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Abstract

The minimum theoretical emittance that can be achieved in a compact synchrotron radiation source is investigated and suitable lattice designs put forward. A comparison is made between superconducting and conventional magnet rings that are suitable sources for x-ray lithography.

Introduction

The concept of a very compact synchrotron radiation source for industrial and research applications was first suggested several years ago [1]. The proposed "Klein Erna" source used a single weak focusing 5 T superconducting magnet and had a circumference of only 1.8 m. There are, however, difficulties associated with such a design, such as beam injection and installation of r.f. and other components. Since then attention has focused on less compact but more flexible designs using two or more dipole magnets, either superconducting or conventional, separated by straight sections in which injection and other components can be installed. There is presently great interest in the development of such rings, resulting mainly from the demand for a source for industrial x-ray lithography [2-4], but with possible research applications also.

In this report the minimum theoretical emittance and beam dimensions that can be achieved in a compact ring with two or four bending magnets with arbitrary field index are investigated, taking as an example a superconducting ring for producing soft x-rays for lithography. Two different lattice designs are presented capable of producing the required lattice functions. Finally, a comparison is made between the superconducting designs and a compact ring based on conventional magnets.

Parameter Optimization

The specification of a source suitable for lithography was considered in refs. [2] and [3]. The wavelength range of interest is 6-12 Å. For the following examples an energy of 0.7 GeV and dipole field of 4.5 T have been chosen, giving $\lambda_c = 8.4$ Å, and meeting the required power density criterion with a beam current of $\lesssim 400$ mA. Additionally it was stated [3] that for lithography r.m.s. beam sizes and divergences should be $\lesssim 1$ mm and 1 mrad respectively in both planes. For other applications even smaller dimensions may be required. To achieve this in general requires a small natural beam emittance, ϵ_{x0} , and so this will be the main aim of the optimisation procedure. We also examine the horizontal beam size (σ_x), since the horizontal beam dimensions are generally larger than the vertical and this quantity can be calculated directly. In fact the horizontal beam divergence (σ_x') is not a relevant parameter for the synchrotron radiation brightness [5] but is included in the final results for completeness.

For small bend angles (θ) and zero field gradient the minimum emittance has been derived analytically [6]:

$$\epsilon_{x0} = \frac{C_q \gamma^2 \theta^3}{J_x 12\sqrt{15}}$$

where $C_q = 3.8319 \times 10^{-13}$
 J_x = horizontal damping partition number
 γ = relativistic parameter

The minimum occurs when the lattice functions are symmetric about the mid point of the dipole with values at this point given by:

$$\beta_0 = \frac{\rho\theta}{2\sqrt{15}} \quad \eta_0 = \frac{\rho\theta^2}{24}$$

with ρ = dipole bend radius. The energy spread is given by:

$$\left(\frac{\sigma_E}{E_0}\right)^2 = \frac{C_q \gamma^2}{J_E \rho}$$

where J_E = damping partition number for energy oscillations, $(3-J_x)$. The horizontal beam size combines the effects of emittance and energy spread:

$$\sigma_x^2 = \epsilon_{x0}\beta + \left(\frac{\sigma_E}{E_0}\right)^2 \eta^2$$

and so is given at the centre of the dipole, for optimum emittance by:

$$\sigma_{x0}^2 = \frac{C_q \gamma^2 \rho \theta^4}{24^2} \left(\frac{1.6}{J_x} + \frac{1}{J_E} \right)$$

At the end of the magnet the lattice parameters are related to those at the centre by the following:

$$\beta_e = 16 \beta_0 \quad \eta_e = 4 \eta_0 \quad \sigma_{xe} = 4 \sigma_{x0}$$

Using these formulae we therefore predict for a two-cell design with the present parameters, assuming $J_x = 1$, $J_E = 2$:

$$\begin{aligned} \epsilon_{x0} &= 4.8 \times 10^{-7} \text{ m rad} & \sigma_E/E_0 &= 8.3 \times 10^{-4} \\ \beta_0 &= 0.210 \text{ m} & \eta_0 &= 0.213 \text{ m} \\ \sigma_{x0} &= 0.36 \text{ mm} \end{aligned}$$

Teng [7] included gradient fields in an analytic approach and showed that the emittance can be reduced by including a defocusing gradient; however, his results are still only approximate at large θ since the variation of J_x was not included in the emittance optimization. To overcome this difficulty a numerical approach has therefore been adopted. Table 1 shows the variation of lattice parameters in the present case as a function of the gradient field n -value: $n = -(dB/dr)(\rho/B_0)$. It can be seen that the results are in broad agreement with the predictions of the simple model above and the emittance decreases with increasing n as expected. A limit is reached at $n \approx 1.5$ when the energy oscillations become antidamped ($J_E < 0$). The beam size at the centre of the magnet is minimized for $n \approx 0.6$; however, at the end of the magnet the size increases with n . For $n \lesssim 0.3$ the beam size can be maintained less than 1 mm anywhere in the dipole.

To achieve smaller emittance and beam sizes a larger number of bending magnets is required. For example, the expressions above suggest that a four-cell design would have a factor of 8 lower emittance and factor 4 smaller beam size than the two-cell geometry. Table 2 shows the result of optimizing the emittance as a function of field index in this case. By comparison with Table 1 it can be seen that the scaling of the various parameters with θ is roughly in accordance with

Table 1. Optimized parameters for a two-cell ring with $E_0 = 0.7$ GeV and $B_0 = 4.5$ T.

n	β_0	η_0	$\epsilon_{x0} \times 10^{-7}$	$\sigma_E/E_0 \times 10^{-4}$	J_x	σ_{x0} (mm)	σ_{xe} (mm)	σ_{x0}' (mrad)	σ_{xe}' (mrad)
0.0	0.25	0.11	7.0	7.4	0.50	0.43	0.95	1.67	1.00
0.2	0.23	0.16	5.4	7.7	0.65	0.37	0.98	1.54	0.86
0.4	0.22	0.19	4.3	8.1	0.87	0.34	1.02	1.40	0.91
0.6	0.22	0.21	3.5	8.6	1.15	0.33	1.08	1.28	1.04
0.8	0.22	0.23	3.0	9.6	1.49	0.34	1.18	1.18	1.30
1.0	0.21	0.25	2.6	11.2	1.90	0.37	1.36	1.10	1.78
1.2	0.21	0.27	2.3	15.0	2.38	0.46	1.78	1.03	2.82
1.4	0.21	0.29	2.0	51.5	2.95	1.49	5.93	0.98	11.59

Table 2. Optimized parameters for a four-cell ring with $E_0 = 0.7$ GeV and $B_0 = 4.5$ T.

n	β_0	η_0	$\epsilon_{x0} \times 10^{-7}$	$\sigma_E/E_0 \times 10^{-4}$	J_x	σ_{x0} (mm)	σ_{xe} (mm)	σ_{x0}' (mrad)	σ_{xe}' (mrad)
0.0	0.11	0.047	0.66	8.0	0.82	0.092	0.33	0.78	0.56
0.2	0.11	0.049	0.62	8.1	0.89	0.091	0.33	0.76	0.58
0.4	0.11	0.052	0.59	8.2	0.96	0.090	0.33	0.74	0.61
0.6	0.11	0.053	0.55	8.4	1.04	0.089	0.34	0.72	0.64
0.8	0.11	0.055	0.52	8.6	1.12	0.088	0.34	0.70	0.67
1.0	0.11	0.057	0.50	8.8	1.21	0.088	0.35	0.69	0.71
1.2	0.11	0.059	0.47	9.0	1.31	0.088	0.35	0.67	0.76
1.4	0.11	0.060	0.45	9.3	1.40	0.089	0.36	0.65	0.81

Table 3. Comparison of major parameters for several different lattices.

	L1	L2	L3	L4	L5	L6
Energy (Gev)	0.7	0.7	0.7	0.7	1.0	1.0
Dipole field (T)	4.5	4.5	4.5	4.5	1.6	1.6
No. of cells	2	2	4	4	4	4
Lattice type	singlet	doublet	singlet	doublet	singlet	doublet
Circumference (m)	8.70	10.30	14.14	17.34	23.98	27.18
Field index, n	0.235	0.8	0.597	0.8	1.33	0.8
Tunes (Q_r, Q_v)	1.59, 0.59	1.60, 0.60	2.25, 1.25	3.25, 2.25	1.75, 1.25	2.75, 1.75
F Quad Strength (T/m) ⁺	10.6	23.8	14.4	30.3	8.1	25.6
D Quad strength (T/m) ⁺	-	18.1	-	19.5	-	19.1
Chromaticity (ξ_r, ξ_v)	-0.6, -2.6	-2.7, -1.5	-1.6, -6.7	-7.3, -3.4	-1.5, -1.3	-3.0, -4.8
Momentum compaction	0.22	0.28	0.074	0.039	0.26	0.091
J_x	0.69	1.52	1.06	1.12	1.79	1.11
Radial damping time (ms)	1.44	0.77	1.52	1.76	2.11	3.84
Natural emittance (m rad $\times 10^{-7}$)	5.21	3.13	0.99	0.53	2.67	1.38
R	1.01	1.05	1.78	1.01	2.87	1.29
σ_{x0} (mm)*	0.37	0.34	0.18	0.08	0.68	0.35
σ_{x0}' (mrad)*	1.35	1.28	0.59	0.73	0.65	0.38
σ_{y0} (mm)*	0.67	0.14	0.29	0.07	0.48	0.11
σ_{y0}' (mrad)*	0.07	0.21	0.03	0.07	0.05	0.11

⁺ quad length = 0.2 m

* 10% coupling

the expressions above. In particular, the emittance is reduced by between 5 and 10 and σ_{x0} by approximately a factor of 4 in the region $0 < n < 1$. In general there is a much weaker dependence on n-value in the four-cell ring as expected, since in large rings it is known that very large field indices are required to produce appreciable changes in emittance [7].

Lattice Design

Having derived the optimum lattice functions at the centre of the dipole to produce minimum emittance it is then necessary to devise a lattice structure which will allow these to be realised. Two schemes have been investigated involving either a single F-quadrupole or a quadrupole doublet at the end of each straight. Figure 1 shows the two types of lattice and

typical β and η functions. Suitable straight section lengths have been chosen and kept constant in each of the examples presented. In each case a working point has been selected close to that at which the minimum emittance is produced but taking into account other parameters such as beam size, quadrupole strength, chromaticity, etc. Results are presented in Table 3.

The most compact lattice consists of two cells of the type shown in fig. 1a. It has been found that for this lattice (L1) the range of n-values over which it is stable is restricted to $n \leq 0.28$. The choice of working point is also rather constrained, nevertheless it can be seen from Table 3 that the ratio (R) between the minimum theoretical emittance at the selected n-value and that obtained is very close to unity.

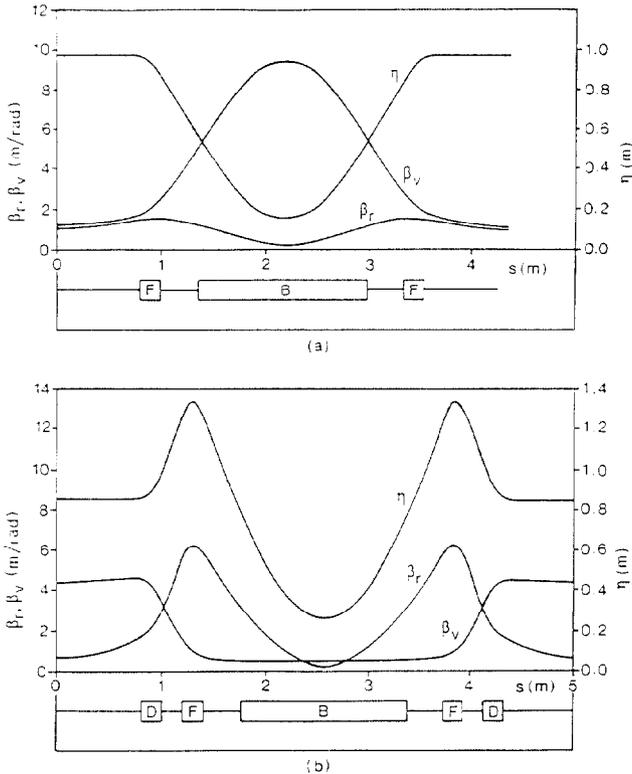


Fig. 1. Singlet (a) and doublet (b) lattice structures and typical lattice functions.

To achieve lower emittance in a two-cell structure demands a higher n -value and this has been investigated with a doublet lattice (L2). It was found that useful emittance reductions could be achieved for $0.7 < n < 1.0$, above which J_x becomes unacceptably large. An n -value of 0.8 was therefore selected and a suitable working point derived. It can be seen from Table 3 that the expected emittance can be achieved; however, the main benefit is in terms of the vertical beam size and divergence since in the horizontal plane there is an increased contribution from energy spread.

To achieve a greater emittance reduction a four-cell lattice is needed. Both singlet (L3) and doublet (L4) lattices have been investigated and results are given in Table 3. The singlet lattice is not able to approach the theoretical minimum as closely as L1, nevertheless a reduction of a factor of 5 is achieved. Lattice L4 achieves a factor of 10 lower emittance but quadrupole gradients are large and radial chromaticity also.

Conventional Magnet Ring

For comparison a ring employing conventional magnets has been investigated based on a maximum dipole field of 1.6 T. To achieve the same power density for lithography with the same beam current then requires a storage ring energy of approximately 1.0 GeV. For a given bend angle and n -value the expressions given above may be used to scale the results of Tables 1 and 2 for the new ρ and γ values. Thus for the same conditions the minimum emittance increases by a factor 2.0 and the horizontal beam size by 2.9. It is clear therefore that a four-cell structure is required in this case to achieve the required beam size. Both types of lattice have been investigated, L5 and L6 in Table 3. In the case of the singlet lattice a higher n -value is required for stability than for the superconducting ring but for the lattice investigated it was

not possible to approach the theoretical emittance very closely and the doublet lattice produced a factor of 2 lower emittance despite the smaller n -value.

Conclusions

It has been shown that in principle the most compact ring meeting a basic specification for x-ray lithography is a two-cell superconducting ring, with a simple quadrupole arrangement (L1), and that this can achieve very close to the theoretical minimum emittance for the given dipole field gradient. The disadvantages of this lattice structure are however the lack of tuneability and the difficulty in correcting the chromaticity because of the form of the lattice functions. These are overcome with the doublet structure at the expense of increased size and higher quadrupole gradients. In general, the doublet lattice with its greater tuneability also allows the minimum emittance to be approached more closely than the singlet, however, in the latter case further optimisation of the examples chosen may be possible, for example by varying straight section lengths.

Lower emittances can be produced by a four-cell structure, however this is likely to introduce significant Touschek lifetime limitations even at the operating energies chosen. For example, in lattices L3 and L4 the Touschek lifetime could be ≤ 10 hours with 400 mA beam current assuming a 500 MHz cavity with 350 kV peak voltage and 10% emittance coupling.

A conventional magnet ring requires a four-cell structure to achieve the same basic specification for lithography, and of course is much less compact. It is also apparent that such a ring has in general a larger radial damping time compared to the superconducting ring, typically a factor of two, which together with the higher operating energy could lead to the need for a higher injection energy - by a factor 1.8 for comparable damping time at injection.

Acknowledgements

This work has been supported in part by Oxford Instruments. Helpful comments have been received from A. van Steenbergen.

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