of transverse focussing in the limits of focussing small or large compared to the defocussing effect of beam breakup. Simulations show that the maximum bunch number and exponent vary smoothly with the focussing parameter given in Eq. (18). The scaling parameter ρ directly determines the focussing strength necessary to suppress transient beam breakup to the desired level.

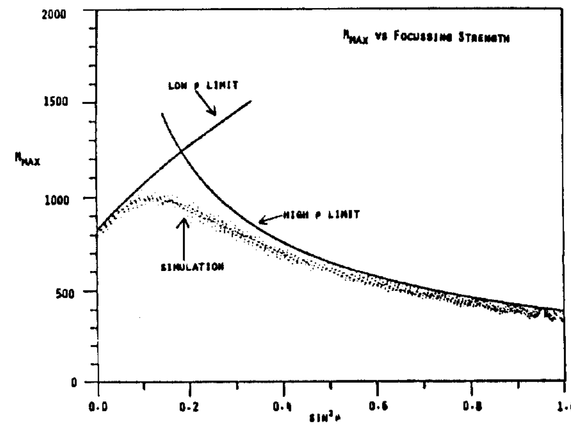

Fig. 1. Bunch number at maximum vs. focussing strength for $\omega\tau/2\pi$ = 24/13, Q = 1000, $r = 2.88 \times 10^{-3}$, N = 30.

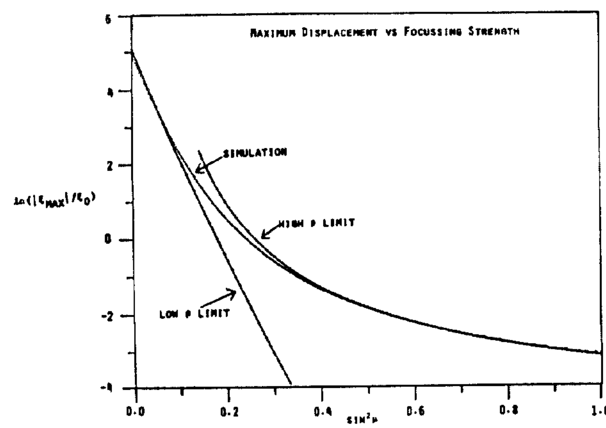

Fig. 2. Maximum exponent $[\ell n(|\xi_{max}|/\xi_o)]$ vs. focussing strength for $\omega\tau/2\pi$ = 24/13, Q = 1000, $r = 2.88 \times 10^{-3}$, N = 30.

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References

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1. See, for example R. Helm and G. Loew, Linear Accelerators, edited by P.M. Lapostolle and A.L. Septier, John Wiley and Sons, 1970, p. 201.

2. K. Yokoya, DESY Report 86-084, August 1986.

3. Gluckstern, Cooper and Channell, Particle Accelerators, 16, 125 (1985).

4. A more detailed derivation of these equations and their consequences for slowly varying parameters is being submitted for publication.