# THEORETICAL AND COMPUTATIONAL ANALYSIS OF IFR BEAM TRANSPORT ON CURVED CHANNELS

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## Introduction

With the successful demonstration at Sandia National Laboratory and at Lawrence Livermore Laboratory of ion focused regime (IFR) electron beam guiding in accelerators there has developed interest in using IFR channels to steer beams along curved paths. We here report the calculations of the expected emittance growth from such channels and the results of the TRACKER code simulations of IFR steering with the assistance of strong focus sector magnets.

## Curved Channel Emittance Growth

For a relativistic beam propagating along a charged channel with a charge per unit length equal to f times the beam charge per unit length, the perpendicular equation of motion is:

$$\ddot{X}_\perp + \omega_o^2 X_\perp = 0$$

where

$$\omega_o^2 \equiv 1/2\omega_p^2 f$$

providing the channel radius is always larger than the beam radius and we have assumed that $\gamma$ is constant.

The first integral of this equation is:

$$\dot{X}_\perp^2 + \omega_o^2 X_\perp^2 = H_o \quad .$$

The particles thus move on constant $H_o$ contours (ellipses) in $\dot{X}_\perp$, $X_\perp$ space. The maximum radial excursion is given by:

$$X_{\perp max} = \frac{\sqrt{H_o}}{\omega_o}$$

while the maximum $\dot{X}_\perp$ is given by:

$$\dot{X}_{\perp max} = \sqrt{H_o} \quad .$$

The area enclosed by these orbits in $X_\perp$, $p_\perp$ space is thus:

$$\epsilon_o = \pi \frac{\gamma H_o}{\omega_o}$$

and is the emittance.

Now if the channel is curved with radius R, the equation of motion in the plane of the turn is:

$$\ddot{X}_\perp + \omega_o^2 X_\perp - \frac{C^2}{R + X_\perp} = 0 \quad .$$

For $\frac{X_\perp}{R} << 1$ this becomes:

$$\ddot{X}_\perp + (\omega_o^2 X_\perp + \omega^2)\, X_\perp - \omega^2 R = 0$$

where $\omega^2 \equiv C^2/R^2$. Defining $\rho$ by:

$$\rho = X_\perp - R \frac{\omega^2}{\omega^2 + \omega_o^2}$$

we obtain as a first integral:

$$\dot{\rho}_\perp^2 + (\omega^2 + \omega_o^2)\, \rho^2 = H_1 \quad .$$

Now in $\dot{X}_\perp$, $X_\perp$ space, these orbits are again ellipses but are displaced along the $X_\perp$ axis by an amount $\delta = R \dfrac{\omega^2}{\omega_o^2 + \omega^2}$ .

In transitioning from the straight to the curved channel, a particle which was on a constant $H_o$ contour now finds itself on a new constant $H_1$ contour. The maximum $H_1$ contour is given by:

$$H_{1max} = (\omega^2 + \omega_o^2) \left[ \frac{\sqrt{H_o}}{\omega_o} + \delta \right]^2$$

so that the new emittance is given by:

$$\epsilon_1 = \frac{\pi\gamma H_{1max}}{\sqrt{\omega_o^2 + \omega^2}} = \sqrt{1 + \omega^2/\omega_o^2} \left\{ \sqrt{\epsilon_o} + \sqrt{\bar{\epsilon}} \right\}^2$$

where $\bar{\epsilon} \equiv \omega_o \delta^2 \pi\gamma$.

This analysis makes sense only if $\delta$ is small compared to the channel radius and the channel radius is small compared to the radius of curvature R. This requires that $\omega^2/\omega_o^2 << 1$ and further that:

$$\frac{a}{R} << \frac{2I_{KA} f}{17\gamma} < 1 \quad ,$$

where a is the beam radius. In this limit we thus get:

$$\epsilon_1 = \left\{ \sqrt{\epsilon_o} + \sqrt{\bar{\epsilon}} \right\}^2 \quad .$$

By exactly the same analysis, in transitioning from the curved channel back to a straight channel the beam picks up another factor of $\sqrt{\bar{\epsilon}}$ in emittance growth so that we have:

$$\epsilon = \left\{ \sqrt{\epsilon_o} + 2\sqrt{\bar{\epsilon}} \right\}^2$$

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as the final emittance of the complete transition. The generalization to multiple turns is obvious.

With the above approximations, we can express $\bar{\epsilon}$ in terms of $\epsilon_o$:

$$\bar{\epsilon} = \epsilon_o \frac{a^2}{R^2} \left( \frac{17\gamma}{2I_{KAf}} \right)^2 < \epsilon_o$$

so that we obtain:

$$\epsilon \cong \epsilon_o \left( 1 + 4 \frac{a}{R} \frac{17\gamma}{2If} \right) .$$

This emittance growth is unacceptably large for very many recirculations, and thus beam steering using the channel forces alone does not seem promising.

If however we apply a transverse magnetic field to help the beam through the turn, the situation improves by another factor of a/R. To see this we merely note that for a transverse field that satifies

$$B_{kg} R_{cm} = 1.7\beta\gamma$$

the displacement $\delta$ in the preceeding analysis vanishes and we obtain for the emittance:

$$\epsilon = \epsilon_o (1 + .5\omega^2/\omega_o^2)$$

$$= \epsilon_o (1 + 17\gamma/(4If)*(a/R)^2) .$$

Strong Focus Sector Magnet Guiding

In previous schemes for turning an electron beam through a 180 degree angle turn, use was made of a constant dipole magnetic field. While this scheme works well at a fixed beam energy, it has little energy bandwidth since the radius of curvature of the beam is given by:

$$R_{cm} = 1.7*\beta\gamma/B_{kg}$$

where $\beta\gamma$ is the normalized electron momentum. Thus for the same entry position into the system, subsequent beams with higher energy will exit the $180^\circ$ turn at different spatial locations. From an experimental viewpoint, this is undesirable.

The use of a strong focus sector magnet can greatly increase the energy bandwidth such a turning system. A strong focus sector magnet is one in which the primary bfield component ($B_z$) is perpendicular to the plane of the turn and has a first order (in r/R) spatial variation given by:

$$B_z = B_0 (r/R)^3 , \qquad (1)$$

where r is the distance from the center of curvature of the sector and R is the scale radius of curvature of the sector. Such a magnet can be at least locally produced by shaped pole pieces and perhaps in other ways. This field is spatially focusing in the median plane for a $180^\circ$ turn in the sense that for a fixed entry point the exit point is independent of the beam energy. This property is illustrated in Figure 1 where the single particle orbits in the median plane for several energies are shown.

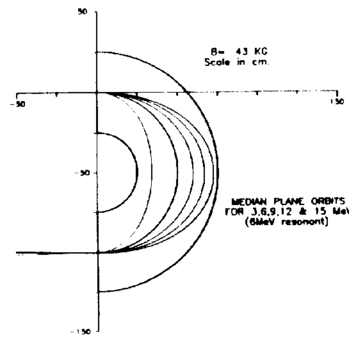


FIGURE 1. STRONG FOCUS SECTOR MAGNET

From the figure it is clear how this sector works. For a "resonant" particle (i.e., one whose energy satisfies $B_0 R = 1.7\beta\gamma$) the orbit is a semi-circular arc of radius R. Particles with higher energy move into the regions where $B > B_0$ and are pushed back to the resonant exit point. Particles with less energy experience an average field which is less than $B_0$ and similarly exit with the resonant particle.

Unfortunately, the magnetic field cannot be described by the strong focusing field given by 2 since the field is not curl free. Using the curl equation we find that to first order in z/R there must be a radial component of the field given by:

$$B_r = 3B_0 (r/R)^2 (z/R) .$$

This field component is unfortunately defocusing in the vertical plane although it is weak for z/R<1.

The introduction of a curved IFR channel of radius R into the sector can however provide the desired focusing in the median plane. As was shown in Section 1, the curved channel by itself leads to unacceptable emittance growth in the beam. By combining the strong focusing sector magent with a curved IFR channel we may be able to obtain large energy bandwidth turning with small emittance growth.

To investigate this possibility, the TRACKER code was configured to simulate an electron beam propagating in equilibrium along a straight IFR channel which is connected to a curved IFR channel which turns through $180^\circ$ and then connects back onto a straight channel. In the region of the curved channel a strong focusing sector magnetic field is applied which is curl and divergence free to eighth order in a/R.

In Figures 2, 3 and 4 are shown the particle trajectories for this system for energies of 2, 10 and 12 MeV, respectively. In all cases the beam current is 10 kiloamps, the initial beam and channel radii are 1 cm and the fractional neutralization is .5. The magnetic field was taken to be 430 gauss while the radius of the curved channel was chosen to be 50 cm. These values give a BR product which is resonant for 6 MeV particles.

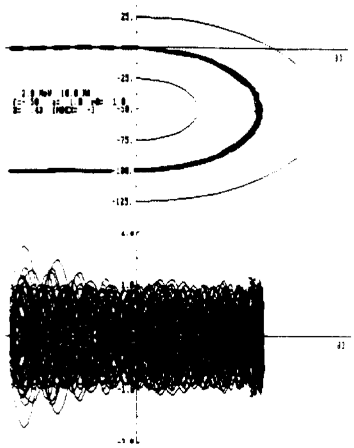

FIGURE 2. PARTICLE TRAJECTORIES

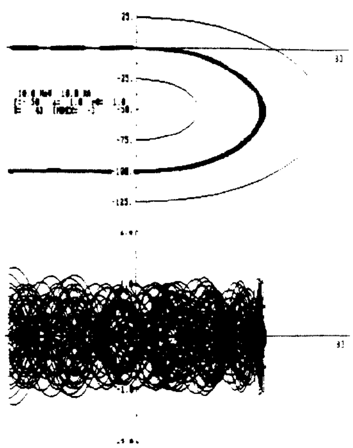

FIGURE 3. PARTICLE TRAJECTORIES

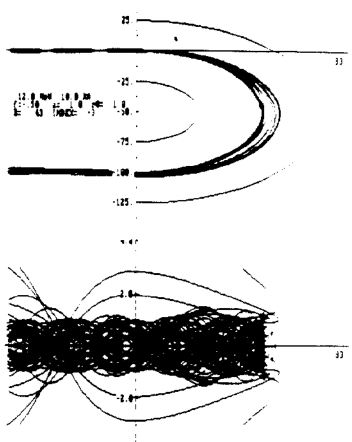

FIGURE 4. PARTICLE TRAJECTORIES

In each of the figures, the top plot is the orbits in the plane of the turn (median plane) while the bottom plot is the orbits in the plane perpendicular to the plane of the turn (vertical plane).

From these figures it is apparent that this configuration gives good results from 2 MeV up to 10 MeV. The 12 MeV run (Figure 4) shows that particles are starting to be lost and that beam expansion is becoming a problem.

Figure 5 gives the results (in the median plane only) of a 6 MeV run where the magnetic field was turned off. This run demonstrates that the channel alone is not sufficient to turn the beam.

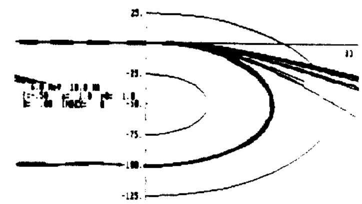

FIGURE 5. PARTICLE TRAJECTORIES

Figure 6 demonstrates the influence of the strong focusing sector magnet. In this run a uniform dipole magnetic field was applied and not the strong focus field. Compare this with Figure 2.

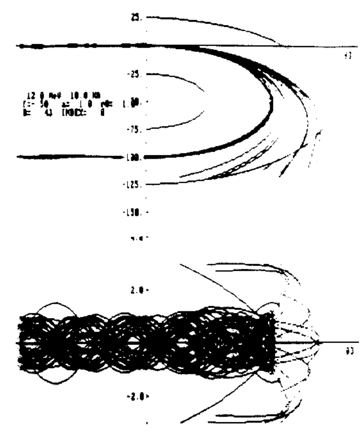

FIGURE 6. PARTICLE TRAJECTORIES

The results of these runs are encouraging for development of wide energy bandwidth beam steering systems. By scaling the magentic field and turning radius up we can obtain configurations useful for recirculation. However, the energy bandwidth is finite (perhaps a factor of 10) so that it might be necessary to switch from one configuration to a higher valued one as the energy increased. It might also be possible to sweep the magnetic field up in time but this looks dubious for the recirculation times of interest at the present time.