

## OPTIMIZATION OF COMPACT SYNCHROTRON OPTICS FOR X-RAY LITHOGRAPHY\*

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### Abstract

The production of integrated circuits having sub-micron component dimensions has motivated the development of compact electron storage rings to provide synchrotron x-ray radiation [1-9]. This paper presents considerations for optimizing the optics for small radius rings using superconducting dipole magnets. The key parameters are the sizes and angular divergences of the source points illuminating the different ports. Two ring designs are compared in terms of theoretical beam parameters achievable using idealized optics.

### Background

The horizontal beam size  $\sigma_x$  and angular divergence  $\sigma_x'$  in a storage ring are given by

$$\sigma_x(s) = [\beta_x(s) \epsilon_x + (\eta(s) \sigma_\epsilon)^2]^{1/2} \quad (1)$$

and

$$\sigma_x'(s) = \left[ \frac{1 + \alpha_x^2(s)}{\beta_x(s)} \epsilon_x + (\eta'(s) \sigma_\epsilon)^2 \right]^{1/2} \quad (2)$$

There are contributions from the emittance  $\epsilon_x$  and the rms energy spread  $\sigma_\epsilon$ . The horizontal beta function  $\beta_x$ , the dispersion function  $\eta$ , and  $\alpha_x \equiv -\beta_x'/2$  are in general dependent on the longitudinal coordinate,  $s$ . This dependence leads to differing source characteristics at the various light extraction ports around the ring.

Initial studies conducted by lithographers [6] indicate that the desired limits on horizontal spot size and divergence are roughly

$$\sigma_x \leq 1mm \text{ and } \sigma_x' \leq 1mr. \quad (3)$$

The desired critical wavelength of the source is taken to be  $\lambda_c = 10\text{\AA}$ .

### Cylindrically Symmetric Design

The cylindrically symmetric ring design [1,8] is characterized by critical wavelength  $\lambda_c$ , dipole field strength  $B$ , and horizontal field gradient index  $n$  as defined by

$$n = -\frac{\rho}{B} \frac{\partial B}{\partial x}, \quad (4)$$

where  $\rho$  is the radius of curvature. The horizontal rms beam size and angular divergence for this design are given by

$$\sigma_x = 19.8 \lambda_c^{-3/4} B^{-5/4} \left[ \frac{1}{n(1-n)} + \frac{1}{(1-n)(3-4n)} \right]^{1/2} \quad (5)$$

and

$$\sigma_x' = 1.38 \lambda_c^{-1/4} B^{1/4} \left[ \frac{1}{n} \right]^{1/2} \quad (6)$$

For  $\lambda_c$  in units of angstroms and  $B$  in units of Tesla,  $\sigma_x$  and  $\sigma_x'$  are in units of millimeters and milliradians, respectively.

An interesting feature of Eqs. (5) and (6) is that dimensional parameters  $\lambda_c$  and  $B$  have been separated from the dimensionless geometrical factors involving  $n$ . In general, the geometrical factors would be more complex expressions involving focal lengths of quadrupoles and drift lengths, but the overall factor would be dimensionless (see Appendix A). This clearly indicates the effects of varying  $\lambda_c$  and  $B$ . As  $\lambda_c$  is decreased, both spot size and beam divergence increase. Though a cylindrical machine may produce acceptable spot characteristics at  $\lambda_c = 20\text{\AA}$ , the original Klein-ERNA design value [1], both spot size and divergence are at best marginal, in the absence of coupling, when scaled to  $\lambda_c = 10\text{\AA}$ .

As the field strength  $B$  is reduced, the beam divergence

improves slightly but the spot size increases significantly. Machine designs utilizing conventional dipoles can counteract this  $B^{-5/4}$  dependency of the beam size only by resorting to more complicated geometries.

### Symmetric Racetrack Design

The lattice considered here has two  $180^\circ$  dipoles with uniform and equal field index which are connected by straight sections, containing quadrupoles and drifts, which are symmetric about their midpoints. Besides allowing several mathematical simplifications due to symmetry, this lattice is expected to produce a local minimum of emittance in parameter space.

For the purpose of optimization, three different functions-of-merit are considered. In order to take advantage of the scaling properties indicated in Appendix A, the functions chosen maintain the homogeneity of the scaling parameters. Explicitly, the functions used are

$$\langle \bar{\sigma}_x \rangle \langle \bar{\sigma}_x' \rangle, \quad \langle \bar{\sigma}_x \rangle \langle \bar{\sigma}_x' \rangle^5, \quad \text{and} \quad \bar{\sigma}_x(0^\circ) \quad (7)$$

where  $\bar{\sigma}_x$  and  $\bar{\sigma}_x'$  are dimensionless quantities. The source size and divergence are calculated by multiplying these quantities by

$$l \equiv \sqrt{C_q \rho} \quad \text{and} \quad \theta \equiv \sqrt{C_q / \rho}$$

respectively. Here  $\langle \rangle$  means average over port locations at  $0^\circ$ ,  $\pm 20^\circ$ , and  $\pm 40^\circ$  in the dipole. The third form indicates a single point sample at the center of the dipole. From Eqs. (1) and (2), these functions depend on the betatron and dispersion functions, their derivatives and the longitudinal and horizontal emittances. The beta function in the dipole of a lattice with the assumed symmetry depends only on  $\beta_0$ , the value in the center of the dipole, and  $n$ . Similarly, the dispersion function depends only on  $\eta_0$  and  $n$ . With parameterizations of  $\beta$  and  $\eta$  in the dipoles, the emittances can be calculated. The above functions-of-merit, referred to generically by  $F_m$ , can therefore be written as

$$F_m = F_m(\beta_0, \eta_0, n). \quad (8)$$

As shown in the first half of Appendix B, the phase advance through the straight section is determined by the values of the betatron and dispersion functions and their derivatives at the exit of the dipole. Since the horizontal phase advance through the dipole depends only on  $\beta_0$  and  $n$ , the horizontal tune can be written as

$$\nu_x = \nu_x(\beta_0, \eta_0, n). \quad (9)$$

Since the tune of a storage ring is a critical operating parameter, it is desirable to have it be an independent rather than dependent variable during the optimization process. Eq. (9) can be rewritten to give

$$\eta_0 = \eta_0(\nu_x, \beta_0, n). \quad (10)$$

This can be substituted in Eq. (8) to yield

$$F_m = F_m(\nu_x, \beta_0, n). \quad (11)$$

Using the first form in Eq. (7), the behavior of Eq. (11) for  $\nu_x = 1.4$  is shown in Figure 1 where contours are spaced by 0.1. Table 1 indicates the dependence of the optimization on  $\nu_x$  and details the resulting normalized beam spot characteristics.

Considering now the dipole strength, Eqs. (A.10) and (A.11) show that increasing  $B$  improves the value of the function-of-merit indefinitely but at the expense of  $\sigma_x'$ . The value of  $B = 3.5$  Tesla satisfies the lithographer's requirements as stated in Eqs. (3). The radius of curvature for  $\lambda_c = 10\text{\AA}$  is then 0.69 meters and the resultant source characteristics for  $\nu_x = 1.4$ , in the absence of coupling, are tabulated in Table 2.

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## Appendix A: Scaling

Here the source size and angular divergence are shown to scale in a simple way with radius of curvature  $\rho$  and beam energy  $\gamma$ . Using the notation of [10], the differential equations for the betatron and dispersion functions are

$$\frac{1}{2} \beta \beta'' - \frac{1}{4} \beta'^2 + K \beta^2 = 1 \quad (\text{A.1})$$

and

$$\eta'' = K \eta + G \quad (\text{A.2})$$

where  $G$  specifies the dipole field strength and has units of  $m^{-1}$  and  $K$  describes the quadrupole strength and has units of  $m^{-2}$ . If  $\rho$  is scaled, then  $G$  scales like  $\rho^{-1}$  and  $K$  scales like  $\rho^{-2}$ . Since differentiation with respect to  $s$  scales like  $\rho^{-1}$ , inspection of Eqs. (A.3) and (A.4) indicates that  $\beta$  and  $\eta$  scale like  $\rho$ .

Considering next the damping partition numbers given by

$$J_x = 1 - D \quad \text{and} \quad J_z = 2 + D, \quad (\text{A.3})$$

where

$$D = \frac{\int \eta G (G^2 + 2K) ds}{\int G^2 ds}, \quad (\text{A.4})$$

it is clear they are independent of scaling  $\rho$ . Consequently the relative energy spread or longitudinal emittance

$$\sigma_\epsilon = \left[ \frac{C_q \gamma^2 \langle G^3 \rangle}{J_\epsilon \langle G^2 \rangle} \right]^{1/2} \quad (\text{A.5})$$

where  $C_q$  is a constant [10], scales like  $\rho^{-1/2}$ . Similarly, the horizontal emittance given by

$$\epsilon_x = \frac{C_q \gamma^2 \langle G^3 H \rangle}{J_x \langle G^2 \rangle} \quad (\text{A.6})$$

where

$$H = \frac{1}{\beta} \left[ \eta^2 + \left( \beta \eta' - \frac{\beta'}{2} \eta \right)^2 \right] \quad (\text{A.7})$$

can be seen to be independent of scaling  $\rho$ .

Combining the above results and referring to Eqs. (1) and (2) in the main text leads to the deduction that  $\sigma_x$  scales like  $\rho^{1/2}$  and  $\sigma_x'$  scales like  $\rho^{-1/2}$ . Considering again Eqs. (A.3) and (A.4) one finds that both  $\sigma_x$  and  $\sigma_x'$  scale like  $\gamma$  as the energy is varied. These results can be summarized by

$$\sigma_x = \sqrt{C_q \rho} \gamma F(\text{lattice}) \quad (\text{A.8})$$

and

$$\sigma_x' = \sqrt{C_q / \rho} \gamma G(\text{lattice}) \quad (\text{A.9})$$

where the dimensionless functions  $F$  and  $G$  reflect the geometric properties of the lattice independent of the electron energy and the scale of the bending radius.

The utility of Eqs. (A.8) and (A.9) is realized when lattice design optimization is considered. For a useful class of functions-of-merit, the above factorization effectively reduces the dimensionality of the parameter space by two because the lattice geometry, represented by  $F$  and  $G$ , can be optimized independent of  $\rho$  and  $\gamma$ . Useful alternatives to Eqs. (A.8) and (A.9) can be realized by expressing  $\rho$  and  $\gamma$  in terms of  $\lambda_c$  and the main dipole field strength  $B$ . This results in the proportionalities

$$\sigma_x \propto \lambda_c^{-3/4} B^{-5/4} \quad (\text{A.10})$$

and

$$\sigma_x' \propto \lambda_c^{-1/4} B^{1/4} \quad (\text{A.11})$$

In this form, the constraint on  $\lambda_c$  is trivially imposed and the effect of varying the dipole field strength is shown.

## Appendix B: Matching

Matching, the design of an optical transport system using quadrupole focusing elements and drift spaces, is a common practice in lattice design. In this appendix, two examples particularly relevant to the design of symmetric racetrack machines are

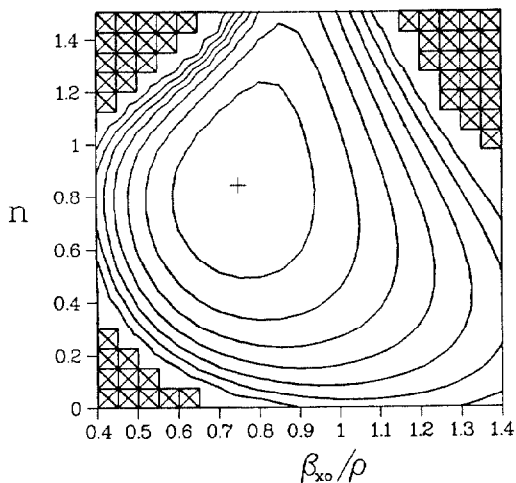


Figure 1. Dependence of  $F_m$  on  $\beta_0$  and  $n$

Table 1. Dependence of  $F_m$  on  $\nu_z$

$\nu_z$	$n$	$\frac{\beta}{\rho}$	$\frac{\eta}{\rho}$	0°		40°		$F_m$
				$\bar{\sigma}_x$	$\bar{\sigma}_x'$	$\bar{\sigma}_x$	$\bar{\sigma}_x'$	
1.05	0.74	1.32	1.02	1.36	0.79	1.51	0.88	1.20
1.10	0.75	1.27	0.89	1.23	0.77	1.40	0.87	1.08
1.15	0.77	1.20	0.77	1.11	0.76	1.29	0.87	0.98
1.20	0.79	1.11	0.66	1.00	0.75	1.20	0.88	0.90
1.25	0.81	1.02	0.57	0.91	0.76	1.12	0.89	0.83
1.30	0.82	0.93	0.50	0.82	0.77	1.05	0.90	0.78
1.35	0.83	0.84	0.44	0.75	0.78	0.99	0.92	0.74
1.40	0.84	0.75	0.39	0.68	0.80	0.95	0.94	0.72
1.45	0.85	0.67	0.36	0.63	0.83	0.92	0.97	0.70
1.50	0.86	0.59	0.33	0.58	0.88	0.91	1.00	0.70
1.55	0.86	0.51	0.31	0.54	0.93	0.90	1.05	0.72
1.60	0.86	0.45	0.29	0.50	1.00	0.92	1.10	0.76
1.65	0.86	0.38	0.28	0.47	1.09	0.95	1.18	0.83
1.70	0.86	0.32	0.27	0.45	1.22	1.01	1.30	0.94
1.75	0.87	0.26	0.27	0.43	1.41	1.12	1.47	1.15

Table 2. Source size for  $\nu_z = 1.4$

Port Angle	$\sigma_x$ (mm)	$\sigma_x'$ (mr)
0°	0.50	0.85
20°	0.56	0.89
40°	0.70	1.00

To complete the design, the straight sections must be developed and the vertical tune considered. From Appendix B, if a triplet is used to match the horizontal machine functions, two of the four parameters defining the straight section,  $l_1, l_2, f_1, f_2$ , remain free parameters. Imposing a constraint on the vertical tune leaves only one. Taking  $l_1$  to be the free parameter determined by the cryostat of the dipole and injection considerations, a value of 0.5m is assumed. For  $B=3.5T$  and  $\nu_z=1.4$ , fixing  $\nu_y=1.4$  yields  $l_2=1.22m$ ,  $f_1=.88m$ , and  $f_2=1.79m$ . The resultant orbital circumference is 11.2m.

## Conclusions

The cylindrical design is marginal in its performance with respect to the requirements of x-ray lithographers as stated above. Although coupling can improve the low current source characteristics, current-dependent effects such as anomalous bunch lengthening will certainly degrade performance. The symmetric racetrack, besides allowing independent tune control and relative simplicity in implementation of rf and injection, satisfies the lithography requirements without coupling.

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presented. If  $R$  is used to represent the linear transport matrix that takes  $x$  and  $x'$  from location 1 to location 2, then the betatron and dispersion functions evolve according to

$$\beta_2 = R_{11}^2 \beta_1 - 2 R_{11} R_{12} \alpha_1 + R_{12}^2 \frac{1 + \alpha_1^2}{\beta_1}, \quad (\text{B.1})$$

$$\eta_2 = R_{11} \eta_1 + R_{12} \eta_1' \quad (\text{B.2})$$

Taking locations 1 and 2 to be at the beginning and end, respectively, of the straight section in a symmetric racetrack lattice implies that  $\beta_2 = \beta_1$  and  $\eta_2 = \eta_1$ . Using this and dropping the subscript 1, Eqs. (B.1) and (B.2) become

$$R_{11} = 1 - \frac{\eta'}{\eta} R_{12}, \quad (\text{B.3})$$

$$(R_{11}^2 - 1) \beta - 2 R_{11} R_{12} \alpha + R_{12}^2 \frac{1 + \alpha^2}{\beta} = 0. \quad (\text{B.4})$$

The betatron phase advance through the symmetric straight section can be shown [11] to be given by

$$\sin(\delta\phi) = \frac{R_{12}}{\beta}. \quad (\text{B.5})$$

Combining Eqs. (B.3) and (B.4) to eliminate  $R_{11}$  and substituting into Eq. (B.5) yields, upon trigonometric simplification

$$\tan\left(\frac{\delta\phi}{2}\right) = \frac{\eta'}{\eta} \beta + \alpha. \quad (\text{B.6})$$

This result implies that it is impossible to construct a straight section, regardless of complexity, that allows adjustment of the horizontal tune without affecting the horizontal betatron and dispersion functions in the dipoles.

As a second, more practical example of matching, consider matching a triplet into the straight section of a symmetric racetrack. To achieve a "match", the betatron and dispersion functions are developed from one end of the straight section and the constraint that their derivatives vanish in the center of the straight is imposed. This results in functional relationships between drift lengths, quadrupole focal lengths, and the machine functions desired at the ends of the straight section.

The specific case considered is indicated by the following thin-lens matrix representation  $R$  for the first half of the straight section.

$$R = \begin{pmatrix} 1 & 0 \\ 1/f_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & l_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f_1 & 1 \end{pmatrix} \begin{pmatrix} 1 & l_1 \\ 0 & 1 \end{pmatrix} \quad (\text{B.7})$$

The derivatives of the machine functions evolve according to

$$\alpha_2 = -R_{11} R_{21} \beta_1 + (1 + 2R_{12} R_{21}) \alpha_1 - R_{12} R_{22} \frac{1 + \alpha_1^2}{\beta_1} \quad (\text{B.8})$$

and

$$\eta_2' = R_{21} \eta_1 + R_{22} \eta_1' \quad (\text{B.9})$$

Imposing the condition  $\eta_2' = 0$  and  $\alpha_2 = 0$  reduces equations B.8 and B.9 to

$$f_2 = \frac{f_1}{\left(1 - \frac{f_1}{\eta/\eta' + l_1}\right)} - l_2 \quad (\text{B.10})$$

and

$$f_1 = \frac{(\beta - l_1 \alpha) t - l_1}{(\beta - (l_1 + l_2) \alpha) t - (l_1 + l_2)} l_2 \quad (\text{B.11})$$

where the abbreviation  $t = \tan(\delta\phi/2)$  has been used. The subscript 1 on the machine functions entering the straight section has been dropped.

Eqs. (B.10) and (B.11) describe the correlation between focal lengths and drift space lengths required to achieve a match into the horizontal machine functions inside the dipoles. Two of the parameters are constrained by the match and two remain free.