

THRESHOLD CERENKOV RADIATION AND BEAM DIAGNOSTICS

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ABSTRACT

Cerenkov and transition radiation occur for single particles or for beam bunches; for the latter, the intensity may be strong at long wave lengths as a result of the coherence of emission. Also, if the medium has finite length, the Cerenkov radiation is diffracted. Not only is the angle of emission broadened but the emission threshold is not sharp. At measurement angles in the vicinity of 90°, the Fourier transforms of the charge distribution within a bunch may be determined for both the Cerenkov and sub-Cerenkov cases.

INTRODUCTION

In previous work¹, the NPS group has calculated in detail the coherent microwave radiation pattern from an electron beam composed of periodic bunches and compared these predictions with experiment². We also generalize these results to a beam consisting of a single electron bunch. The important results are: a) at frequencies such that the wavelength of the emitted radiation is greater than the size of an individual bunch the electrons in an individual bunch radiate coherently; (b) the radiated intensity is modulated by the Fourier transform of the charge distribution of a single bunch; c) when the beam interaction length is of finite size the radiation is spread in angle rather than occurring only at the Cerenkov angle.

In this paper we discuss other aspects: the threshold for emission, and the determination of the bunch charge distribution from observation of sub threshold Cerenkov radiation.

COHERENT CERENKOV RADIATION

The energy radiated per unit solid angle per unit frequency by a single bunch of charge q traveling a distance L is

$$E(\nu, \vec{k}) d\nu = QR^2 d\nu \quad (1)$$

where Q is a constant defined by

$$Q = \mu c q^2 / 8\pi^2 \quad (2)$$

and μ is the permeability of the medium. The velocity of light in vacuum is c_0 and the velocity of light in the medium is $c=c_0/n$ where n is the index of refraction. The coherent power per unit solid angle, W , radiated at the frequency ν by a periodic charged particle beam of charge q per bunch is given by $W = \nu_0^2 QR^2$ where ν_0 is the fundamental frequency of the beam generator. Since W and E differ only by a constant, they have identical spatial distributions. The radiation function R is

$$R = 2\pi\eta \sin\theta I(u) F(\vec{k}) \quad (3)$$

and the parameters are

$$I(u) = \sin u / u \quad (4)$$

$$u = \pi\eta[(n\beta)^{-1} - \cos\theta] \quad (5)$$

where θ is the angle between the direction of travel of the charged particle beam and the direction of propagation of the emitted radiation, $\eta=L/\lambda$ is a dimensionless parameter, and $I(u)$ is the diffraction function. The wave vector of the emitted radiation is k ($k=\omega/c$), and $F(\vec{k})$ is the dimensionless form factor, i.e., the Fourier transform of a charge bunch is $qF(\vec{k})$. When the spatial distribution of a bunch is a line charge the form factor is a function of k_z only where $k_z = k \cos \theta$. The form factor is identically one for a point charge, and for a finite distribution $F(\vec{k})=1$ for $k=0$.

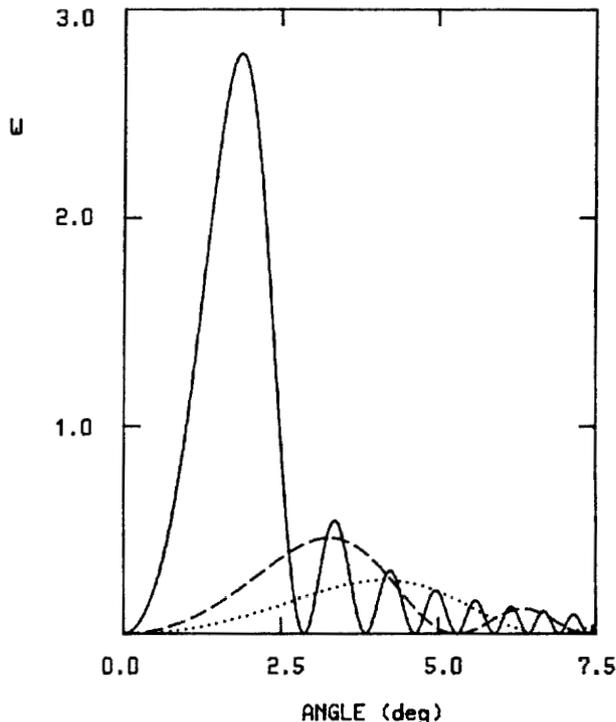


Fig. 1. Calculated radiation intensity as a function of angle for an electron beam issuing from an S-band, 100 MeV linac. The dotted, dashed and solid curves are for beam path lengths of 150, 250, and 1000 respectively.

It is difficult to deal analytically with the maxima of E even if $F(\vec{k})$ has a relatively simple form. However, the minima are evenly spaced in u since the diffraction function always has zeroes at $u=m\pi$. The corresponding θ values are given by

$$\cos\theta_m = \frac{1}{n\beta} - \frac{m}{\eta} \quad (6)$$

where m is an integer. As η increases the successive zeroes become more closely spaced in angle.

For $m=0$, $\theta_0=\theta_c$ and no minimum occurs since the limit of $I(u)$ is 1. For $m = \pm 1$, these limits restrict the value of the principal peak of W to lie between the θ values determined by these zeroes in $I(u)$; assuming that these values of u correspond to physical values of θ .

The behavior of the main radiation lobe, bounded by the angles θ_{-1} and θ_1 , depends on the constants $n\beta$ and η . As $\eta \rightarrow \infty$, the lobe narrows and both θ_{-1} and θ_1 approach θ_c , assuming, of course, that $n\beta > 1$ and θ_c is defined. In the limit of an infinite medium, the radiation all appears at the Cerenkov angle. In the other extreme, as η becomes smaller, diffraction spreads out the main lobe, and θ_1 increases to eventually become 180° .

EMISSION THRESHOLD

At some finite η the radiation pattern is spread into a diffraction lobe bounded by θ_{-1} and θ_{+1} . As the beam energy, and thus β , is reduced, θ_{-1} , θ_c , and θ_{+1} become smaller. The angles may become non-physical because the governing equations contain $\cos \theta$ which formally may exceed unity. Since the inequality $\theta_{-1} < \theta_c < \theta_{+1}$ is always satisfied, it is possible to have only θ_{-1} be non-physical or to have both θ_{-1} and θ_c non-physical. In either case, the resulting main lobe of radiation extends from zero degrees to θ_{+1} and this phenomenon may be termed sub-Cerenkov radiation because it occurs for $n\beta$ less than (but usually close to) unity.

We define the onset or threshold of the emission of Cerenkov radiation to be the situation when θ_{+1} begins to enter the physical range and setting $\theta_{+1}=0$ in (6) gives the threshold relation in terms of β . Using the usual relation between β and γ , this relation can be written in terms of γ_t , the value of γ necessary for the onset of sub-Cerenkov radiation.

$$\gamma_t(\eta) = \left[1 - \frac{1}{n^2(1+\eta^{-1})^2} \right]^{-1/2} \quad (7)$$

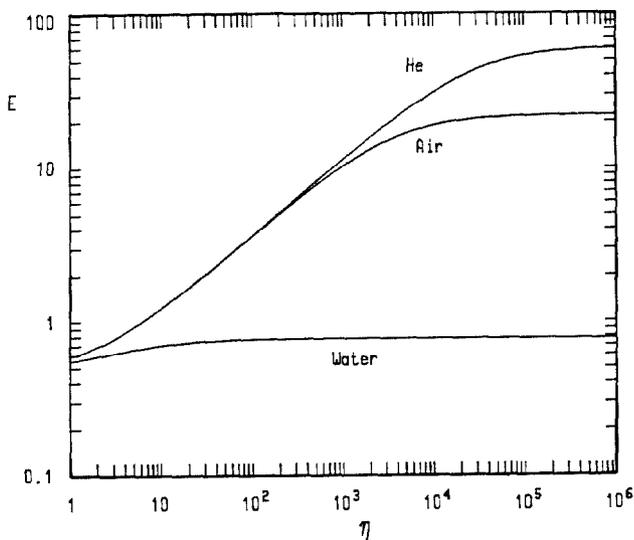


Fig. 2. Threshold electron bunch energies a function of η . The units of E are MeV.

The energy required for onset of emission is then given by $E_t = \gamma_t E_0$, where E_0 is the electron rest energy. Limiting values of (7) can be obtained for very long and very short path lengths.

ENVELOPE OF THE RADIATION

An understanding of the sub-Cerenkov radiation patterns and their development into the Cerenkov shape is more easily forthcoming with a different formulation. The radiation function can be written $R=F(k) \sin u \cdot G$ where

$$G(n\beta, \phi) = [(n\beta)^{-1} - \cos\theta]^{-1} \sin\theta \quad (8)$$

If variation of R with $F(k)$ is neglected, then aside from some constants, $(G(n\beta, \theta))^2$ is the envelope of the oscillating $\sin^2 u$ function which takes on values between zero and one. The form of the envelope depends upon the value of $n\beta$. Either there is a peak at $\cos\theta_s = n\beta$, or there is a singularity at $\cos\theta_c = 1/n\beta$; only one possibility is allowed. Regardless of the value of $n\beta$, the function has zeroes at $\theta=0$ and $\theta=\pi$.

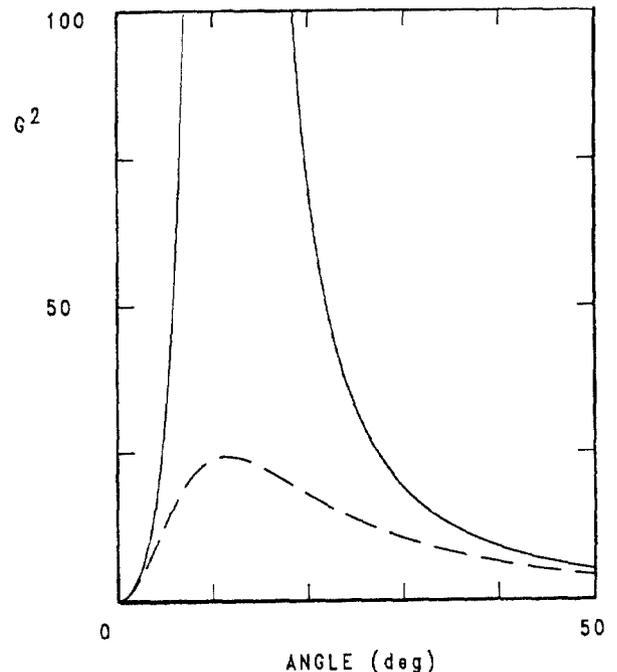


Fig. 3. Envelope function as a function of angle. The solid curve (Cerenkov) is for $n\beta=1.02$. The dashed curve is for $n\beta=0.98$.

DISCUSSION

Radiation is in the sub-Cerenkov regime when $n\beta < 1$. Then the envelope function has a peak at θ_s ($\cos\theta_s = n\beta$) of height equal to $(\cot^2\theta_s)$. In this case, the height of the peak does not depend upon the path length of the charged particle beam. As $n\beta$ increases, the peak angle θ_s decreases and the height of the peak grows. When $n\beta = 1$, the peak angle goes to zero, the envelope function has a pole (at $\theta = 0$), and the Cerenkov regime is attained.

In the Cerenkov regime ($n\beta > 1$) the envelope has a singularity at the Cerenkov angle ($\cos \theta_c = 1/n\beta$). However, since $\sin u$ is always zero at the Cerenkov angle, the radiation function remains finite with a height which depends explicitly upon the value of the beam length.

In the sub-Cerenkov case there are two types of radiation pattern; those without a single dominant peak, and those that have a dominant peak giving the appearance of diffracted Cerenkov radiation. The difference is one of path length. When the oscillating $\sin^2 u$ function has a maxima near the maximum of the envelope function, that peak will be largest. The positions of the maxima in $\sin^2 u$ are found by setting u in (5) to be an odd multiple of $\pi/2$.

In media with index of refraction near 1, the radiation described here has the same spatial characteristics as transition radiation. If the index is set equal to 1, the peak of the radiation envelope occurs at $\cos \theta_s = \beta$. Then $\sin \theta_s = \gamma^{-1}$, or for small θ_s , $\theta_s = \gamma^{-1}$ which is a characteristic of transition radiation.

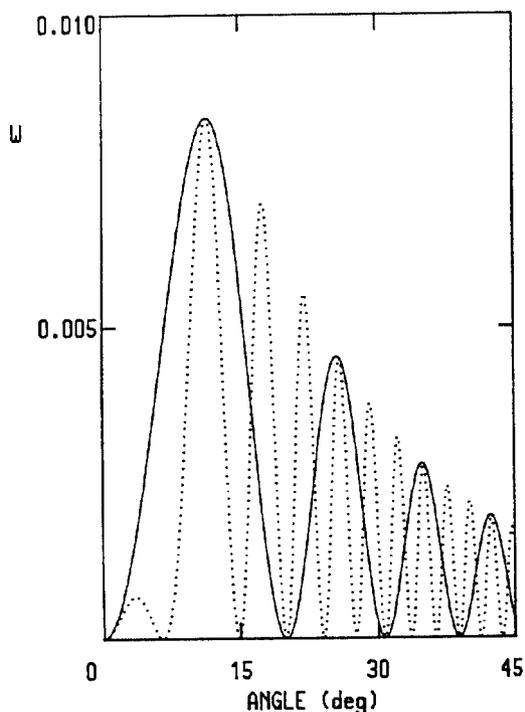


Fig 4. Sub-Cerenkov radiation patterns for $n\beta = 0.98$. The solid curve, calculated for $n=12$, has the appearance of diffracted Cerenkov radiation. The dotted curve, calculated for $n=37$ shows a minor maxima preceding the largest lobe and a maxima of nearly the same intensity following it.

We have already pointed out³ that an angular map of the radiation in the vicinity of 90 degrees could be used to measure $F(k)$ in the Cerenkov case. Here we wish to note that the same techniques could be utilized with sub-Cerenkov radiation. Fig. 5. shows the pattern calculated for 100 MHz radiation from a

single trapezoidal 2 MeV electron bunch propagating in air. Both the diffracted sub-Cerenkov radiation in the forward direction and the radiation near 90 degrees with an envelope characteristic of the Fourier transform of the charge bunch distribution are shown. At higher frequencies only the latter appears.

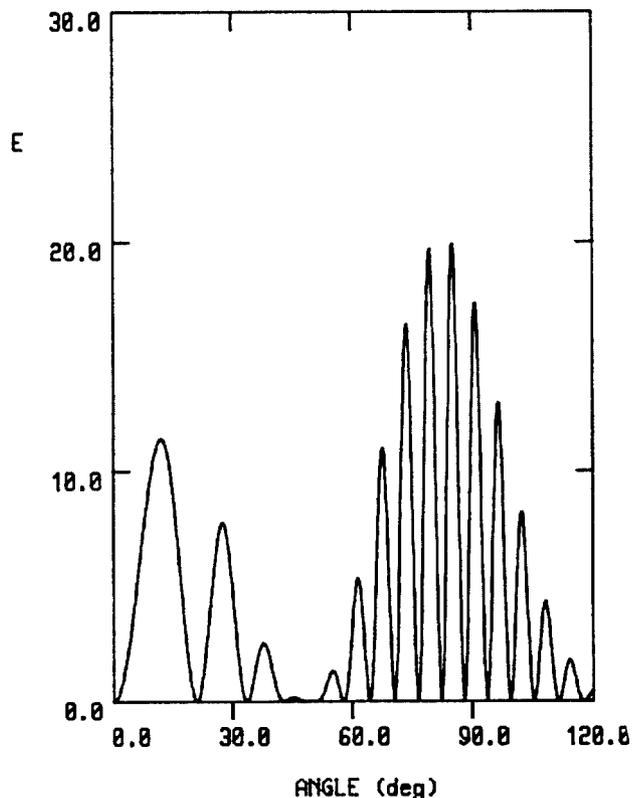


Fig. 5. Energy per solid angle for 100 MHz radiation from a 2 MeV trapezoidal charge pulse traveling a distance of 30 m. The pulse has a base length of 4.8 m and a top length of 4.2 m.

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