IMPROVEMENT OF THE DYNAMIC APERTURE IN CHASMAN GREEN LATTICE DESIGN LIGHT SOURCE STORAGE RINGS*

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I. Introduction

The half cell shown in Fig. 1 illustrates a typical Chasman-Green electron or positron storage ring lattice specifically designed for photon beams from undulators and wigglers located in each dispersion-free straight section. The need for a small particle beam emittance requires that the horizontal phase advance per cell should be in the neighborhood of $0.9 \times 2\pi$. Necessary chromaticity correcting sextupoles, S_D and S_F, located in the dispersion straight section introduce non-linear perturbations which limit the dynamic aperture because of amplitude dependent tune shifts. Two families of sextupoles, S_1 and S_2, can be introduced into the dispersion-free region to moderate the more harmful effects of S_D and S_F. The following sections discuss the nature of the perturbations and provide some guidelines for the adjustment of S_1 and S_2.

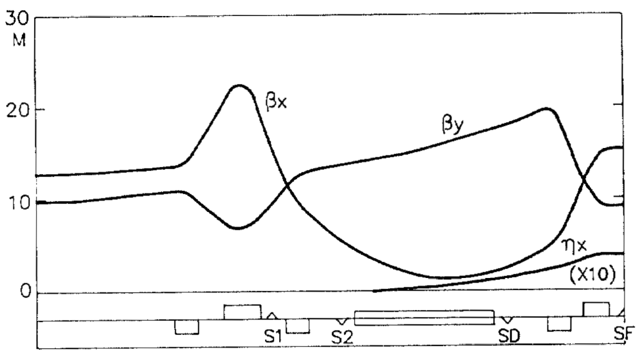

Fig. 1. Lattice and dispersion functions for Chasman-Green half cell. Reflective symmetry about either end.

II. Distortion Functions

The Hamiltonian used to describe the motion of charged particles in a lattice containing sextupole fields may be written as

$$H = \frac{J_x}{\beta_x} + \frac{J_y}{\beta_y} + V(J_x,\ J_y,\ \phi_x,\ \phi_y,\ s) \tag{1}$$

where

$$V=f(s)J_x^{3/2}(\cos 3\phi_x + 3\cos\phi_x)$$

$$-3\bar{f}(s)J_x^{1/2}J_y(2\cos\phi_x + \cos\phi_+ + \cos\phi_-)\ ;$$

$$f(s) = \frac{\sqrt{2}}{12} S_k\delta(s-s_k);\ \bar{f}(s) = \frac{\sqrt{2}}{12}\bar{S}_k\delta(s-s_k)\ ;$$

$$S_k=\left(\frac{\beta_x^{3/2}B''(x)\ell}{B\rho}\right)_k;\ \bar{S}_k=\left(\frac{\beta_x^{1/2}\beta_y B''(x)\ell}{B\rho}\right)_k;$$

$$\phi_\pm = \phi_x \pm 2\phi_y$$

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For V=0, J_x, J_y are constants of the motion. Treating V as a perturbation, one introduces the generating function

$$F(\phi_x,\phi_y,J_{1x},J_{1y},\ s)=\phi_xJ_{1x}+\phi_yJ_{1y}+G(\phi_x,\phi_y,J_{1x},J_{1y},s) \tag{2}$$

to produce new variables

$$\phi_{1z}=\phi_z+G_{J_{1z}}\ ,\ J_{1z}=J_z-G_{\phi_z}\ ,\ G_a\equiv\frac{\partial G}{\partial a} \tag{3}$$

If G satisfies the equation[1]

$$\frac{G_{\phi_x}}{\beta_x} + \frac{G_{\phi_y}}{\beta_y} + G_s + V = 0 \tag{4}$$

the new Hamiltonian is

$$H_1 = \frac{J_{1x}}{\beta_x} + \frac{J_{1y}}{\beta_y} + V_{J_{1x}}G_{\phi_x} + V_{J_{1y}}G_{\phi_y} \tag{5}$$

and J_{1x}, J_{1y} are constants of the motion to the first order in V.

The function that satisfies Eq. (3) is

$$G = -\int_s^{s+C}\frac{ds'}{2}\{f(s')J_{1x}^{3/2}\left(\frac{\sin 3a_x}{\sin 3\nu_x} + \frac{3\sin a_x}{\sin\pi\nu_x}\right)$$

$$-3f(s')J_{1x}^{1/2}J_{1y}\left(\frac{2\sin a_x}{\sin\pi\nu_x} + \frac{\sin\phi_+}{\sin\pi\nu_+} + \frac{\sin a_-}{\sin\pi\nu_-}\right)\} \tag{6}$$

where

$$a_x = \phi_x + \psi_x(s') - \psi_x(s) - \pi\nu_x;$$

$$a_\pm = \phi_\pm + \psi_\pm(s') - \psi_\pm(s) - \pi\nu_\pm;$$

$$\psi_z = \int_0^s\frac{ds}{\beta_z}\ ;\ \nu_z = \int_0^C\frac{ds}{\beta_z}\ ;\ z = x,\ y;$$

$$\psi_\pm = \psi_x \pm 2\psi_y\ ;\ \nu_\pm = \nu_x \pm 2\nu_y;$$

C = cell length

From Eq. (6), one can derive directly

$$G_{\phi_x} = -(2J_{1x})^{3/2}\left(C_3\cos 3(\phi_x+\alpha_+) + C_1\cos(\phi_x+\alpha_1)\right)$$

$$+(2J_{1x})^{1/2}2J_{1y}\left(2\bar{C}\cos(\phi_x+\bar{\alpha})\right. \tag{7}$$

$$\left.+ C_+\cos(\phi_++\alpha_+)+C_-\cos(\phi_-+\alpha_-)\right)$$

$$G_{\phi_y} = (2J_{1x})^{1/2}4J_{1y}\left(C_+\cos(\phi_++\alpha_++\alpha_+)-C_-\cos(\phi_-+\alpha_-)\right) \tag{8}$$

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where typically the various C's and α's are defined as

$$C = \sqrt{A^2+B^2} \; ; \; \tan j\alpha = \frac{A}{B} \; ; \; j = 1,\ 3;$$

and the corresponding A's and B's as

$$B_j(j\psi_x) = \sum_k \frac{S_k}{16\sin j\pi\nu_x} \cos j\left(|\psi_x(k)-\psi_x|-\pi\nu_x\right) \quad j=1,\ 3$$

$$\bar{B}(\psi_x) = \sum_k \frac{\bar{S}_k}{16\sin \pi\nu_x} \cos\left(|\psi_x(k)-\psi_x|-\pi\nu_x\right)$$

$$B_\pm(\psi_\pm(\psi_\pm) = \sum_k \frac{\bar{S}_k}{16\sin \pi\nu_\pm} \cos\left(|\psi_\pm(k)-\psi_\pm|-\pi\nu_\pm\right) \; ;$$

with $A(Z) = B'(Z)$.

These functions have been called Distortion Functions.[2] They are determined by the lattice structure and the distribution of sextupoles. The magnitude of the C's determines the maximum variations of amplitudes through the lattice about a constant value determined by J_{1x}, J_{1y}. The first three distortion functions, B_3, B_1, $\bar{B}$ for the lattice of Fig. 1 with the chromaticity correcting sextupoles turned on are shown in the top half of Fig. 2.

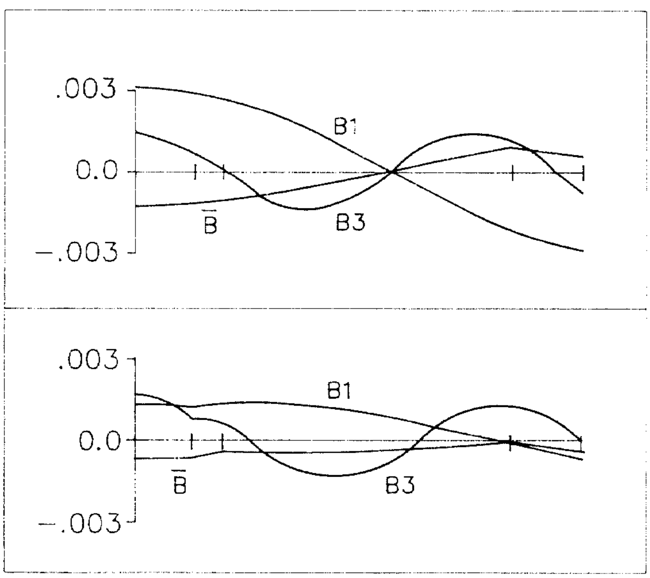

Fig. 2. Three distortion functions before (top) and after (bottom) improvement by turning on the harmonic suppression sextupoles. The chromaticity correcting sextupoles are at the right. The harmonic suppression sextupoles are at the left.

The amplitude dependent tune shifts are determined by the magnitude of the B functions at the sextupole multiplied by the corresponding sextupole strengths. This can be shown as follows:

$$\Delta\nu_z = \frac{1}{2\pi} \int_0^C \frac{\partial}{\partial J_{1z}} < G_{\phi_x} V_{J_{1x}} + G_{\phi_y} V_{J_{1y}} > ds \tag{9}$$

where the average is taken over the particle phases. Using Eqs. (7) and (8), one arrives at

$$\Delta\nu_x = \frac{1}{4\pi} \sum_k \left[-J_{1x}S_k(B_3+3B_1)_k + 2J_{1y}\bar{S}_k(2B_1-B_++B_-)_k\right] \tag{10}$$

$$\Delta\nu_y = \frac{1}{4\pi} \sum_k \left[2J_{1x}\bar{S}_k(2B_1-B_++B_-)_k - J_{1y}\bar{S}_k(4\bar{B}+B_++B_-)_k\right] \tag{11}$$

To reduce these shifts additional sextupoles must be added to the lattice in such a way as to either reduce the B functions at all sextupoles or to achieve cancellations by taking advantage of their phases. Since the tune shifts depend quadratically on the sextupole strengths, this is not easy to do in practice. The solution is easier if one knows which harmonics are causing most of the problem.

III. Harmonic Expansion

The expansion of V into harmonic components is accomplished by means of five sets of coefficients having the general form

$$\frac{Rf}{\sqrt{8}} e^{i(\psi-\nu\theta)} = \sum_m A_m e^{-i(m\theta-\alpha)} \tag{12}$$

$$R d\theta = ds \qquad 2\pi R = C$$

The result (using a reflective symmetry point as reference) is

$$V = \frac{\sqrt{8}}{R} \sum_m \Big[J_x^{3/2}(A_{3m}\cos Q_{3m}+3A_{1m}\cos Q_{1m}) \\ -3J_x^{1/2} J_y(2B_{1m}\cos Q_{1m}+B_{+m}\cos Q_{+m}+B_{-m}\cos Q_{-m})\Big] \tag{13}$$

where

$$Q_{jm} = j(\phi_x-\psi_x)+(j\nu_x-m)\theta \quad j = 1,3;$$

$$Q_{\pm m} = \theta_\pm-\psi_\pm+(\nu_\pm-m)\theta;$$

$$A_{jm} = \sum_k \frac{S_k}{48\pi} \cos[j(\psi_x-\nu_x\theta)+m\theta]_k \; ;$$

$$B_{1m} = \sum_k \frac{\bar{S}_k}{48\pi} \cos[\psi_x-\nu_x\theta+m\theta]_k \; ;$$

$$B_{\pm m} = \sum_k \frac{\bar{S}_k}{48k} \cos[\psi_\pm-\nu_\pm\theta+m\theta]_k \; ;$$

Substitution of the components into Eq. (6) yields the following harmonic expansion for the Distortion Functions;

$$B_j = w_m \frac{3A_{jm}}{j\nu_x - m} \cos[j(\psi_\nu-\nu_x\theta)+m\theta] \quad j=1,3 \tag{14}$$

$$\bar{B} = \sum_m \frac{3B_{1m}}{\nu_x - m} \cos[\psi_x-\nu_x\theta+m\theta]$$

$$B_\pm = \sum_m \frac{3B_{\pm m}}{\nu_\pm - m} \cos[\psi_\pm-\nu_\pm\theta+m\theta]$$

(K. Y. Ng[3] started from this form of the distortion Functions and worked backwards to arrive at the Collins formula given by Eqs. (7) and (8)). Finally by direct substitution of these expansions into Eqs. (10) and (11), and identification of the sums over k, one arrives at the harmonic expansion of the amplitude-dependent tune shifts

$$\Delta\nu_x = M_{11}J_{1x}+M_{12}J_{1x} \tag{15}$$

$$\Delta\nu_y = M_{21}J_{1x} + M_{22}J_{1y}$$

with $$M_{11} = -36 \sum_m \left[\frac{A_{3m}^2}{3\nu_x - m} + \frac{3A_{1m}^2}{\nu_x - m}\right]$$

$$M_{12} = M_{21} = 72 \sum_m \left(\frac{2B_{1m}A_{1m}}{\nu_x - m} - \frac{B_{+m}^2}{\nu_+ - m} + \frac{B_{-m}^2}{\nu_- - m}\right)$$

$$M_{22} = -36 \sum_m \left[\frac{4B_{1m}^2}{\nu_x - m} + \frac{B_{+m}^2}{\nu_+ - m} + \frac{B_{-m}^2}{\nu_- - m}\right]$$

IV. Predictions and Improvements by Harmonic Suppression

The lattice shown in Fig. 1 has tunes of $\nu_x = 0.88$, $\nu_y = 0.36$ per cell. With the chromaticity correcting sextupole turned on, the calculated amplitude dependent tune shifts per cell are

$$\Delta\nu_x(\text{units } 10^{-6}) = 7.18\ N_x^2 + 1.5\ N_y^2$$

$$\Delta\nu_y(\text{units } 10^{-6}) = 3.02\ N_x^2 + 0.98\ N_y^2$$

$$N_x^2 = \frac{2J_x}{\varepsilon},\ N_y^2 = \frac{4J_y}{\varepsilon},\ \varepsilon = \text{natural emittance}$$

Tracking programs show that the limit of stability on the median plan is at $N_x = 41$, where the tune per cell approaches 1. The tune shift increases rapidly as the one approaches the fundamental instability.

In fact, it is possible to predict the total dynamic aperture for this lattice using only the harmonic components A_{33}, A_{11}, and B_{11}. It can be shown that for constant N_y^2, the horizontal stable aperture should extend from N_{x1} to N_{x2} where

$$N_{x1} = -\frac{X_o}{3}(F+1)\ ; \qquad (16)$$

$$N_{x2} = \frac{X_o}{3}(2F-1);$$

$$X_o = \frac{(\nu_x - 1)}{2\sqrt{\varepsilon}(3A_{11} + A_{33})}$$

$$F = \sqrt{1 + \frac{36B_{11}\varepsilon N_y^2(3A_{11} + A_{33})}{(\nu_x - 1)^2}}$$

The prediction of Eq. (16) and the dynamic aperture obtained by tracking are shown in Fig. 3.

The amplitude dependent tune shifts for the lattice with the chromaticity correcting sextupoles turned on is much too large for stability in a ring containing uncorrected orbit errors. Inspection of Eqs. (15) and (16) suggest that the most efficient use of the harmonic suppression sextupoles S_1 and S_2 is to reduce A_{11} and B_{11}. It turns out that if these sextupoles are tuned such that A_{11} and B_{11} are reduced to about 30% of their original values, the coefficient of J_{1x} in Eq. (10) is reduced to zero. The resultant increase in dynamic aperture is shown in Fig. 3.

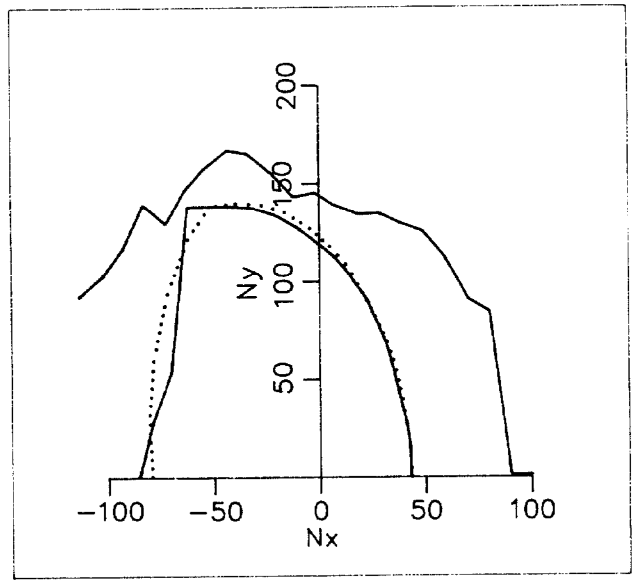

Fig. 3. Inner curves are the measured (solid) and calculated (dotted) dynamic aperture without harmonic suppression sextupoles. Outer is the measured dynamic aperture with the harmonic suppression sextupoles.

(Eq. (10) is no longer valid, since with A_{11} and B_{11} reduced, other higher order resonances become effective). The corresponding decrease in the B_1 and B distortion function is shown in the lower half of Fig. 2. The amplitude dependent tune shifts are reduced to

$$\Delta\nu_x(\text{units } 10^{-6}) = 0.00\ N_x^2 + 0.18\ N_y^2$$

$$\Delta\nu_x(\text{units } 10^{-6}) = 0.36\ N_x^2 + 0.24\ N_y^2$$

The above analysis demonstrates that if the causes of stability limitations are known, the corrections necessary to improve stability can be calculated. The results have also been tested using dynamic aperture searches for a 40-cell ring with uncorrected orbit distortion errors. The results show that, because attention has been given to reducing the amplitude tune shifts, the improvement provided by the harmonic sextupoles is maintained.

References

[1] E. Courant, R. Ruth and W. Weng, "Stability in Dynamical Systems I," Summer School on High Energy Particle Accelerators, Upton, NY, July 1983, SLAC PUB-3415(August 1984).

[2] T. L. Collins - Fermilab TM-128 (1984).

[3] K. Y. Ng, Fermilab Report TM-1281 (1984).