ALLOWABLE FIELD ERRORS IN UNDULATORS USED AS SYNCHROTRON RADIATION SOURCES

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Summary

High brightness synchrotron sources based on undulator radiation require both low emittance electron beams and high precision undulators. Under many circumstances, loss of brightness due to wiggler field errors is dominated by e-beam steering from dipole errors. A simple method to calculate the allowed dipole errors is presented. The error tolerance differs depending on whether or not the emittance is large enough to preclude full spatial coherence. In the large emittance, spatially incoherent limit the storage ring betatron function can be adjusted to optimize either the spectral coherence or spatial coherence, but not both simultaneously. The calculated tolerances are compared to measured field errors of the Spectra Technology THUNDER undulator.

Introduction

There is increasing interest in using synchrotron radiation from electron storage rings as a source for the XUV spectral region. The source properties are often optimized by use of an undulator, as opposed to other insertion devices, because of high coherence possible. [1] An important aspect of the system design is the way in which magnetic field errors of the undulator limit coherence and reduce source brightness. [2] Errors causing steering within the wiggler are often the most important problem. A criteria often quoted for quality of an undulator is the RMS field error. This tends to emphasize phase errors rather than the trajectory errors, since the latter cannot be calculated with only knowledge of the RMS errors. Accordingly, it is useful to calculate the allowed field errors directly. In this paper a simple scheme is presented for identifying permissable error fields. The relationship between the storage ring betatron function and coherence is also discussed.

There are several basic requirements that must be met if the undulator radiation is to be as coherent as possible, both spectrally and spatially. Spatial coherence refers to the E-field phase with respect to lateral movement, and spectral coherence refers to E-field phase with respect to longitudinal movement. These parameters are completely equivalent to other figures of merit, namely focusability and bandwidth, respectively. Together with the total number of emitted photons, these parameters are the most fundamental figures of merit for the source. However, it is important to note many applications of undulator radiation are sensitive to other, less fundamental, figures of merit. For example, an optics beam line may require a small photon beam, independent of the divergence angle, for irradiation of a sample. In that case the focusability of the beam, which is the product of size and divergence, would be unimportant.

Full spectral coherence for an undulator of N periods requires that the radiation through an on-axis pinhole has a bandwidth (FWHM) that does not exceed 1/N. Full spatial coherence requires that the radiation cone has a size and spread angle not exceeding the diffraction limited values. Equivalently, at any waist the beam obeys the condition

$$\sigma_R \, \sigma_R' \approx \frac{\lambda_R}{4\pi} \qquad [1]$$

where σ_R and σ_R' are the radius and far field half angle points of the $e^{-1/2}$ intensity. The radiation wavelength is λ_R. In Eq. 1 the radius and far field half angle would be measured with a monochromator set to λ_R, viewing through a pinhole.

With the product $\sigma_R \, \sigma_R'$ defined for the diffraction limited photons, the particular apportionment between the angular and spatial components depends on the wiggler length. A starting point for finding the angular extent σ_R', is to note that an e-beam energy shift of 1/4N will reduce the on-axis intensity at the original centerline wavelength by a factor of two. The basic resonance equation can then be used to show that the equivalent shift results when the observation angle changes by $(\lambda_R/L)^{1/2}$ where L is the length of the undulator. This is independent of the undulator K value. [2] Then by the above definitions, we have

$$\sigma_R' = \left(\frac{\lambda_R}{L}\right)^{1/2} . \qquad [2]$$

Then from Eq. 1

$$\sigma_R = \frac{(L\lambda)^{1/2}}{4\pi} \qquad [3]$$

which is in agreement with other work. [3]

This coherence, as defined above, can be destroyed by emittance, undulator steering errors, or energy spread. The basic criteria to avoid coherence loss for planar undulators are given in Table 1.

Table 1

COHERENCE REQUIREMENTS

a) Small E-Beam Angular Content ($\sigma_e' < \sigma_R'$)

$$\sigma_e' < \left(\frac{\lambda_R}{L}\right)^{1/2}$$

b) Small E-Beam Size ($\sigma_e < \sigma_R$)

$$\sigma_e < \frac{(L\lambda_R)^{1/2}}{4\pi}$$

c) Additional Constraint in Vertical Plane

$$\sigma_e < \frac{\lambda_u}{2\pi \, N^{1/2}}$$

d) Energy Spread

$$\frac{\Delta\gamma}{\gamma} < \frac{1}{2N}$$

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where λ_u is the undulator wavelength. The e-beam $e^{-1/2}$ points in radius and half angle are σ_e and σ_e' respectively. Condition (c) results from frequency shift due to the vertical gradient in the magnetic field. The inequality is based on an undulator K value of $(2)^{1/2}$. There are also additional considerations for straightness of the undulator centerline and undulator phase errors. The centerline constraint is equivalent to (c) in Table 1, and also applies only to vertical straightness. A phase error constraint, relating to both $\Delta\lambda_w$ and ΔB_w (when $K \approx (2)^{1/2}$) are essentially equivalent to the energy spread requirement (d) in Table 1. In condition (d) $\Delta\gamma/\gamma$ is properly considered a full width.

For typical synchrotron parameters, constraints (a) and (b) are generally the most difficult to satisfy. These angle and size limitations place a limit on steering errors in the undulator. These errors offset the trajectory angle according to

$$\Delta\theta = 5.85 \times 10^{-4} \frac{\Delta(B\ell)}{\gamma} \qquad [4]$$

where $\Delta\theta$ = angular error in radians, $\Delta(B\ell)$ = integrated dipole error in G-cm, and γ = energy of the e-beam in units if the rest mass. Using this relationship and the basic undulator resonance equation, constraints (a) and (b) of Table 1 can be related to the $\Delta(B\ell)$ dipole errors. This leads to the field error limits given in Table 2. The angle σ_e' is in radians and σ_e and L must be in the same length unit. As seen by the definitions of the table, the size requirement is derived from an angular error acting over a lever arm of half the wiggler length. This corresponds to dipole errors which occur in the central portion of the undulator. Errors which occur at either end are equivalent to entrance or exit errors and can be compensated externally. The internal errors cause beam curvature and cannot be corrected externally.

The table is divided into coherent and incoherent cases. The coherent cases apply to $K \approx (2)^{1/2}$ and the error tolerance changes only slightly at lower K values. The use of harmonic emission makes the condition more stringent. Here the number of periods N is replaced by nN, where n is the harmonic number, defined so that n=1 for the fundamental. The incoherent cases refer to the situation where the e-beam phase space greatly exceeds that of the photons, i.e.,

$$\sigma_e \sigma_e' >> \frac{\lambda_R}{4\pi} \qquad [5]$$

and the error limits given refer to maximizing the spatial coherence. Note that $\sigma_e\sigma_e'$ is simply the emittance, and this condition can be applied separately in each plane. As is discussed below, the betatron function can be adjusted (in the incoherent case) to maximize either the spatial coherence, or spectral coherence, but not both. In the incoherent case, the σ_e and σ_e' values for the e-beam in Table 2 can be written in terms of the storage ring betatron function, β, and emittance, ϵ, using

$$\sigma_e = (\epsilon\beta)^{1/2}$$

and

$$\sigma_e' = \left(\frac{\epsilon}{\beta}\right)^{1/2}$$

The conditions for the incoherent case then become

$$\text{error} < 1700\ \gamma \left(\frac{\epsilon}{\beta}\right)^{1/2} \text{ G-cm} \qquad [6]$$

for the angular tolerance, and

$$\text{error} < 3400\ \frac{\gamma}{L}\ (\epsilon\beta)^{1/2} \text{ G-cm} \qquad [7]$$

for the size tolerance.

It is useful to note that whenever Eq. 5 is satisfied the system is not fully spatially coherent, but it still may be spectrally coherent depending on the choice of the ring betatron function, β. The β value which does not increase the homogeneous radiation bandwidth is also that which meets condition (a) in Table 1. For maximum spectral coherence (i.e., minimum bandwidth) we then require

$$\beta > \frac{L\epsilon}{\lambda_R} \qquad [8]$$

The constraint (c) of Table 1 can provide an upper bound on β. For many applications, even when the system is spatially incoherent (Eq. 5), the size and divergence of the source is more important than the bandwidth. For these cases the spatial coherence may be dominated by magnetic field errors rather than the emittance. That is, the Table 2 requirements for the incoherent case are not met. Then the effect of magnetic field errors is minimized and the spatial coherence is maximized when

$$\beta = \frac{L}{2} \qquad [9]$$

TABLE 2
ALLOWABLE INTEGRATED DIPOLE ERRORS

	COHERENT	INCOHERENT
Angle	$\Delta\theta < (\lambda/L)^{1/2}$	$\Delta\theta < \sigma_e'$
Requirement	error < $1700/N^{1/2}$ G-cm	error < $1700\ \gamma\sigma_e'$ G-cm
Size	$\Delta\theta\ L/2 < (L\lambda)^{1/2}/4\pi$	$\Delta\theta\ L/2 < \sigma_e$
Requirement	error < $270/N^{1/2}$ G-cm	error < $3400\ \gamma\sigma_e/L$ G-cm

as seen from Eqs. 6 and 7.

The importance of integrated dipole field errors can be understood by considering as an example the planned Advanced Photon Source [4] at Argonne and the newly constructed THUNDER undulator [5] built by Spectra Technology, Inc. for visible wavelength free-electron laser [6] use. This undulator has characteristics common to insertion devices planned for the Advanced Photon Source. The expected ring emittance of [7] 7×10^{-9} mrad horizontal and 7×10^{-10} mrad vertical, together with the approximate 0.6A fundamental wavelength, can be used in Eq. 5 to show that system operates in the incoherent limit in each plane. The allowed dipole errors in each plane can be estimated assuming 2 cm period, 5 m wiggler, 7 GeV beam energy, and beta function of 10 m and 20 m in the vertical and horizontal planes respectively. The calculated numbers based on Table 2 (and Eqs. 6 and 7) are shown in Table 3.

Table 3

INTEGRATED DIPOLE ERROR LIMITS FOR ADVANCED LIGHT SOURCE USING ASSUMED PARAMETERS

	Vertical Plane	Horizontal Plane
Angle Requirement	200 G-cm	440 G-cm
Size Requirement	780 G-cm	3500 G-cm

All are based on the incoherent limit. In both planes the angle requirement dominates. These requirements can be compared with performance of the THUNDER system, where the random error in each of ten 50 cm segments is 100 G-cm or less in each plane. If the errors fall in random fashion, the integrated error for 5 m overall length would be less than 300 G-cm, satisfying the horizontal requirement, but not the vertical. Similar considerations for the visible wavelength FEL led to the segmented design of THUNDER. Electron beam diagnostics and steering stations are located between sections so that accumulation of random errors can be avoided. The steering corrections are based on measurements of the e-beam trajectory rather than measured dipole errors, because of the difficulty in measuring field integrals to the required accuracy. This method for acquiring proper e-beam trajectories by real time adjustment may be required in high brightness synchrotron undulators as well.

References

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