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CANTED-POLE TRANSVERSE GRADIENTS IN PLANAR UNDULATORS

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Abstract

Two-plane focusing in planar undulators can provide significant emittance-acceptance enhancement for free-electron laser applications. A quadrupole focusing field is superimposed by angularly canting the magnets in a pure rare-earth cobalt (REC) undulator or by canting the pole pieces in a hybrid (permanent magnets plus steel) undulator. Important differences exist between canting in the two types of undulators. For the pure-REC undulator, direct field calculations are directly applicable but a this-large calculations are directly applicable, but a thin-lens model of the pole pieces is more amenable to calculation for the hybrid undulator. Measurements are presented which verify the magnitude of the gradients and focusing obtained by canting in both undulator trues undulator types.

Introduction

Electron beam emittance requirements for free-electron lasers (FELs) are quite stringent [1] at visible wavelengths, but the emittance requirements can be substantially relaxed if the undulator focuses in both planes. Angular canting of the undulator poles is one promising scheme to produce this equal two-plane focusing in planar undulators. This is especially important in light of the difficulty in producing high e-beam current with small emittance using linear accelerators.

There are several methods to produce two-plane focusing in planar undulators. One technique involves the superposition of a quadrupole field upon the undulator field. This can be achieved either with external quadrupoles or by angular rotation of the undulator magnets. The former method allows a readily adjustable focal strength but relies upon precise alignment to ensure that the undulator and focusing elements are coaxial. The latter method guarantees that the focal properties are properly aligned and could be used with hybrid undulators which do not allow linear superposition of external fields. Angular canting can involve either longitudinal carting [2] of those magnets with magnetization vector parallel to the e-beam propagation direction or transverse canting [3] of the magnets with magnetization vector perpendicular to the

propagation direction or transverse canting [3] of the magnets with magnetization vector perpendicular to the e-beam. The latter form allows closest packing of the magnets with no degradation of the on-axis magnetic field strength and has been demonstrated [3] in the geometry of the first Spectra Technology undulator, shown in Fig. 1. The alternating cant angles together with the alternating field orientations at each pair of canted magnets produce a non-alternating transverse field gradient along the undulator bore. The cant angle for two-plane focusing is modest, causing minor perturbations of the undulator field. Resonance between the synchrotron motion of electrons transped in the ponderomotive potential well and transverse betatron motion may result in loss of

electrons trapped in the ponderomotive potential well and transverse betatron motion may result in loss of FEL interaction strength. The betatron-synchrotron coupling may be enhanced [4] by quadrupole focusing in a planar undulator. Modeling shows, however, that the loss of interaction is not severe [1] in short-wavelength FEL oscillators. Other investigators [5] are examining alternative pole-tip shaping to produce sextupolar focusing, for application in high-gain amplifier geometries in which

shaping to produce sextupolar focusing, for application in high-gain amplifier geometries in which the coupling is significant. As part of the Boeing/Spectra Technology free-electron laser program, a canted-pole design has been developed for two-plane focusing in a planar hybrid undulator. Measurements in a scaled-size mockup verify the magnitude of the gradients and focusing obtained by canting. The technique has been used on the Tapered Hybrid Undulator (THUNDER) and performance verified [6,7]. Important differences exist between canting in hybrid and pure-REC types of undulators. Section II presents and contrasts the equations and scaling of the canting for both types of undulators, and results are summarized in undulators andresults are summarized Section III.



Figure 1. End view of first Spectra Technology undulator showing canted magnet configuration. The e-beam is directed into the page.

Canted-Pole Physics

Major differences exist in the formulation of transverse canting for pure-REC and hybrid undulators. Superposition of the fields of individual magnets is the most direct approach for determining the cant angle in the pure-REC undulator, whereas a thin-lens model, incorporating the effective longitudinal magnetic length of the pole pieces, is more useful for a hybrid geometry. Measurements are shown which confirm the magnitude of the cant angles required to provide a given focal strength in each undulator type.

Two-Plane Focusing

A planar undulator with two-plane focusing will have a field distribution which is closely approximated near the axis by the form

$$\begin{split} B_{\mathbf{x}} &= -B_{\mathbf{o}} \left(\mathbf{a}\mathbf{y} + \mathbf{b}\mathbf{y} \cos 2\mathbf{k}_{\mathbf{w}}\mathbf{z} \right) \\ B_{\mathbf{y}} &= B_{\mathbf{o}} \left(\cos \mathbf{k}_{\mathbf{w}}\mathbf{z} - \mathbf{a}\mathbf{x} - \mathbf{b}\mathbf{x} \cos 2\mathbf{k}_{\mathbf{w}}\mathbf{z} \right) \\ B_{\mathbf{z}} &= -B_{\mathbf{o}} \left(\mathbf{k}_{\mathbf{w}}\mathbf{y} \sin \mathbf{k}_{\mathbf{w}}\mathbf{z} - 2\mathbf{b}\mathbf{k}_{\mathbf{w}}\mathbf{y}\mathbf{x} \sin 2\mathbf{k}_{\mathbf{w}}\mathbf{z} \right) , \end{split}$$
[1]

where $B_{o}^{}$ is the peak magnetic field amplitude, $k_w = 2\pi/\lambda_w$ is the undulator wavenumber, and λ_w is the undulator wavelength. The coefficient "a" describes the constant transverse gradient in By produced by the canting; this term provides the two-plane focusing. The coefficient "b" characterizes an off-axis second-harmonic term which is a by-product of canting. These two quantities are functions of various geometrical factors, including linear dependences on the cant angle, θ . The field distribution is qualitatively similar for the pure-REC and the hybrid undulator, but the relative magnitudes of the "a" and "b" terms differ dramatically. In either case, focusing is introduced in the nominally free-expanding wiggle plane if the canting is arranged so that the coefficient

$$a = \frac{1}{B_{o}^{\lambda} w} \int_{z}^{z+\lambda} \frac{\partial B_{y}}{\partial x} dz, \qquad [2]$$

is positive. The focal strength is given by

$$k_{\beta x}^{2} = 2^{1}/2 k_{\beta 0}a$$
 [3]

where $k_{\beta x}$ is the x-plane wavenumber for the betatron where $k_{\beta\chi}$ is the x-plane wavenumber for the betation motion [8] resulting from the canting, $k_{\beta 0} = a_w k_w / \gamma$ is the nominal y-plane betatron wavenumber in the absence of canting, $a_w = eB_0\lambda_w/\pi mc^2 2^3/^2$, and γ is the electron energy in units of the rest mass mc². We note that the sign of "a" depends upon the direction of canting and is positive provided that the canting is done as shown schematically in Fig. 1. As focusing is introduced in the wiggle plane, the focal strength in the nominally focusing plane is reduced according to

reduced according to

$$k_{\beta y}^{2} = k_{\beta 0}^{2} - k_{\beta x}^{2} . \qquad [4]$$

For equal emittance in each plane, the best spatial match to a cylindrically symmetric photon beam is given by equal two-plane focusing. In this case,

$$k_{\beta x} = k_{\beta y} = \frac{a_{w}^{k} k_{w}}{2^{1/2} \gamma}$$
 [5]

and "a" must have the value

$$\mathbf{a} = \frac{\mathbf{a}_{\mathbf{w}}^{\mathbf{k}}\mathbf{w}}{2^{3}/^{2}\gamma} .$$
 [6]

Pure-REC Undulators

<u>Pure-REC Undulators</u> In the pure-REC undulator, determination of the cant angle required for equal two-plane focusing is simplified by the fact that the field distribution can be calculated directly by superposing the fields from the individual magnets. It is imperative that the contribution of the off-axis second harmonic term be included in the analysis. Examination of Eq. 1 shows that the transverse gradient, $\partial B_y/\partial x$, is an oscillatory function of z. Since "b" typically exceeds "a" in magnitude, the gradient actually changes sign between each pair of canted magnets. The "a" term results only from the finite length of the magnet bars, which allows some flux to return around the ends of the bars. The end-effect dependence of the focusing in pure-REC undulators is clearly seen by examination of the "a" and "b" coefficients as a function of magnet bar length. The primary result of canting is off-axis second harmonic; the coefficient "b" is virtually independent of the bar length. But the constant gradient term "a" decreases exponentially with the bar length. Table 1 lists the pertinent parameters for the first pure-REC Spectra Technology undulator [9]. The undulator is 2.3 m long, has 100 periods, and is constructed of SmCo5 permanent magnets. This undulator has demonstrated large e-beam energy

constructed of SmCo₅ permanent magnets. This undulator has demonstrated large e-beam energy extraction [10] using a 9 percent resonant-energy taper. The taper is achieved by decreasing the period 13 percent and the peak field 8 percent along the length of the undulator, at fixed gap. Following the successful extraction experiments, the magnets were canted as shown in Fig. 1. As indicated by Table 1, a gradually decreasing cant angle is required to provide equal two-plane focusing along the entire length of the undulator, because of the taper. But for simplicity, a uniform cant angle of 2.7 degrees was selected. This gives approximately equal focusing along the full length, on the average. The focal

			Table 1		
PARAMETERS	$0\mathbf{F}$	FIRST	SPECTRA	TECHNOLOGY	UNDULATOR

Length	Lw	2.3 m	
Full Gap	g	1.27 cm	
Magnet Bar Dimensions		0.56×0.83	x 5.0 cm ³
Remanent Field	Br	8800 G	
Resonant Gamma	γ_{r}	37.9	
	-	Entrance	Exit
Undulator Period	λ _w	2.54 cm	2.22 cm
Peak Field	В	2.64 kG	2.44 kG
Cant Angle for Equal Two-Plane Focus	θ	3.1 deg.	2.4 deg.

strength was measured using pop-in fluorescent screens located within the undulator. Approximately equal betatron oscillations were observed to occur in both planes when the e-beam was steered off-axis at the undulator entrance, thus demonstrating [3] canted-magnet two-plane focusing.

Hybrid Undulators

Hybrid Undulators The canting of a hybrid undulator is fundamentally different from a pure-REC undulator because of the presence of the highly permeable pole pieces. The two-dimensional modeling often used for designing hybrid undulators is not applicable to canting since it is inherently a three-dimensional effect. Several simplistic models can provide insight into the anticipated scaling of the required cant angle with gap and pole thickness but, unlike the pure-REC case, no simple analytic formulation exists. Whereas an REC magnet acts as fixed magnetic charge sheets, the permeable pole pieces in a hybrid geometry behave more like magnetic equipotential surfaces. The hybrid system is complicated in that the operating point of the permeable material is often not far from saturation. The approximation of true equipotential surfaces is rigorously valid only in the limit that the material is far from saturation. Making the equipotential surface

Making the equipotential surface approximation, a simple two-dimensional model can be constructed. Two equipotential surfaces are tilted by a full angle 2θ . The gradient of such a system which has a nominal magnetic field B₀ and full gap g is

$$\frac{dB}{dx} = B_{o} \frac{2\theta}{g}$$
 [7]

to first order in θ and g. An additional model for the expected gradient can be constructed from the empirical formula for the peak magnetic field of a hybrid undulator [11].

$$B_{o}(\text{tesla}) = 3.33 \exp\left[\frac{-g}{\lambda_{w}}\left[5.47 - 1.8 \frac{g}{\lambda_{w}}\right]\right] . [8]$$

By differentiating Eq. 8 with respect to the gap, an expression can be obtained for the gradient. For undulator designs with optimized gap/wavelength ratio, agreement is obtained with Eq. 7. A tapered hybrid undulator, THUNDER, has been built and measured at Spectra Technology. In order to determine the proper cant angle for equal two-plane focusing, we initially constructed a small four-period hybrid-undulator mockup employing steel pole pieces and surplus REC magnets. This mockup allows evaluation of the transverse gradient for various cant angles and gaps in the presence of saturation and three-dimensional effects. This short undulator is sized to have the nominal gap to undulator-wavelength ratio of THUNDER but is built at twice scale to allow greater adjustment. A Hall probe is used to map the magnetic field at various longitudinal and transverse magnetic field at various longitudinal and transverse positions.

For the provided for the electron beam, and the densities for the product of the electron product of the electron product of the two-dimensional canted plane model. The gradient is constant to better than one percent over the maximum possible excursion of the electron beam, and

is constant to better than one percent over the maximum possible excursion of the electron beam, and the deviations from linearity are as anticipated based on the finite pole length. Measurements at various gaps confirm that the gradient at the center of the pole piece is independent of the pole length as long as the end effects are small. By constructing a two-dimensional magnetic field map of the canted hybrid undulator, the gradient that is available for focusing may be determined. Figure 3 is the measurement of the gradient as a function of longitudinal position along the undulator for a cant angle $\theta = 25.4$ mrad. The schematic under the curve shows the relative location of the permeable pole pieces. As is expected, the focusing is concentrated under the canted pole pieces. This substantiates use of a thin-lens model when examining the transverse gradients in a hybrid undulator. Each of the pole-piece pairs can be considered a thin dipole magnet with a gradient having an effective length and strength, and isolated from its neighbors. Figure 3 also suggests that, consistent with the thin-lens model, the ratio of the local peak gradient



Figure 2. Transverse field profile of hybrid canted undulator. Cant angle $\theta = 25.4$ mrad, dB/dx = 358 G/cm.

to the total average gradient should only be a function of the pole thickness to undulator-wavelength ratio and independent of the pole length. This is in sharp contrast to the canting of the pure-REC undulator where the effective focusing gradient is purely an end effect. Additionally, the local gradient is always positive in the hybrid. This indicates that a \approx b, again in contrast to the pure-REC undulator where a < b except for extremely short magnet length.

pure-REC undulator where a < b except for extremely short magnet length. Figure 4 is a plot of the measured peak gradient as a function of angle for three different gaps. Shown as well are values predicted by a simple two-dimensional model. The measurements show scaling of the gradient with gap and angle which is consistent with the simple model. Possible saturation effects were investigated and found to be negligible for the design regions of interest. Table 2 lists the measured parameters of THUNDER. The cant angle of 6.75 mrad is consistent

design regions of interest. Table 2 lists the measured parameters of THUNDER. The cant angle of 6.75 mrad is consistent with Eq. 6 for equal two-plane focusing in the undulator. The angle is determined by scaling, consistent with Eqs. 7 and 8, to the design period and gap. Since the final design is scaled down from the mockup, the effective width of the individual pole pieces remains proportionally the same and the ratio of the peak to average focusing is preserved. A subset of poles in each undulator subsection is canted in order to make the cant angle large enough so that machining errors will cause insignificant deviations.

Summary and Conclusions

The utility of transverse canting for providing two-plane focusing has been demonstrated in both hybrid and pure-REC undulators. In many situations, this additional focusing enhances FEL operation by increasing the emittance acceptance of the undulator. These canting techniques may also find application in gain-expanded transverse-gradient undulators [12] for storage ring FELs or in other situations where two-plane focusing or gradients are required for specific undulator applications.

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Figure 3. Measured transverse gradient of canted hybrid undulator, $\theta = 25.4$ mrad. The schematic indicates the location of the pole pieces.



Figure 4. Peak transverse gradient of the canted hybrid undulator as a function of cant angle. Dashed lines are corresponding 2-D model.

 Table 2

 PARAMETERS OF THUNDER HYBRID UNDULATOR

Total Length	L,	5 m
Full Gap at Entrance	g	0.48 cm
Peak Field On Axis	B	10.1 kG
Resonant Gamma	γ_r	248
Undulator Period	λ	2.18 cm
Cant Angle for [*] Equal Two-Plane Focus	θ	6.75 mrad
Energy Taper	$\Delta \gamma_r / \gamma_r$	Variable, 0+12%

* 7.5 of 22 periods canted/subsection

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