APERTURE DETERMINATION AND OPERATION SIMULATION IN LARGE STORAGE RINGS

Lindsay Schachinger
SSC Central Design Group*, Lawrence Berkeley Laboratory
Berkeley, CA 94720

Dynamic aperture is a well known and much used tool for collider design. However, for the SSC with its strong magnetic nonlinearities it is necessary to examine the nonlinear behavior of the beam more closely to minimize cost while assuring essentially linear behavior during routine operations. Thus the concept of linear aperture has been developed, and extensively applied in the design of the SSC. In order to explore this concept in a realistic situation, accelerator experiments have been proposed at the Tevatron which will test the parameters of the linear aperture definition chosen.

Introduction

In designing a superconducting collider, one of the major tradeoffs to be considered is magnet coil diameter *vs.* beam stability aperture. The cost of the machine decreases as the coil diameter of the superconducting magnets decreases [1]

$$\frac{\text{dipole cost}}{\text{meter of bend}} \sim 1 + 2\frac{d}{d_0} \tag{1}$$

and

$$\frac{\text{quad cost}}{\text{unit quad}} \sim 10 + 3\frac{d}{d_0} + 5\left(\frac{d}{d_0}\right)^2 \tag{2}$$

where d is the coil diameter. But the higher order multipole moments increase like

$$a_n, b_n \propto \frac{1}{d^{n+\frac{1}{2}}} \tag{3}$$

where a_n, b_n are defined by

$$(B_y + iB_x) = B_0 \sum_{n=0}^{M} (b_n + ia_n)(x + iy)^n. \tag{4}$$

The maximum stable amplitude (and thus the maximum usable aperture of the machine) is limited by the relatively high order multipoles ($\approx b_8$ for the SSC), which increase rapidly as d is decreased.

For the Tevatron design we have the following parameters: [2]

coil radius = 38mm
beam pipe radius = 35mm
beam size at 150 GeV = 3mm
dynamic aperture at 150 GeV = 37mm

With a coil radius almost a factor of two smaller, we get for the SSC: [3] [4]

coil radius = 20mm
beam pipe radius = 16mm
beam size at 1 TeV = 1.3mm
dynamic aperture at 1 TeV = 14mm

For the SSC, the dynamic aperture is an order of magnitude greater than the beam size, a somewhat smaller margin than for the Tevatron, but one which might be workable. Note, however, that the dynamic aperture is determined by tracking runs of 400 turns, and thus is an upper limit on the stable aperture. Beam size is not the right quantity to use for comparisons, but rather some quantity which is the maximum amplitude we expect particles to reach in normal beam operations. Unfortunately we also don't really know how things scale. What damage are the multipole errors doing at amplitudes we want to visit routinely in the course of operating the machine?

How much aperture do we need? For the SSC, there are several components [3] [5]

beam size ($\sqrt{6}\sigma$) = 1.3mm at 1 TeV, 0.3mm at 20 TeV
beam position fluctuation at injection = 1.5mm
orbit distortion = 1.25mm
miscellaneous = 0.5mm.

None of these numbers are strong functions of cell length or phase advance, within the ranges considered here. So at injection, we need approximately 5mm, and at collision only 2.5mm. The corresponding number for the Tevatron at injection is approximately 7mm. Since much of this aperture will be sampled during correction of the machine (orbit correction, injection damping, etc.) we must be able to understand the motion of the beam inside this aperture in order to make the appropriate corrections. If this motion is approximately linear, then simple linear theory will be a good predictor of the beam motion.

Linear Aperture

We now introduce the Courant-Snyder invariant amplitude [6]

$$W = \sqrt{\frac{x^2 + (\alpha x + \beta x')^2}{\beta}}. \tag{5}$$

In a linear machine, this quantity is an invariant. However, for a machine with nonlinearities, W will deviate from a constant and the extent of that deviation will be a measure of the nonlinearity of the motion. In practice, we track particles with equal x and y emittances for 400 turns, calculate the Courant-Synder "invariant" at each turn, and define the smear as

$$\text{smear} = \frac{\sqrt{2}}{3} \times \max\left(\frac{\hat{W}_x - \check{W}_x}{\sqrt{\bar{W}_x^2 + \bar{W}_y^2}}, \frac{\hat{W}_y - \check{W}_y}{\sqrt{\bar{W}_x^2 + \bar{W}_y^2}}\right), \tag{6}$$

where $\hat{W}_x$ and $\check{W}_x$ are the maximum and minimum horizontal amplitudes, and similarly for the vertical amplitudes. The denominator is the average amplitude. We then define the linear aperture as the amplitude (measured at $\hat{\beta}$ in the arcs) for which the smear is less then 10%, and the tune shift is less than 0.005. [7] The same limits define the linear aperture off-momentum, at $\delta = \pm 0.001$.

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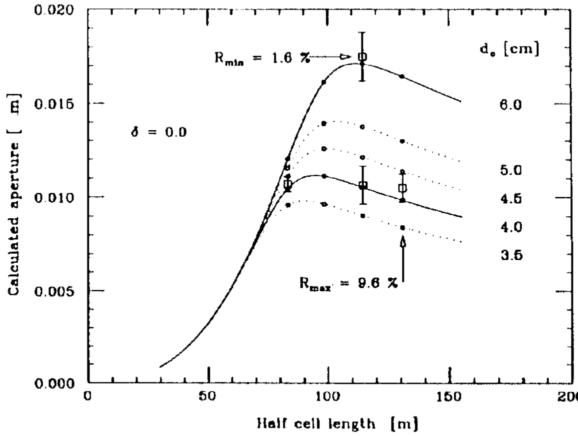

Figure 1. Linear Aperture for $\mu = 90°$.

In the fall of 1986 a study of the linear aperture as a function of cell length, phase advance per cell, and coil diameter was performed by a group at the Central Design Group of the SSC, with the objective of finding the design with the largest linear aperture for the smallest cost.[8] The cell length was varied, keeping the number of dipoles per half cell an integer between four and seven. Two phase advances, 60° and 90° were considered, and the coil diameter was varied between 3.5cm and 5.0cm in steps of 0.5cm. Using a lattice of cells only, an analytic calculation predicted the linear aperture for all points on this grid,[9] and several points were checked by tracking.[10] The analytic calculation included random sextupole and octupole errors in the dipoles and first and second order effects due to chromatic sextupoles. Since the value of b_2 was reduced by binning, cross terms in b_2 could be ignored. Very good agreement was obtained between this analytic calculation and the tracking which was done as a check. Figure 1[11] shows the results of this study for the 90°, on-momentum case. In the figure, the points with error bars are the tracking results, while the other points are the results of the analytic calculation. The aperture at small cell lengths is dominated by the chromatic sextupoles, and at at large cell lengths by the random errors. The conclusion, based on comparing this data with the cost figures, was that 90° and six dipoles with 4cm coil diameter per half cell was the most cost-effective option that could be obtained without changing the coil diameter, which would require magnet redesign.

For this lattice, then, we have measured the linear aperture at both injection and collision. The layout of the injection lattice is shown in Figure 2. In the arcs each 90° cell contains twelve dipoles, and there are four interaction regions, two with $\beta^* = 0.5$m and two with $\beta^* = 10$m in the collision lattice, two utility sections for injection, abort and RF, and two regions reserved for future IR's. The multipole errors in the arc dipoles are the dominant source of nonlinearity in the injection lattice, while the multipole errors in the IR triplets, where amplitudes are large, play a major role in the collision lattice. The low beta IR optics is shown in Figure 3. Contrast the maximum β here of $\approx$ 8km to the 340m maximum β in the arcs.

The smear criterion is the most stringent of the linear aperture criteria for the random multipole errors. Figure 4 shows the smear *vs.* amplitude for the injection lattice averaged over ten random number distributions of multipole errors in the dipoles. For these data the b_2 error is reduced by 80% by using a "binning" scheme with seven bins.[12] The average linear aperture for these

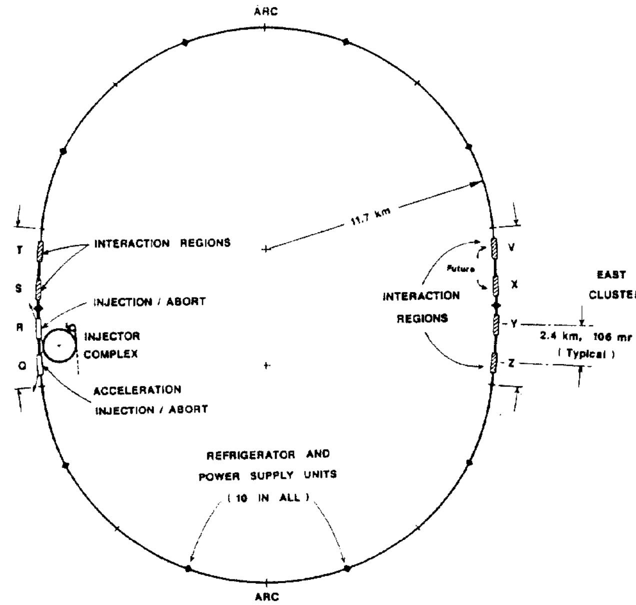

Figure 2. SSC layout

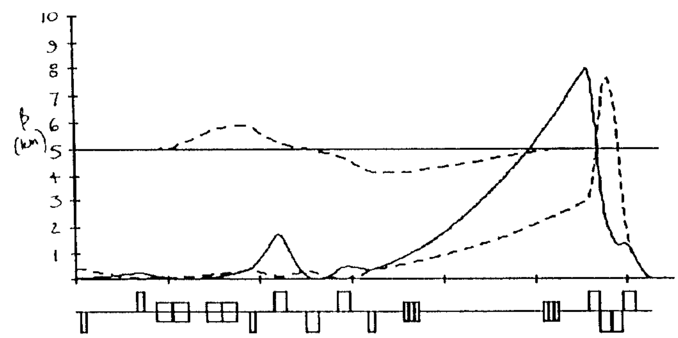

Figure 3. SSC low β optics

ten machines is 10.05 ± 0.98mm, and the dynamic aperture is 13.97 ± 0.61mm.[4] Thus the linear aperture is 5σ greater then the needed aperture, and the smear at the needed aperture is 1.5 ± 1.0%. This, then, is the largest smear we expect to see during routine operation of the SSC.

The situation at collision is shown in Figure 5, for one random seed.[13] In this case we have corrected the multipole errors in the IR triplet quadrupoles so that a_n, b_n are $0.05 \times 10^{-4}\text{m}^{-n}$ for $n = 2, 5$. Here, due to the rather large amplitudes in the IR triplets, the dynamic aperture has moved in, and is quite close to the linear aperture. When the dynamic aperture is obtained by averaging the results for five random error distributions, the result is 3.81 ± 0.16mm. The smear at the needed aperture is still quite small, $\approx$ 5%.

For the older 60° lattice in the CDR[3], we have checked that the linear aperture does not degrade with the inclusion of closed orbit errors. Rms orbit distortions of 0.43mm are produced when arc quadrupole misalignments of 0.5mm and arc dipole field errors are included. Particles have also been tracked at the linear aperture for 100 synchrotron periods, with no increase in the smear. We are currently studying the effects of power supply ripple.

When we turn to the systematic errors due to persistent currents, we find that the tune shift criterion is most strin-

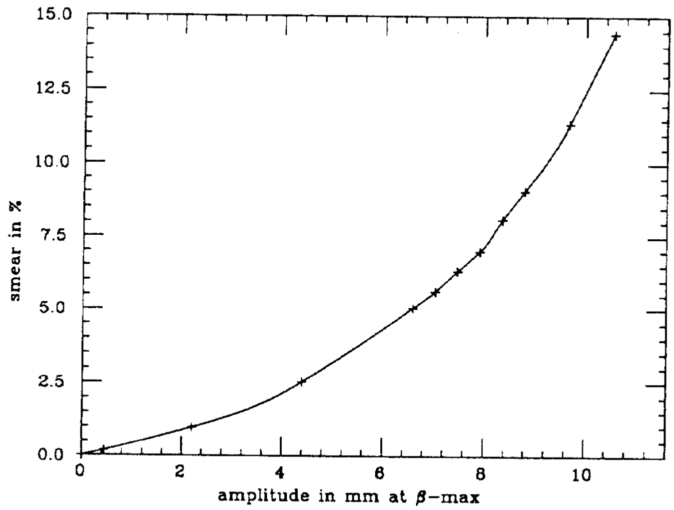

Figure 4. Smear *vs.* amplitude for SSC injection optics

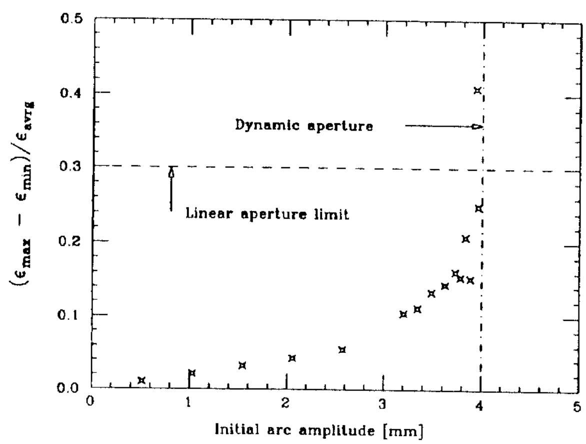

Figure 5. Smear *vs.* amplitude for SSC collision optics

gent. Persistent current multipoles are strongest at injection where, for 5μm filaments, the most troublesome moments are $b_2 \approx 5\text{m}^{-2}$ and $b_4 \approx 0.4 \times 10^4\text{m}^{-4}$. Most affected here is the off-momentum linear aperture which is currently required to be 5mm at $\delta = \pm 0.001$. Correction of these multipoles to within the limits given by the linear aperture criteria would require local correctors in the form of bore-tube windings on each dipole for b_2 and b_4.[14]

At this time, however, the values of the parameters in the linear aperture criteria are under review. The value chosen for δ is rather large, considering the fact that in the real machine the momentum spread ($\sqrt{6}\sigma$) at injection will be 3.7×10^{-4}, and injection energy errors will be less than 1.5×10^{-4}. The limit on the tune shift, ± 0.005, was chosen early on, and with new tools and understanding it will be reexamined.

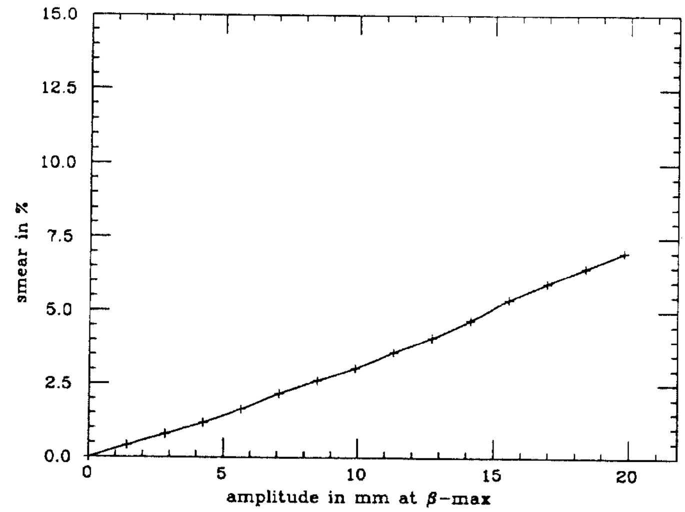

Figure 6. Smear *vs.* amplitude for Tevatron injection optics

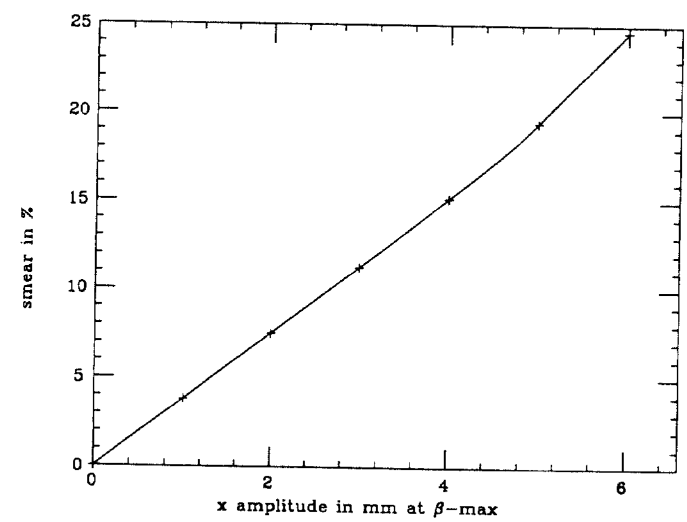

Figure 7. Tevatron smear *vs.* amplitude with strong sextupoles

Tevatron Aperture Experiments

The 10% value for smear seems reasonable, but is somewhat arbitrary. In order to further explore the concept of linear aperture, we have proposed a series of aperture experiments at the Tevatron, which are scheduled for a first run in late April. Simulations have been done using the Tevatron fixed target lattice at 150 GeV, with the measured multipole moments in the superconducting dipoles. A plot of smear *vs.* amplitude is shown in Figure 6. It is interesting to note that for the Tevatron the smear at the needed aperture of $\approx$ 7mm is $\approx$ 2%, comparable with that of the SSC at injection. Clearly, due to physical aperture limitations we could not reach a smear of 10% without some additional nonlinearities. Fortunately, there are a series of sixteen rather strong sextupoles ($Lb_2 = 0.175\text{m}^{-1}$) distributed around the Tevatron tunnel, which can be powered to produce smear *vs.* amplitude as shown in Figure 7. With a horizontal kicker, we can achieve amplitudes of 4 to 5mm and produce a smear greater than 15%. Our intention, then, is to compare the results of the simulation with real measurements, and to investigate the behavior of a beam with a smear of $\approx$ 10%.

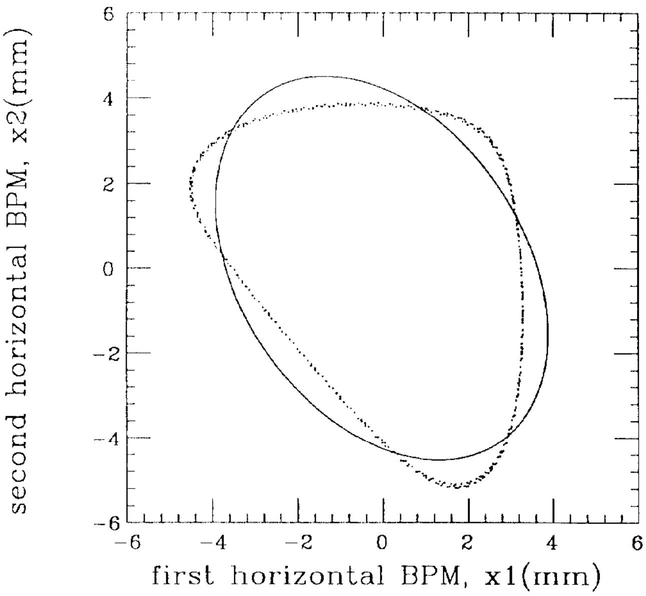

Figure 8. x_1 *vs.* x_2

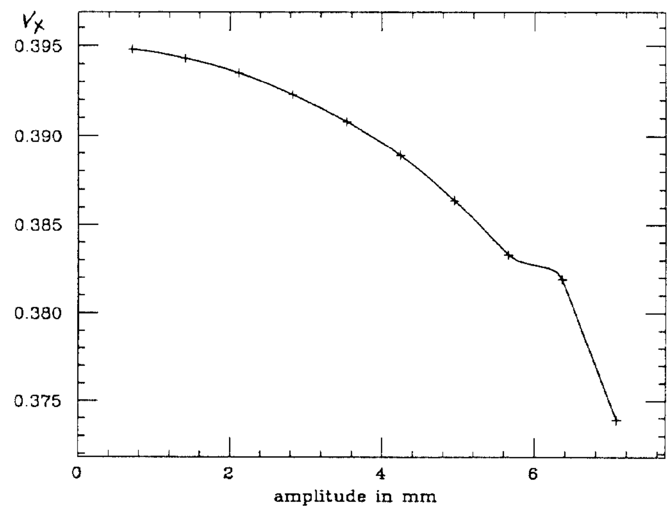

Figure 9. Tevatron tune shift with amp. with special sextupoles

To avoid the reliance on two points in equation 6, for the experimental situation it is necessary to define a new quantity, the rms value of the turn-by-turn invariant measurements, which is related to the smear. This quantity, properly normalized, we call shmere. The shmere is measured using two beam position monitors, which record coordinates, call them x_1^{in} and x_2^{in}. We then plot x_1^{in} *vs.* x_2^{in} as in Figure 8 and fit to an ellipse,

$$a^2 = C_{11}x_1^2 + C_{12}x_1x_2 + C_{22}x_2^2 \qquad (7)$$

where now $x_1 = x_1^{in} - x_1^{off}$ and $x_2 = x_2^{in} - x_2^{off}$, and x^{off} is the closed orbit offset. The fit adjusts C_{11}, C_{12}, C_{22}, x_1^{off}, and x_2^{off} to minimize the shmere. The error in the shmere measurement is a decreasing function of amplitude whose maximum is $\sim 2\%$ for an amplitude of 1mm for beam position monitor resolution of 80μm.

When the these strong sextupoles are turned on, they produce a fairly large amplitude dependent tune shift across the beam, shown in Figure 9, causing the beam to decohere in ~ 200 turns. This makes measurement of the smear more difficult, since the smear is a function of time. So the smear will have to be measured on-the-fly, averaging over ~ 20 turns. In this case the fit above requires another parameter which describes the apparent gaussian decay in amplitude with turn number. This technique has been tested in the Tevatron using a machine with coupling strong enough to cause decoherence within ≈ 100 turns, and appears to work.

Operation Simulation

To further explore the effects of nonlinearities on the operation of the SSC, we plan to simulate the application of various correction algorithms to a model of the machine which is as realistic as possible. This simulation is being implemented on a SUN Microsystems 3/160, an interactive graphics workstation which has approximately half the computational power of a VAX 8600. Many of the tools for the simulation, which uses TEAPOT[15] to model the underlying physics of the accelerator, now exist.

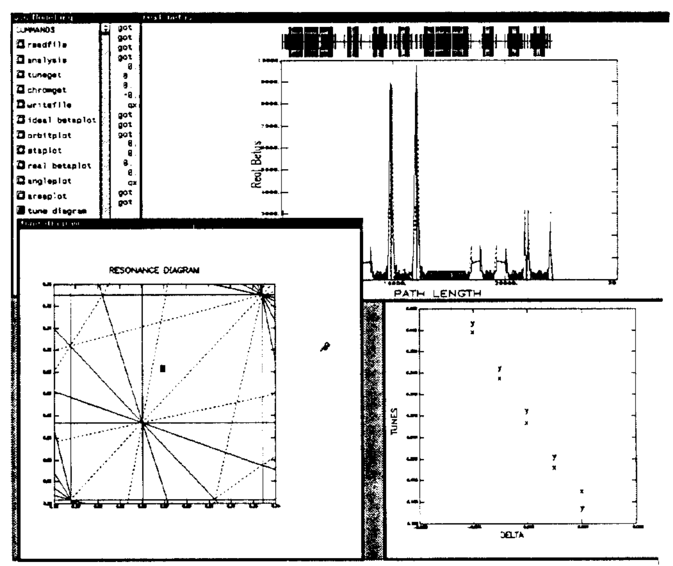

Figure 10

Figure 10 shows what a typical session might look like. Various physics parameters of the machine are available in graphical form, and one can for instance set the tune of the machine by choosing a point on the working diagram.

The issues of coupling and persistent currents will be the first two addressed. We plan to also look at first turn orbit correction, injection in general, setting of the secondary correctors in each arc sector (quad, skew quad, sextupole, and skew sextupole), ramping, and the transition to collision optics. The simulation will be a test-bed for the various corrections, and will use only measurable quantities augumented initially with displays of other variables to aid in the understanding of the corrections. Eventually we will have a model of the machine to which we can add broken devices, deviced which are wired backward, *etc.*, as well as the full complement of machine multipole errors, misalignment errors, and detector resolutions. Under these conditions we can further explore the concepts of needed and linear apertures.

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