MEASUREMENT OF THE BETATRON PHASE ADVANCE AND BETATRON AMPLITUDE RATIO AT THE SP$\bar{P}$S COLLIDER

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Abstract

A technique for the precise measurement of lattice functions in a hadron collider has been developped. The betatron functions on either side of the two low beta insertions of the SPS collider have been determined from the measured amplitude and phase of horizontal beam oscillations with a peak amplitude of 40 µm. Four directional couplers and four synchronous receivers working at 200 MHz monitor the betatron oscillations of the beam excited by the fast deflectors of the damper. A fast Fourier transform of the signals provides the phase and amplitude ratio of the beam oscillations between any pair of monitors. The relative amplitude and phase of the beam oscillations can be measured with an accuracy of < 5% in amplitude and < 2° in phase. For achieving such an accuracy a special calibration method has been implemented to determine the propagation times and amplification factors of the measuring equipment, using the intensity signals of the beam itself. The same equipment can be used also for measuring the beam transfer function by injecting white noise into the beam deflectors.

1. Purpose of the betatron function measurement

For collider operation of the SPS, it is important to measure the betatron phase advance and amplitude ratio on either side of the two low beta insertions of the collider. The transition region between the low beta insertions and the regular lattice of the collider is very sensitive to gradient errors of the low beta quadrupoles. Both the betatron phase advance and amplitude ratio between two points of the lattice can be measured from the phase and amplitude of coherent betatron oscillations. Two pairs of monitors have been installed near to the electrostatic separators of the collider. In this paper, the analysis and measurement technique is presented for the betatron oscillations of the single bunches in the SP$\bar{P}$S - collider. A blockdiagram of the equipment is shown in fig.1.

2. Betatron oscillations and betatron function

The betatron oscillations of a single bunch of protons in the SP$\bar{P}$S - collider are generated by a short horizontal kick. The deflection given by the electrostatics plates of the damper, which are used for this purpose, is limited to δ_o = 0.4 µrad at 315 GeV/c, corresponding to an initial oscillation amplitude of 40 µm at β = 100m. The betatron oscillation is damped slowly with an e-folding time τ = 70 ms because of the Q-spread of the proton beam :

$$\tau = 1/(\pi f_o \partial Q), \text{ wherein}$$

f_o = 43.3 KHz, beam revolution frequency
$\partial Q = 10^{-4}$ at 315 GeV/c, Q-spread for protons in the absence of antiprotons.

The position y_n of a single bunch is sampled at every turn, when the bunch passes through the position monitor :

$$y_n = a + \delta_o \sqrt{\beta_o\beta_1} \sin(\mu+nQ2\pi) \exp(-n/f_o\tau) \qquad (1)$$

y_n = sample of beam position
n = turn number, n = 0, 1, 2 ...
a = closed orbit position at the monitor
δ_o = beam deflexion angle, δ_o = 0.4 µrad at 315 GeV/c
β_o = amplitude of betatron function at the deflector, β_o = 62 m
β_1 = amplitude of betatron function at the position monitor
μ = betatronic phase advance between beam deflector and position monitor
Q = tune of SP$\bar{P}$S - collider, Q = 27.70

The spectrum of the betatron oscillation is calculated from the samples y_n by a fast Fourier transform (FFT). Since the sampling frequency f_o is much lower than the betatron frequency Qf_o, the signal of the betatron oscillation is undersampled. The FFT spectrum analysis of the measurements provides only the aliased betatron frequency $|f_1| \le 0.5 f_o$, wherein

$$f_1 = (Q-28)f_o = -qf_o, \; q = 28-Q = 0.3 \qquad (2)$$

The frequency f_1 of the undersampled betatron oscillation is negative, and therefore the sign of the phase information of the spectrum is reversed with respect to the betatron oscillation Qf_o [1)].

Despite the frequency transposition, the complex FFT spectrum still contains the correct amplitude Y and negative phase information φ of the betatron oscillation :

$$Y = \sqrt{\langle y_n^2 \rangle} = \varepsilon\delta_o \sqrt{\beta_o\beta_1} \qquad (3)$$

$$\varphi = -\mu + \text{cst.} \qquad (4)$$

$$\varepsilon \approx \frac{f_o\tau}{4N}\left(1 - \exp\left(-\frac{2N}{f_o\tau}\right)\right) \qquad (5)$$

ε = e-folding factor, $\varepsilon \le 0.5$
N = total number of beam revolutions measured, typically N = 512

If the deflexion angle δ_o, the betatron amplitude β_o and the e-folding factor ε are known with sufficient accuracy, the absolute value of the betatron amplitude β_1 at the monitor can be determined according to eq (3). In practice however, the e-folding factor ε changes considerably with the Q-spread of the beam, and the deflexion angle δ_o and betatron amplitude β_o at the deflector are not known accurately enough. Better precision is obtained for relative measurements between two monitors at different locations of the lattice :

$$Y_1/Y_2 = \sqrt{\beta_1/\beta_2} \qquad (6)$$
$$\varphi_{12} = -\mu_{12} = \mu_1 - \mu_2 \qquad (7)$$

The signal convention for a positive phase measurement φ_{12} is defined so that signal 1 lags signal 2 in time.

Equations (6) and (7) suppose that the signal gain is equal between the two measurement channels, and that the signals of the two monitors are always sampled within the same beam revolution. This is strictly the case if the sampling frequency of the FFT spectrum analysis is equal to the beam revolution frequency. Most commercial FFT spectrum analysers have an internal sampling clock which runs at a fixed frequency higher than f_0. Due to the asynchronous sampling frequency, the phase φ_{12} between the two monitor signals is modified by the time difference t_1-t_2, at which the signals arrive at the spectrum analyser :

$$\varphi_{12} = -\mu_{12} + (t_1-t_2)\, 2\pi\, f_0\, q. \tag{8}$$

The phase displacement caused by the time difference t_1-t_2 can be measured from the intensity signal Σ of the beam monitors. In order to calibrate the phase displacement due to the propagation delay t_1-t_2 as closely as possible to the betatron frequency qf_0, the intensity signal is sampled every third turn, providing a spectral line at $f_0/3$. The phase difference φ_0 of the intensity signal measured every third turn amounts to :

$$\varphi_0 = (t_1-t_2)\, 2\pi\, f_0/3 \tag{9}$$

The betatron phase advance $\mu_2-\mu_1$ between two monitors can be calculated easily from :

$$\mu_{12} = -\varphi_{12} + 3q\,\varphi_0 \tag{10}$$

The amplitude ratio Y_1/Y_2 and the relative phases φ_{12} and φ_0 can be measured and displayed on line by commercial FFT spectrum analysers which provide the transfer function between channel 1 and 2. The accuracy of the transfer function is better for a relative than for an absolute measurement, because the interpolation errors of the FFT compensate each other in a relative measurement [2].

3. Measurement sensitivity and calibration

The betatron oscillations of the beam are measured by directional couplers and by homodyne receivers working at 200 MHz, as used for the closed orbit measurement of the SPS [3]. The sensitivity of the directional coupler BPCS is reduced due to the large aperture Ø 210 mm of the machine required on both sides of the low beta insertions. Nevertheless, it is possible to measure the spectrum of a single bunch of 10^{11} ppb oscillating horizontally with an amplitude of 30 µm. The signal-to-noise ratio of the spectrum analysis is about 20 dB [4].

An essential point for accurate amplitude and phase measurement is the calibrator module installed in the tunnel. In the calibration mode, the beam intensity signal Σ of the directional coupler is fed to both channels Σ and Δ of the homodyne receiver [3]. In this way, the calibration signal derived from the beam intensity is exactly the same for all monitors in the ring, and the calibration signals arriving in the main control room must all have the same amplitude. As a result of the precise mechanics of the directional coupler, the intensity signal amplitude of different monitors does not deviate by more than 0.5%.

In the calibration mode, time differences in the arrival of the calibration signals from different places in the collider are measured as phase differences φ_0 at the subharmonic frequency $f_0/3$, since the receiver signals are sampled at every third beam revolution. The sampling of the bunch signals in the auxiliary buildings is remotely controlled by the timing system, but the exact instant of the hold is determined by the autotrigger working on the peak amplitude of the bunch intensity signal Σ of the receiver [3]. So both the amplitude and phase response of the complete measurement equipment are calibrated with the best possible accuracy by the intensity signal of the beam itself.

In measuring the transfer functions between any pair among four monitors, there are three independent and three redundant measurements. The two groups of measurements with beam agree within a statistical error of ± 3% in amplitude and ± 1° in phase. This error is due to the noise and the limited resolution of the equipment, and is of the same order as the intrinsic amplitude error of the calibration facility.

4. Measurements

In order to check the calibration of the homodyne receivers and S/H-amplifiers, measurements (photo 1) have been carried out with a regular machine lattice during pulsed collider operation, when the low beta insertions were switched off. The measurements agreed with the theoretical values of the betatron function within 4% for the amplitude and 1° for the phase, see table 1.

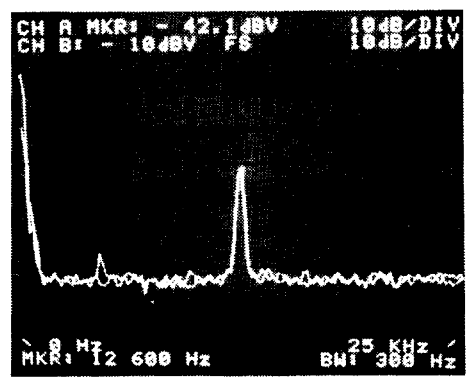

Photo 1 : Amplitude spectrum of hor. oscillations 75 µm at 148 GeV/c of a single bunch at monitors 4.14/4.16.

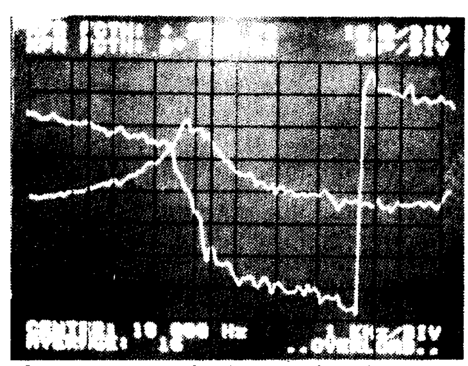
Photo 2 : Vertical transfer function 14-24 KHz of fixed target beam at 10 GeV/c.
10 dB/div, 50°/div, 1 KHz/div.

Subsequently, different measurements have been carried out during the setting-up of the collider with the low-beta insertions squeezed to 1 m x 0.5 m. The discrepancy between the measurements and the simulation program was considerably larger than the errors of the measurement, especially for the phase, which can be very precisely measured. This effect has been confirmed independently and explained in [5].

The transfer function of the beam can be measured easily by the same equipment, if the beam deflector is driven by the noise generator of the spectrum analyser, see photo 2.

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References

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Table 1 Relative measurement of betatronic function between two monitors 4.14/4.16 in the pulsed collider mode without low beta insertions.

		Inj. Cycle	Pulsed collider	
		148 GeV/c	100 GeV/c	450 GeV/c
Calibration at $f_0/3$				
Gain ratio	c_{12}	0.982	0.960	0.960
Phase delay	φ_0	1°	2°	2°
Oscillation measurement				
Amplitude ratio	a_1/a_2	0.944	0.971	0.990
Oscillation phase	φ_{12}	- 88°	- 87°	- 87°
Betatronic Function				
Amplitude ratio	$\sqrt{\beta_1/\beta_2}$	0.96	1.01	1.03
Betatronic phase	μ_{12}	89°	89°	89°
Theoretical values				
Amplitude ratio	$\sqrt{\beta_1/\beta_2}$	1.00	1.00	1.00
Betatronic phase	μ_{12}	89°	89°	89°
Measured tune	Q	26.71	26.69	26.69

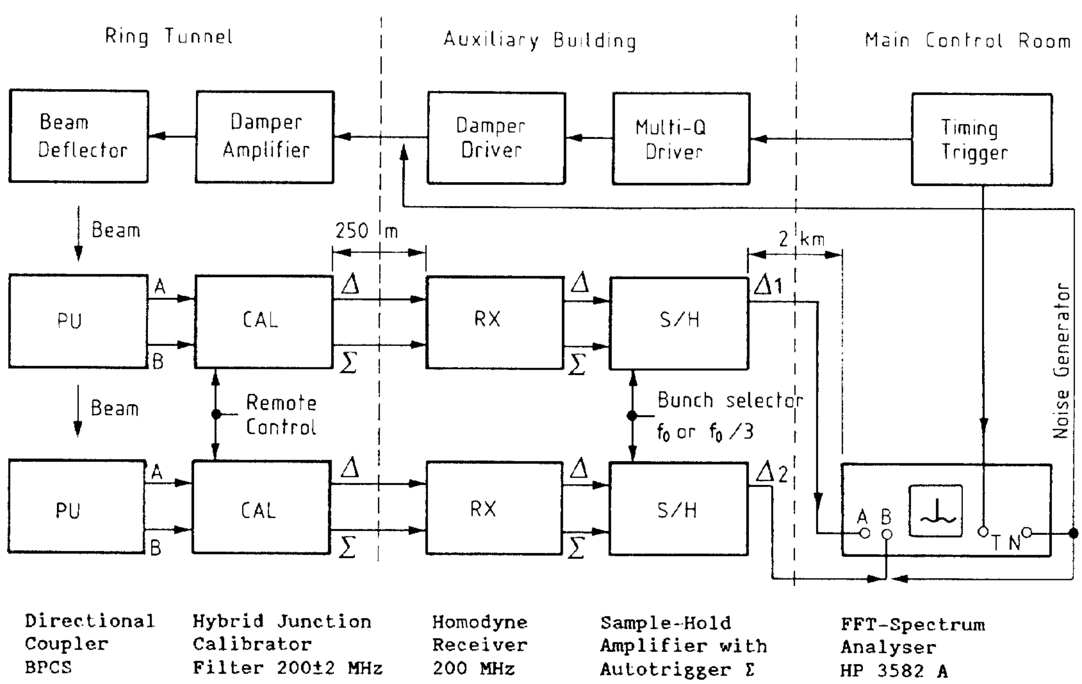

Fig. 1 - Block diagram for measuring the transfer function Δ1/Δ2