FFAG SYNCHROTRONS FOR HEAVY ION ACCELERATION

G. Wüstefeld*
Kernforschungsanlage, Jülich, West Germany
P. F. Meads, Jr.
Oakland, California 94611-1631

INTRODUCTION

We here describe the main parameters for a heavy ion FFAG synchrotron that can accelerate about 10^{10} ions to an energy in excess of 360 MeV/A. The pulse rate is 1 Hz, although this could be increased tenfold by simply increasing the RF accelerating voltage. For this study, we assume the injection of singly or doubly charged ions with a total kinetic energy of 10 MeV to 30 MeV for the lighter ions. The injected current for the lighter ions is assumed to be up to 15 mA, and the injection efficiency is assumed to be 50%. Multiturn injection of 8 turns into the transverse phase space yields phase areas of 450×10^{-6} m-rad in the radial plane and 340×10^{-6} m-rad in the vertical plane. A first pulse is injected, captured, accelerated, and stacked at an orbit where the square of its momentum is just twice that of the injected pulse. A second pulse is then injected, captured, accelerated, and combined with the first pulse, at which point the combined pulses are then accelerated to the final energy.

We also assume for simplicity that no further ions are injected until these combined pulses are delivered to the experimentor. In order to keep the magnets from getting too complicated (too large a range between the minimum and maximum field) and to retain an adequately long straight section (for a spiral machine), we restrict the synchrotron to an increase in momentum that is threefold that injected for each pass. Then prior to reinjection, we must increase the charge state by a factor of 3.0 by passing the beam through a thin stripper. This process is lossy as only a fraction of the beam will emerge with the desired charge state. However, a total of four cycles through the FFAG is practical for the heavier ions, thereby yielding an overall increase in momentum to 81 times the injected momentum, which is equivalent to an energy gain of about 6000 times for nonrelativistic ions. Many variations of this scheme exist with differing degrees of additional complication. For example one could either extract at an intermediate radius or inject at an intermediate radius to obtain a factor of two multiplication in momentum for one of the acceleration stages.

In each stage, we strip to a charge state below the equilibrium charge state so that we can maximize the yield of the desired charge state by optimizing the thickness of the stripper. This is easily done with a (pulsed) gas stripper, but could also be done with rotating solid strippers. We assume here that the efficiency for the desired charge state will be 15%, although for the lighter ions or the injection of singly charged molecules, the yield will be higher. The loss of intensity through multiple passages of strippers is a general problem in the acceleration of heavy ions and is not by any means unique to this method. Losses by interaction of the partially stripped ions with the background gas will be ignorably small, because the constant field FFAG would have vacuum tanks with thick metal walls that can be easily designed to maintain an excellent vacuum.

Changing to different ion species, where the initial magnetic rigidity is different, can be accommodated by either changing the location of the injection and extraction lines or by changing the average magnetic field as is routinely done in cyclotrons.

Subject to the complications mentioned above and to increased cost, the accelerator could be designed to achieve a larger gain in momentum, thereby providing the same overall energy gain with fewer acceleration stages. Another option would be the insertion of dispersion-free straight sections that would allow the nearly continuous accumulation of injected beam and also the pulse-to-pulse switching of ion species. Another complication that would yield increased current is the simultaneous acceleration of ions in different stages, using multiple RF systems.

Other Tandem Heavy Ion Approaches

There have been many proposals and several facilities built to accelerate heavy ions in several stages. Most of these use multiple accelerators although a few are based on the combination of a normal synchrotron and a storage ring or on two passes through one cyclotron. In the last decade, Oak Ridge had proposed a machine where the low energy beam was extracted and then reinjected, being stripped at the center so that it could be reaccelerated.[1] Bennet has proposed a method where the beam is not extracted between the two stages, but moved toward the center and reinjected on a centered orbit by stripping at two places.[2] Common to all of these approaches are the severe conditions to be satisfied in order to otain a second simultaneous acceleration.

Subsequent acceleration in a synchrotron, such as was proposed for the Omnitron, requires a separate storage ring of excellent vacuum due to the long time required to store partially stripped ions waiting for the synchrotron to return to its injection field. Moreover, being tied to the rate of change of the magnetic field, the acceleration time is long, and thus the synchrotron must also have an excellent vacuum, which is in conflict with the design requirements on the vacuum chamber to keep Eddy currents down to acceptable levels.

Space Charge Limit

The space charge limit for an FFAG is determined by the same considerations as that for a conventional synchrotron using the method of Laslett. The limit on the number of ions is determined by the electrical self fields that depress the betatron frequency to the nearest destructive resonance line. Let us limit this discussion to the nonrelativistic case where we may ignore image currents. Let us also assume the injection of a single turn so the radial and vertical emittances are the same (this is easily generalized) and assume the tunes are the same in the two planes, then we may write:

$$N = -2\,\frac{BA\,\Delta Q}{r_0}\,[\beta\gamma W\pi]\,\left[\frac{\beta\gamma}{q}\right]^2\frac{1}{\beta}$$

Here N is the number of ions that will depress the tunes by ΔQ, A is the mass number, B is the bunching factor, r_0 is the classical radius of the proton, $[\beta\gamma W\pi]$ is the normalized emittance, $[\beta\gamma/q]$ is proportional to the magnetic rigidity, which is constant at the injection radius, and beta is the relativistic parameter. For a constant bunching factor and no dilution, the space charge limit is inversely proportional to β and thus has an asymptotic minimum. Generalizing to different emittances and different tunes in the two planes will simply change the constant, but not the other relationships. Creating an intermediate stack, which is then subsequently accelerated, will change the space charge limit.

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Stripping

The mean equilibrium charge state attained with passage through a thick enough medium increases with ion velocity, remaining nearly proportional to the reduced velocity over the range of interest, particularly for gas strippers.[3] The equilibrium charge state is far greater than the charge state of ions extracted from the FFAG. If we pass these ions through a stripping medium they will gradually approach the equilibrium charge state with each successive charge state increasing in population up to some depth where its relative fraction peaks; deeper into the stripping medium, that charge state decreases in abundance as the next higher charge states grow in abundance. We show such a progression in stripping charge states in Figure 1 where +6 gold ions are injected at 5.4 MeV; the +18 charge state is reached quickly. We note that the thin stripper greatly reduces the scattering and energy spread as compared to the thick stripper necessary to reach equilibrium. This is a definite advantage where multiple stripping operations are used. Note also that the fraction of the beam with the desired charge state is greater than if we were to strip to equilibrium. We can thus choose a stripper thickness to maximize the population of the desired charge state.

Simple Formulae for the Geometric Size

It is possible to obtain quite good approximate formulae for the FFAG by starting with the smooth approximation for the radial and vertical tunes:

$$Q_x^2 = k + 1 + k^2G^2$$

$$Q_y^2 = -k + k^2G^2 + F^2(1 + \tan^2\xi)/2$$

where k is the field index, ξ is the spiral angle, F^2 is the flutter, and G, which can usually be ignored, is the normalized variance in the orbit deflection angle. We consider a lattice comprising N identical cells each of which contains a positive magnet (that deflects the orbit toward the center) of length P and field B_0 and a negative magnet of length M and field $-mB_0$; there is a drift length L between each pair of magnets: [L P L M]. With this hard edged magnet approximation, we can easily calculate the mean and mean squared magnetic field and thereby obtain the flutter. The average field is

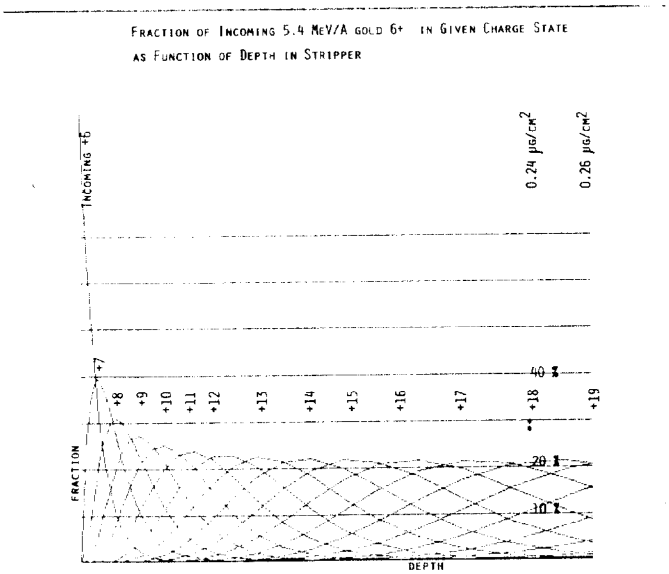

Figure 1. Charge states as Funtion of Depth in Stripper

$$\langle B\rangle = B_0(P - mM)/2\pi R,$$

where now P is the total azimuthal length of all plus magnets, and M is the total length of all negative magnets. Now the product of the mean magnetic field, $\langle B\rangle$, and the average radius, R, is just the magnetic rigidity of the particle, so we have

$$B\rho = B_0(P - mM)/2\pi.$$

In a similar way, we can integrate the square of the magnetic field to get the mean square:

$$\langle B^2\rangle = B_0^2(P + m^2M)/2\pi R$$

The expressions for both the mean field and the mean square field do not contain the number of cells or the location of the magnets within the cells. We have now a relationship between M and P that must be observed in order that we have a closed equilibrium orbit.

Let us indicate the ratio of the total negative bending to positive bending by α: $\alpha = mM/P$, then we obtain the flutter from

$$F^2/2 + 1 = \langle B^2\rangle/\langle B\rangle^2.$$

Using the above expressions for the mean and mean square fields, we find

$$\langle B^2\rangle/\langle B\rangle^2 = \frac{B_0R}{B\rho}\,\frac{1 + m\alpha}{1 - \alpha}$$

We solve this for R:

$$R = (\tfrac{1}{2}F^2 + 1)\,\frac{B\rho}{B_0}\,\frac{1 - \alpha}{1 + m\alpha}$$

We next eliminate the flutter from the expression for the average radius using the expressions for the tunes. We have

$$Q_x^2 + Q_y^2 - 2G^2k^2 = \tfrac{1}{2}F^2(1 + \tan^2\xi) + 1.$$

Thus

$$R = \frac{B\rho}{B_0}\,\frac{1-\alpha}{1+m\alpha}\left[\frac{Q_x^2+Q_y^2-2k^2G^2-1}{1+\tan^2\xi} + 1\right].$$

We next expand α and use the expression for the average magnetic field to obtain

$$P + m^2M = \frac{2\pi}{R}\left(\frac{B\rho}{B_0}\right)^2\left[\frac{Q_x^2+Q_y^2-2k^2G^2-1}{1 + \tan^2\xi} + 1\right].$$

Now the left hand side is closely related to the total magnet length in the accelerator (it is the same for the case m=1), and it shows how sensitive this is to the peak magnetic field, B_0. This form has now been extended to include the spiral magnet FFAG.

For completeness, we include the well known exact relationship between the minimum and maximum average radius and the minimum and maximum momentum in terms of the field index, k:

$$\frac{R_{max}-R_{min}}{R_{max}} = 1 - \left[\frac{P_{min}}{P_{max}}\right]^{\frac{1}{k+1}}$$

This of course gives us the other dimension of the magnets, the radial width, for a given momentum gain from P_{min} to P_{max}.

These expressions, which assume combined function magnets, yield the crucial parameters for the machine cost. These expressions also hold for a cyclotron. We first emphasize the importance of obtaining a high peak field to make a compact machine. The FFAG synchrotron can be generally much smaller than a comparable cyclotron because of the freedom in choosing the radial tune allowing us to use much larger values of k and thus greatly compress the radial width of the magnets.

Parameters for a Very Compact Heavy Ion FFAG

The above formulae were used to obtain the approximate parameters for several heavy ion FFAG lattices; these were then refined using an exact solution of the equations of motion for estimated azimuthal field profiles.[4] A summary of the parameters for one of these is given in Table I. The ring consists of 14 superconducting magnets. The radial focusing is sufficiently strong to keep the radial extension of the orbits to 2 meters. The vertical focusing is achieved by a modest spiral angle of 30° and by an enhanced flutter due to a negative field overshoot of 45%. The layout of the ring is shown in Fig. 2, and the magnetic field profile is shown in the insert in the center of that figure.

We describe in Table II the acceleration of gold in four stages, starting with the injection of 0.5 pmA at 13.71 MeV and assuming 15% is stripped to the correct charge state at each of the three stripping operations.

Estimate of Acceleration Voltage

Table I
Heavy Ion FFAG Ring Parameters

Magnetic rigidity (max)	11.32 T-m
Injection / extraction radii	15.76 m / 17.87 m
Field index k = (R/B)(dB/DR)	7.76
Spiral angle ξ	30.4°
N, number of sector magnets	14
Betatron tunes Q_x/Q_y	3.25/2.25
Maximum magnetic field	4.0 T
Horizontal/vertical acceptance	450/340 π mm mr

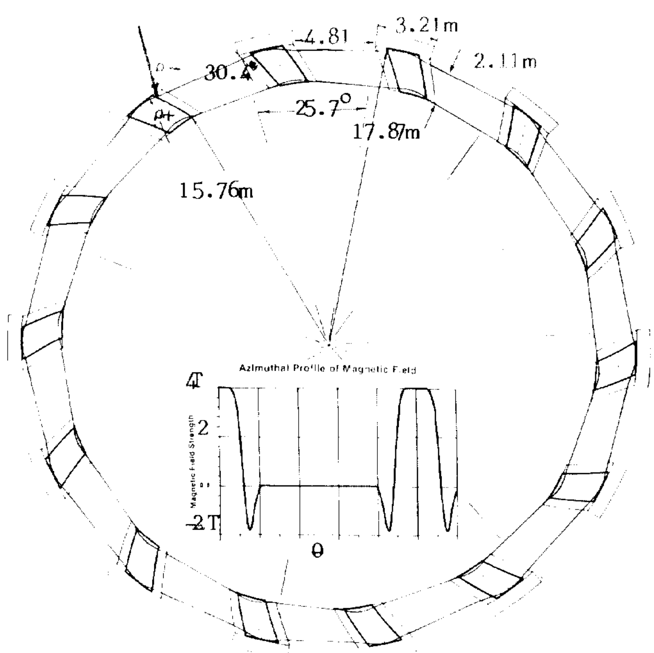

Figure 2. Heavy Ion FFAG Lattice
(The Azimuthal Field Profile is in the Insert.)

Table II
Acceleration of Gold

q	$\beta\gamma_{inj}$	$\beta\gamma_{ext}$	N 10^{10}	T_{ext} MeV/A
2	0.0120	0.0360	66	0.67
6	0.0360	0.1079	9.8	5.40
18	0.1079	0.3237	1.5	47.55
54	0.3237	0.971	0.22	366.60

The rate of energy gain is given by the relation:

$$\frac{dT}{ds} = \frac{1}{\beta c}\frac{dT}{dt} = \frac{eqV\sin\phi}{2\pi R}.$$

This can be integrated to obtain the time required to accelerate from the injection energy to the extraction energy. If V and ϕ are assumed to be constant, we can integrate analytically to obtain the acceleration time. Because of the narrow radial band, we can further simplify this by using an average radius, whereupon we find the integration time for one stage to be

$$\Delta t = \frac{2\pi R}{V}\left[B\rho_{extr} - B\rho_{inj}\right].$$

Letting ϕ = 0.5, we need 6.0 kV per turn to accomplish the total acceleration in four sequential stages in one second. If we stack during the first stage as described above, some additional voltage will be required in order to keep the same total time, but then we will double the number of ions accelerated. The efficiency can be further improved by more complicated stacking methods.

Dispersion-free Insertions

The changing of particle species would be done by scaling the overall magnetic field so the injection and extraction orbits remain unchanged.

It is possible to place insertions in a radial sector FFAG such that ions of all energies and all charge states, within the range of magnetic rigidity that can be contained in the FFAG, will pass along the same axis within the straight section.[5] With such an arrangement it is possible to change energies or ion species at will--on a pulse-to-pulse basis.

With this insertion, we would not have to provide any particular ratio of charge states between adjacent stages, and we could thus easily accelerate the different stages simultaneously.

The dispersion-free insertion also provides for the possibility of injecting such that the beam passes through the stripper several times thereby allowing many turns to be injected and an equilibrium distribution to be achieved, before one particular charge state is then accelerated. This would be necessary to obtain adequate beams of some rare charge states. It it may be possible to arrange affairs such that injection can continue while previously injected pulses are being accelerated, being suspended briefly during the capture operation and shortly thereafter.

The size of the rf cavities would be greatly reduced if they were placed in a dispersion-free drift space.

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