THE EFFECT OF NONLINEAR FORCES ON COHERENTLY OSCILLATING SPACE-CHARGE-DOMINATED BEAMS*

C.M. Celata
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

Abstract

A particle-in-cell computer simulation code has been used to study the transverse dynamics of nonrelativistic misaligned space-charge-dominated coasting beams in an alternating gradient focusing channel. In the presence of nonlinear forces due to dodecapole or octupole imperfections of the focusing fields or to image forces, the transverse rms emittance grows in a beat pattern. Analysis indicates that this emittance dilution is due to the driving of coherent modes of the beam near their resonant frequencies by the nonlinear force. The effects of the dodecapole and images forces can be made to effectively cancel for some boundary conditions, but the mechanism is not understood at this time.

Introduction

The high power density needed on target for a heavy ion fusion reactor implies the need for extremely intense beams. In the parameter range of interest such beams are "space-charge-dominated"--i.e., the space charge forces, rather than the emittance, determine the beam envelope radii. Indeed, about 99% of the focusing field strength is cancelled by the space charge force, giving a space-charge-depression of the betatron frequency to ~ 0.1 of its zero-current value.

The general problem addressed in part by this paper is the question of what phenomena limit the intensity of a coasting beam which can be stably transported without emittance degradation in an alternating gradient focusing channel. This question has been investigated by Hofmann, Laslett, Smith, and Haber[1] for the case where image forces are neglected and no external nonlinearities are present. In that paper it was shown that coherent modes of the beam exist. However, these modes were shown by simulation to be nonlinearly stabilized with negligible emittance growth when $\sigma_0 \leq 80°$,[1,2] where σ_0 is the zero-current value of the betatron phase advance per lattice period for a particle. This was confirmed by experiment.[3]

In this paper we examine the effect of external nonlinear forces and image forces, i.e., the forces due to charge induced on the conductors around the beam, on space-charge-dominated beams. Note that for intense beams the image forces can become of the order of the focusing field forces. As discussed below, no important effect on beam quality is found for beams which are centered in the focusing channel. However, for misaligned beams significant emittance increase is seen for all of the nonlinearities that have been studied.

Emittance Growth Due to Misalignments

We will first consider the effect of forces due to charge induced on nearby conductors, the so-called "image forces", on the beam. We have studied two different boundary conditions: the round conducting pipe concentric with the center of the focusing channel, and the case pictured in Fig. 1, which we shall refer to as the "cylindrical quadrupole" geometry. Figure 1 shows four conducting cylindrical electrostatic quadrupole electrodes surrounding the beam. This models the geometry presently used in the Single Beam Transport Experiment and Multiple Beam

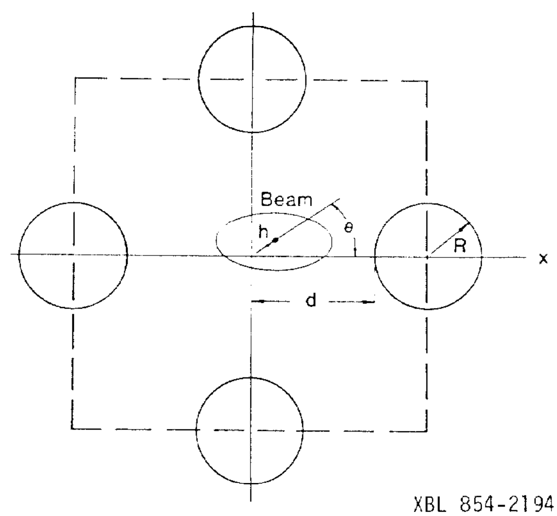

Fig. 1. The geometry of the model, showing the transverse plane, with conducting electrodes (present at all z) and the boundary of the simulation grid (dashed).

Experiment of the Heavy Ion Fusion Program at Lawrence Berkeley Laboratory. Though in the experiments the electrodes occupy only about 60% of the longitudinal space available, we have made the simplifying assumption in the simulation code that the electrodes are present at all longitudinal locations. The error implicit in this assumption is ameliorated by the fact that the beam radius is smaller between electrodes, and therefore image forces are less important. The simulation imposes periodic boundary conditions along the dashed line shown in Fig. 1. This simulates the case of an (infinite) array of beams in a regular array of focusing electrodes. All conductors are assumed by the code to be perfect conductors.

The particle-in-cell code used for this study was the code SHIFTXY, written by I. Haber. It is a two dimensional (x-y) nonrelativistic code. Space charge forces are computed self-consistently. For the studies presented in this paper, alternating gradient focusing has been modeled using a thin lens approximation, though the code is capable of calculating for finite-length focusing elements. With only a few exceptions, which will be noted below, the initial distribution function used was gaussian in v_x and v_y, with uniform temperature and density.

For the case of a beam in a square conducting pipe, it was shown by I. Haber[4] that although the shape of the beam is changed, the emittance of a centered beam is unaffected by image forces unless the beam fills more than 85% of the aperture radius. For larger filling factors particle loss occurs. We also observe very little effect on beam emittance for centered beams for any of the nonlinearities studied, until the beam fills most of the aperture. Phase plots for a centered beam in the cylindrical quadrupole geometry, with no external nonlinearities present, are shown in Figs. 2 and 3 for the beam initially and after 19 periods of transport. In this case, where the beam fills 75% of the aperture radius, there is an initial rise in rms emittance by about 8%, followed by a very slow rise of ~ 1.5% per 100 periods.

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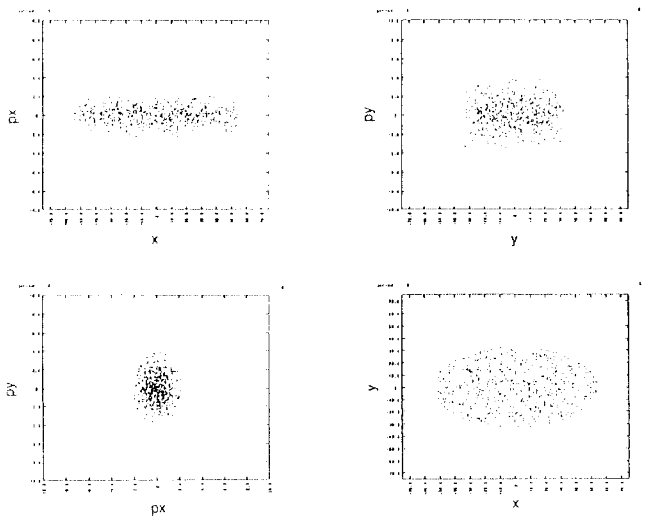

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Fig. 2. Phase plots for initial distribution function for centered beam. $\sigma_0 = 60°$, $\sigma = 6°$, rms major radius/distance to electrode surface = 0.375.

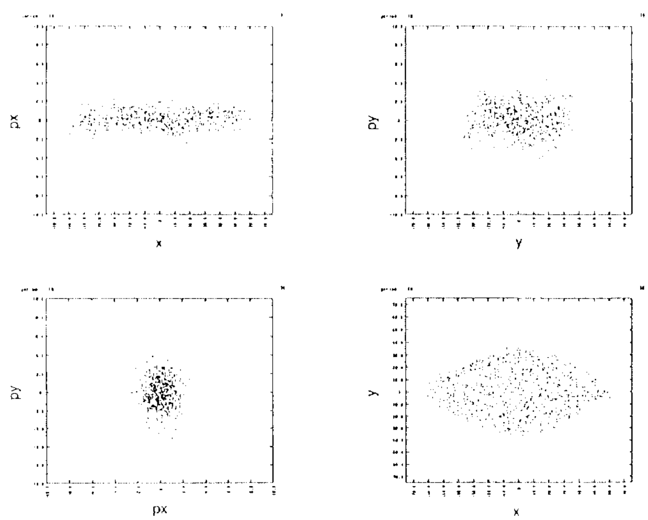

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Fig. 3. Phase plots at period 19 in cylindrical quadrupole geometry for beam shown in Fig. 2.

The growth of transverse rms emittance for a misaligned beam exposed to image forces is shown in Fig. 4 for the round pipe and cylindrical quadrupole geometries described above. (Rms emittance is defined as $\epsilon_x \equiv (\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2)^{1/2}$, and similarly for y, where $x' = dx/dz$.) The emittance in Fig. 4 is recorded once per lattice period at the center of the x focusing lens. The rms y emittance exhibits the same behavior as the x emittance, in phase with it. The beats that can be clearly seen in the cylindrical quadrupole case are not present for the round pipe case shown, though when the parameters are changed to give larger emittance growth the round pipe case also exhibits beats. The sum frequency for the beats is approximately twice the coherent betatron frequency. Note that the emittance growth for the round pipe boundary is much less than for the cylindrical quadrupoles, even though the beam has been displaced by twice as much. This is probably due to the fact that the image forces for the highly

Emittance Growth from Image Forces

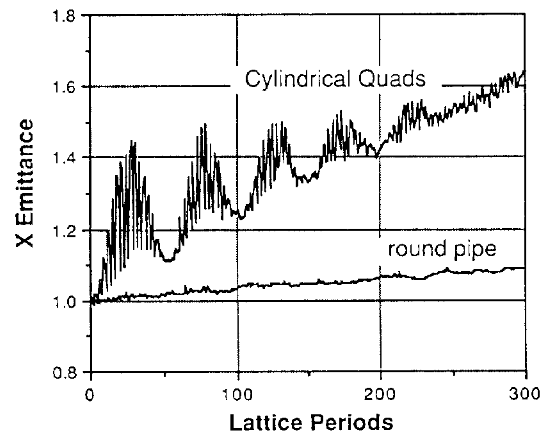

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Fig. 4. x rms emittance, normalized to its initial value, for boundary conditions described in the text. $\sigma_0 = 60°$, $\sigma = 6°$, beam major radius/distance from axis to nearest conductor = 0.42, $\theta = 45°$, h = a/4 (cyl. quads), a/2 (round pipe).

symmetric round pipe are much weaker. A few runs have been done for a Kapchinskij-Vladimirskij distribution function, which shows very similar patterns and magnitude of emittance growth.

Further characterization of the phenomenon shown in Fig. 4 can be obtained by investigating other moments of the distribution. The rms x and y radii of the beam do not vary during the run, while the rms v_x and v_y show the same behavior as the emittance. Thus, the beam is heating, not expanding. No particle loss is seen. Phase plots (see Fig. 5) show some triangularization of the beam in x-y, x-p_x, and y-p_y space, and even symmetry in x and y, indicating that the potential causing the beam deformation is odd in x and y. (Plots for the initial beam are the same as those in Fig. 5, with all axes of that figure scaled down.) This odd symmetry indicates that the distribution function will have non-zero odd multipole components, and therefore

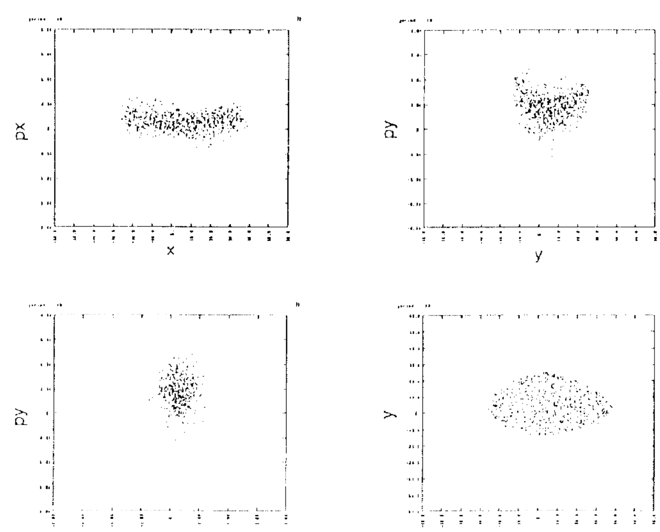

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Fig. 5. Phase plots for period 19 for cylindrical quadrupole parameters of Fig. 4.

non-zero third moments. We have therefore also looked at third order moments of the distribution function. These show beats at the same frequency as the emittance, though the sum frequency is half of that for the emittance. No secular growth of the third order moments is seen.

Thorough scaling studies have not been done, due to the extensive parameter space and the limited computer time available. The emittance growth increases with σ_0, the beam major radius, a, and the offset, h (see Fig. 1), and decreases with increasing aperture or betatron phase advance per lattice period, σ. The secular growth of the emittance is found to be approximately linearly proportional to the amplitude of the emittance oscillations.

Some experimental evidence for emittance growth due to image forces has been obtained in the Single Beam Transport Experiment,[3] in which a 120 keV Cs^+ beam is transported through 40 periods of alternating gradient focusing. Figure 6 shows the rms x emittance vs. z for two beams with different initial offset, h, from the focusing channel axis. The emittance has been calculated from the measured phase space plots using either 100% of the current or only the inner 95% in order to exclude halo effects. The emittance difference between the two beams at a given z location can be measured accurately to a few percent, but comparison at different z locations is hampered by calibration difficulties. As the graph shows, the emittance of the beam with the greater offset grows as a function of z with respect to the other beam. With h = 0 no emittance growth was seen. The magnitude of the emittance growth observed was consistent with simulation calculations, but more data would be needed to prove that the effect observed was the same as that seen in calculations.

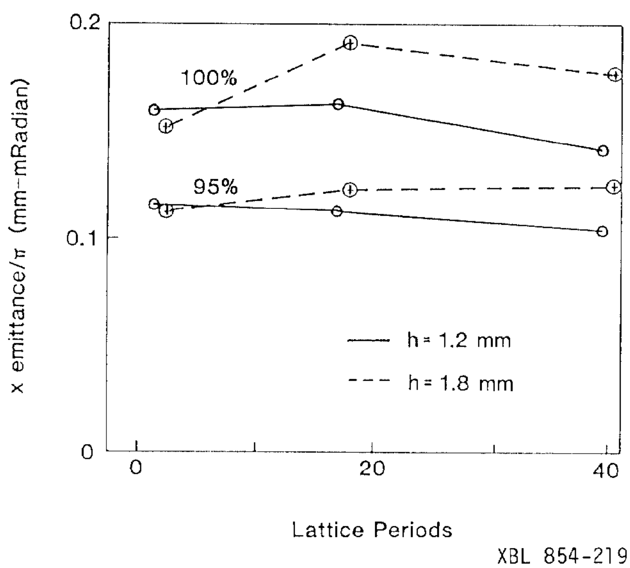

Fig. 6. Experimental measurements of rms x emittance (circled points) for two off-center beams. σ_0 = 60°, σ = 8°.

Similar effects to those just described for image forces are seen in simulation calculations in which external nonlinear forces are present. Figure 7 shows the growth of emittance for a misaligned beam in the presence of a dodecapole potential. If quadrupole symmetry is maintained, this is the lowest order multipole which can occur as an imperfection in the focusing field. The focusing potential applied in the code is therefore $V_0[A_2(r/d)^2 \cos 2\theta + A_6(r/d)^6 \cos 6\theta]$, where d is the distance from the center of the focusing channel to the surface of the cylindrical quadrupole electrode. V_0 alternates in sign and a thin lens

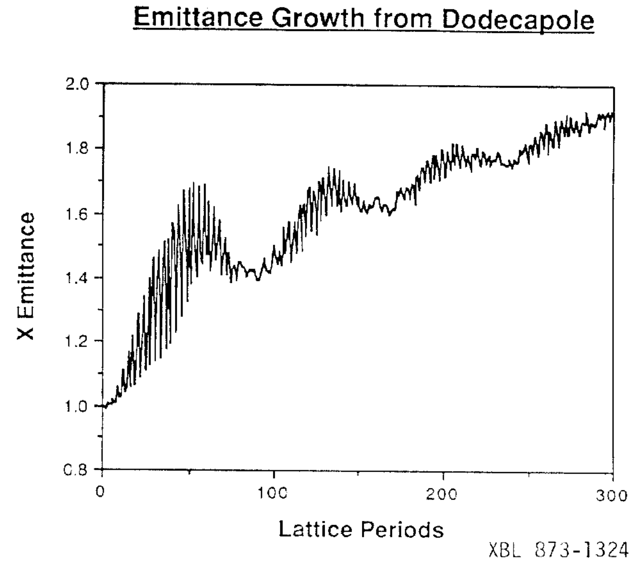

Fig. 7. x rms emittance vs. z with external dodecapole force. σ_0 = 60°, σ = 6°, h = a/2, θ = 0°, $A_6/A_2 = 1.02\times10^{-2}$.

approximation is used. No image forces are applied. Again, if the beam is centered in the focusing channel no emittance growth occurs. For misaligned beams a beat pattern is again seen in the emittance growth, with the sum frequency approximately equal to twice the coherent betatron frequency. The emittance growth, and also the amplitude of the emittance oscillations, appears to be proportional to A_6h, and it increases with increasing σ_0 or decreasing σ. Again the rms beam radii are essentially constant, while the rms v_x and v_y increase in the same manner as the emittance. However phase plots[5] show an odd symmetry in x and y, and therefore an even potential, indicating that while the macroscopic beam changes are similar to the image force case, the process is different in detail.

Similar effects have also been seen for an alternating sign octupole potential used to model end effects of interdigital electrostatic quadrupole structures.[5] It seems reasonable therefore to state the following as a general result: For intense space-charge-dominated ($\sigma/\sigma_0 \sim 0.1$) misaligned beams the presence of a nonlinear force causes emittance growth in a beat pattern. This is due to beam heating, with negligible change in rms beam radii.

A Possible Mechanism

The beat pattern seen in the graphs of emittance vs. z suggests that the nonlinear forces applied may be driving a coherent mode of the beam at a frequency slightly off resonance. As noted above, Hofmann et al. showed that space-charge-dominated beams in alternating gradient focusing systems have coherent modes which stabilize nonlinearly with negligible effect on beam emittance[1]. The potential of these modes is proportional to cos mθ. Figure 8 shows the phase advance per lattice period of the m = 3 or "third order" modes vs. σ for σ_0 = 60°. Note that one of the modes has a phase advance per lattice period which approaches σ_0 as the beam becomes more intense ($\sigma \to 0$). This occurs for all m. The phase advance per period of the coherent betatron oscillation is also nearly the same as σ_0, the deviation being a measure of the strength of the nonlinearity versus that of the linear focusing force. Therefore the mode frequency is close to the coherent betatron frequency. Moreover, the strength of the applied nonlinear force at the beam will be modulated at the coherent betatron frequency, since its value depends on the distance of the centroid from the axis of the focusing

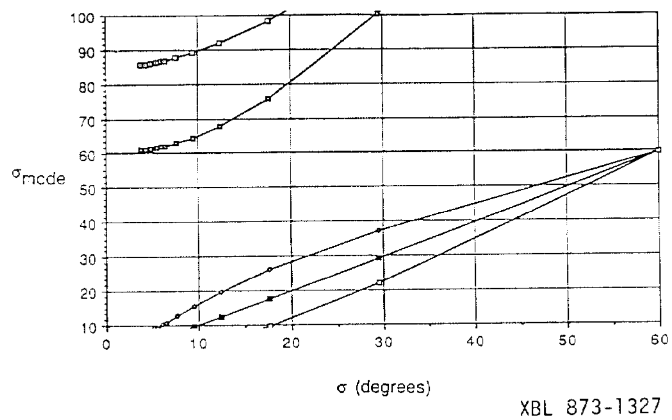

Fig. 8. Phase advance per lattice period of the third order modes of the K-V distribution vs. σ. $\sigma_0 = 60°$.

channel. Therefore the nonlinear force provides a driving force at a frequency near the resonant frequency of these coherent modes of the beam.

The symmetry of the phase plots suggests that in the case of the image force the $m = 3$ mode might be involved, and for the dodecapole, perhaps $m = 4$. In the beam frame the image force potential does contain a $r^3 \cos 3\theta$ multipole component, and the dodecapole contains all multipoles up to 6. However we should remember that though these are the predominant symmetries evident in the phase plots, it is likely that a mixture of modes would be involved, and it is unclear that the most obvious mode in the phase plots is the one that causes the most emittance growth.

L. Smith has studied analytically the mechanism just described, for the case of constant focusing, a Kapchinskij-Vladimirskij (K-V) distribution, and the image forces in a round pipe[6]. In this case, the largest multipole component of the image forces is the third order component. Putting this into the linearized Vlasov equation with a perturbation potential proportional to $r^3 \cos 3\theta$ he found the perturbed potential to be

$$V_1 \approx \frac{\omega_0^2}{2} \frac{a^3 h^3}{R^4} \sin\left(\frac{\Omega+\omega_c}{2} t\right) \sin\left(\frac{\Omega-\omega_c}{2} t\right) \qquad (1)$$

where ω_0 is the coherent betatron frequency with no image forces or external nonlinear forces present, $\Omega = 5\omega^2 + \omega_p^2/4 + \sqrt{\omega_p^4/16 + \omega^2\omega_p^2 + 16\omega^4}$ is the frequency of a K-V third order mode being driven , with ω the betatron frequency and ω_p the plasma frequency, and R is the pipe radius. It is difficult to compare the scaling with ω_0, a, h, and R given in Eq. (1) with simulation results, since for the round pipe the emittance growth is so small. However, one can clearly see from this calculation the beat phenomenon which appears in the simulation results. The frequencies agree fairly well with the simulation calculations. This lends plausibility to the hypothesis that the emittance growth derives from the driving of coherent beam modes near their resonant frequencies by external nonlinearities or image forces, whose strength is modulated by the coherent betatron oscillation. The emittance growth itself cannot be obtained from the linearized Vlasov equation, since to this order it is zero.

Cancellation of Image Effects by the Dodecapole

When both the image forces of the cylindrical quadrupole geometry and a dodecapole force of the proper

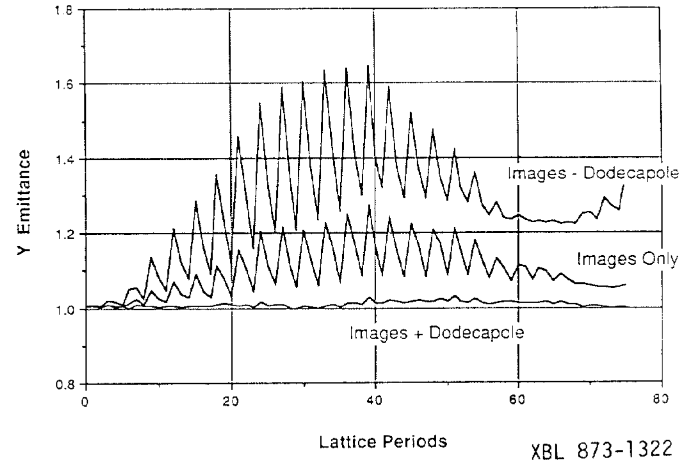

Fig. 9. y emittance vs. z for the cylindrical quadrupole geometry and parameters of Fig. 2, with and without an additional dodecapole force, showing disappearance of emittance growth. Also shown is a case with sign of A_6 reversed. $A_6/A_2 = 1.02\times10^{-2}$.

strength are present, simulation shows that the emittance growth can be reduced to zero. This is shown in Fig. 9 for the same case as Figs. 4 and 7. Though only the first 100 lattice periods are shown in Fig. 9, no emittance growth is seen over the length of the simulation run, 300 periods. This "cancellation" seems to be insensitive to the values of σ and σ_0, and is a gentle function of A_6. Since the modes driven by these two forces seem to be different, one might speculate that this "cancellation" is due to some kind of interference of one mode with the coherence of the other so that the emittance could not grow, rather than a true cancellation of the effect. However, if the sign of the dodecapole term is changed, the emittance growth is seen to double, rather than disappearing. This is shown by the third curve in Fig. 9, marked "images - dodecapole". This seems to indicate some real cancellation.

We have as yet been unable to find the mechanism by which this cancellation occurs. One reasonable hypothesis might be that both forces are driving several modes, with one primarily responsible for the emittance growth. Then if the coefficients of the multipole component of the two forces responsible for the emittance growth were made to cancel, emittance growth would not occur. The most obvious choice might be the third order mode, since it has the symmetry seen in the phase plots for the image forces. Since the image forces are present at all z in the simulation, and the dodecapole only at the lenses, they can not cancel exactly in detail. However, since we are interested in intense beams (low σ) the betatron phase advance of the particles per lattice period is low, and it is reasonable to require the cancellation of the coefficients integrated over one lattice period. We therefore find the third order multipole coefficient of both the image and dodecapole forces as seen in the beam frame (i.e., perform the multipole expansion about the beam centroid), average them over one lattice period, and set the coefficients for the two forces equal and opposite. This has been done analytically, using the image force due to four hyperbolic electrodes rather than the cylindrical ones used in the simulation. The result gives a value of A_6 for cancellation which is a factor of 10 higher than the value which the simulation shows to cause cancellation. Moreover, using in the simulation the value of A_6 calculated analytically causes extreme emittance growth. If we attempt to follow the same procedure, but assume that the mode of interest is the fifth order mode, then we again find disagreement with the simulation, the value of A_6 calculated analytically being a factor of 4 less than the simulation value. So at this time we can only say that the mechanism of cancellation is not understood. It may be that it is not necessary to completely cancel the driving force in order to stop emittance growth, but only to bring it below some threshold for growth--we know, for instance, that nonlinear stabilization of the modes occurs when they are started with a small perturbation and no driving force is present. This could account for the fact that the analysis finds a larger third order dodecapole component necessary for cancellation than the simulation shows is needed. Another possibility is that the mechanism is nonlinear and does not involve straightforward cancellation of the driving forces.

We have been unable to produce similar cancellation to that of the cylindrical quadrupole case in the simulation results for the round pipe image force by any simple scaling of input parameters or analytical calculations. However no systematic variation of the parameters to find a minimum of the emittance growth has yet been done.

Summary and Conclusions

We have shown using particle-in-cell simulation that in an alternating gradient focusing system in the presence of (1) image forces due to either of two different conducting boundary geometries, or (2) dodecapole or octupole imperfections of the focusing fields, transverse emittance growth occurs for misaligned intense ($\sigma/\sigma_0 \sim 0.1$) beams. This appears to be a general phenomenon caused by the driving of stable coherent modes of the beam by the nonlinearity, and we expect that it will occur for other nonlinear forces besides those mentioned above. The frequency of modulation of the nonlinearity strength, the coherent betatron frequency, is nearly coincident with the frequency of some coherent modes for intense beams, enabling the nonlinear force to drive these modes near their resonant frequencies.

For the case of the image forces of cylindrical quadrupole electrodes simulation has shown that emittance growth can be prevented by applying a dodecapole force of the correct strength. Analytical theory shows that this is not due to a straightforward cancellation of the driving force of the most likely modes, and the mechanism for the effect is not yet understood.

Analysis of data from the Single Beam Transport Experiment indicates a growth of rms emittance for misaligned beams, but further experiments are needed to assess the validity of both the growth of emittance and the image-dodecapole cancellation seen in simulation results.

The practical consequences of the effects we have discussed depend on the amount of emittance growth that is tolerable in a given experiment, the length of the accelerator, and the diameter of the beam. The beam diameter is important since it gives the scale length for acceptable misalignments, and technological limits on this length of course exist. In general, for a given allowable emittance growth the phenomena described will specify limits on the amount of misalignment which is permissible, given the strength of the nonlinear forces acting and the value of σ/σ_0. From another point of view, given the technologically achievable alignment of a beam and the lower limit on spurious focusing field nonlinearities, the effects described give a lower limit on the value of σ/σ_0 which can be transported without exceeding limits on emittance growth for a given length accelerator. For Heavy Ion Fusion this does not place unacceptably low limits on the alignment of the beam or the charge per unit length that can be transported. Results like those shown in Fig. 4 indicate that emittance growth due to the image forces of a round pipe is small enough to be tolerable in the ~ 400 periods needed for a reactor, assuming reasonable misalignments of $h \leq a/4$.

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