GAIN OF AN FEL WITH AN UNMATCHED ELECTRON BEAM*

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Abstract

The gain, phase shift, wavefront curvature and radius of the radiation envelope in a free-electron laser amplifier are obtained in the small signal regime. The electron beam is assumed to have a Gaussian density distribution in the transverse direction. When the electron beam envelope is modulated in space, the optical spot size oscillates with an almost identical wavelength but is delayed in phase. In the case of small amplitude long wavelength betatron modulation of the electron beam envelope, generation of optical sidebands in wave number space has a negligible effect on the dispersion characteristics of the primary wave.

Introduction

A well-known feature of the free-electron laser (FEL) is that the refractive index of the medium is a complex function and hence the radiation is amplified and to some extent focused in the vicinity of the electron beam [1,2]. It may then be possible for the electron and radiation beams to interact over an extended length along the wiggler, with the diffractive tendency being compensated by the FEL interaction, thereby enhancing the efficiency of the process.

Considerable progress has been made in studying this process by several authors [3-8]. The purpose of this paper is to apply the formalism of the Gaussian-Laguerre modal source dependent expansion (SDE) of Ref. 8 to examine the propagation and guiding of the optical wave in an amplifier operating in the exponential gain regime, when the envelope of the electron beam is modulated along the wiggler.

Mathematical Formulation

For a planar wiggler, it is appropriate to assume a linearly polarized radiation vector potential

$$\underline{A} = (1/2)\, A(r,\theta,z)\exp\left[i\left(\frac{\omega z}{c} - \omega t\right)\right]\underline{e}_x + \text{c.c.},$$

with angular frequency ω and complex amplitude A. In the slowly varying envelope approximation, the wave equation reduces to

$$\left(\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{2i\omega}{c}\frac{\partial}{\partial z}\right)a = S(r,\theta,z), \quad (1)$$

where $a = |e|A/m_oc^2$, and the source function is given by

$$S(r,\theta,z) = -\frac{8\pi|e|}{m_oc^3}\left\{J_x(r,\theta,z)\exp\left[-i\left(\frac{\omega z}{c} - \omega t\right)\right]\right\}_{slow}. \quad (2)$$

Here e is the charge on an electron of (rest) mass m_o, $J_x(r,\theta,z)$ is the current density and $\{\ \}_{slow}$ indicates that only the spatially and temporally slow part of the quantity in braces is to be retained.

The basic premise of the work presented herein is that the radiation field is azimuthally symmetric and the vector potential is expressible as:

$$a\,(r,\theta,z) = \sum_{m=o}^{\infty} a_m(z)D_m(\xi,z), \quad (3)$$

with $D_m = L_m(\xi)\exp\{-[1-i\alpha(z)]\xi/2\}$, where $\xi = 2r^2/r_s^2(z)$, $r_s(z)$ is related to the radiation spot size, $\alpha(z)$ is proportional to the curvature of the wavefront, and $L_m(\xi)$ is the Laguerre polynomial of order m.

When the lowest order mode dominates one finds that

$$\left(\frac{\partial}{\partial z} + A_o\right)a_o \simeq -i\,F_o, \quad (4)$$

and the spot size and wavefront curvature evolve via

$$\frac{d}{dz}r_s - \frac{2c\alpha}{\omega r_s} = -r_s\left(\frac{F_1}{a_o}\right)_I, \quad (5a)$$

$$\frac{d}{dz}\alpha - 2(1+\alpha^2)\frac{c}{\omega r_s^2} = 2\left[\left(\frac{F_1}{a_o}\right)_R - \alpha\left(\frac{F_1}{a_o}\right)_I\right], \quad (5b)$$

where

$$A_o = \frac{1}{r_s}\frac{d}{dz}r_s + i\left[(1+\alpha^2)\frac{c}{\omega r_s^2} - \frac{\alpha}{r_s}\frac{d}{dz}r_s + \frac{1}{2}\frac{d}{dz}\alpha\right],$$

the F's are given by the following overlap integral:

$$F_m(z) = \frac{c}{2\omega}\int_o^{\infty} d\xi\, S(\xi,z)D_m^*(\xi,z), \quad (6)$$

and the label R (I) indicates the real (imaginary) part.

Noting that $L_o(\xi) = 1$, the normalized vector potential is seen to be given by [Eq. (3)]

$$a(r,\theta,z) \simeq a_o(z)\exp\left\{-[1-i\alpha(z)]\frac{r^2}{r_s^2(z)}\right\}, \quad (7)$$

where, in the exponential gain, small-signal regime,

$$a_o(z) \simeq a(o)\exp\left\{i\int_o^z dz_1\left[\Delta k(z_1) - i\Gamma(z_1)\right]\right\}. \quad (8)$$

Assuming the electron beam profile to be given by

$$n_b(z) = n_{bo}\left[\frac{r_{bo}}{r_b(z)}\right]^2\exp\left[-\frac{r^2}{r_b^2(z)}\right], \quad (9)$$

where $r_b(z)$ is the electron beam radius at z and n_{bo} is the beam density at $r_b(z) = r_{bo}$, the source term in Eq. (1) may be readily evaluated to obtain

$$S(r,z) = f_B^2\frac{\omega_{bo}^2}{2\gamma^3c^2}\left[\frac{r_{bo}}{r_b(z)}\right]^2\exp\left(-\frac{r^2}{r_b^2}\right)\frac{\omega\, k_w a_w^2 a}{c(\Delta k - i\Gamma)^2}, \quad (10)$$

where the vector potential of the planar wiggler of periodicity $2\pi/k_w$ is given by

$$\underline{A}_w = A_w\cos(k_wz)\underline{e}_x,$$

$$a_w = |e|A_w/m_oc^2,$$

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γ is the relativistic mass factor, f_B is the usual difference of Bessel functions, $f_B = J_o(\zeta) - J_1(\zeta)$, $\zeta = (1/4)a_w^2/[1 + (1/2)a_w^2]$, and

$$\omega_{bo} = \left(4\pi|e|^2 n_{bo}/m_o\right)^{1/2}$$

is the plasma frequency of the electron beam with density n_{bo}.

Substituting Eqs. (8) and (10) into Eq. (6) and making use of Eqs. (4) and (5), it is simple to show that the equations reduce to

$$\frac{d\alpha}{d(k_w z)} = 2(1+\alpha^2)\left(\frac{ck_w}{\omega}\right)\frac{1}{(k_w r_s)^2} + 2\left[\left(\frac{F_1}{k_w a_o}\right)_R - \alpha\left(\frac{F_1}{k_w a_o}\right)_I\right], \tag{11a}$$

$$\frac{d\left(k_w r_s\right)^2}{d\left(k_w z\right)} = 4\alpha\left(\frac{ck_w}{\omega}\right) - 2\left(\frac{F_1}{k_w a_o}\right)_I \left(k_w r_s\right)^2, \tag{11b}$$

$$\frac{\Delta k}{k_w} - i\frac{\Gamma}{k_w} + 2\left(\frac{ck_w}{\omega}\right)\frac{1-i\alpha}{\left(k_w r_s\right)^2} + 2\left(\frac{F_1}{k_w a_o}\right)\left[1 + \left(\frac{r_b}{r_s}\right)^2\right] = 0, \tag{11c}$$

where

$$\frac{F_1}{k_w a_o} = f_B^2\left(\frac{\omega_{bo}}{ck_w}\right)^2\left[\frac{r_{bo}}{r_b(z)}\right]^2\frac{a_w^2}{2\gamma^3}\frac{\left(r_b/r_s\right)^2}{\left[1+2(r_b/r_s)^2\right]^2} \times \left(\frac{\Delta k}{k_w} - i\frac{\Gamma}{k_w}\right)^{-2}. \tag{11d}$$

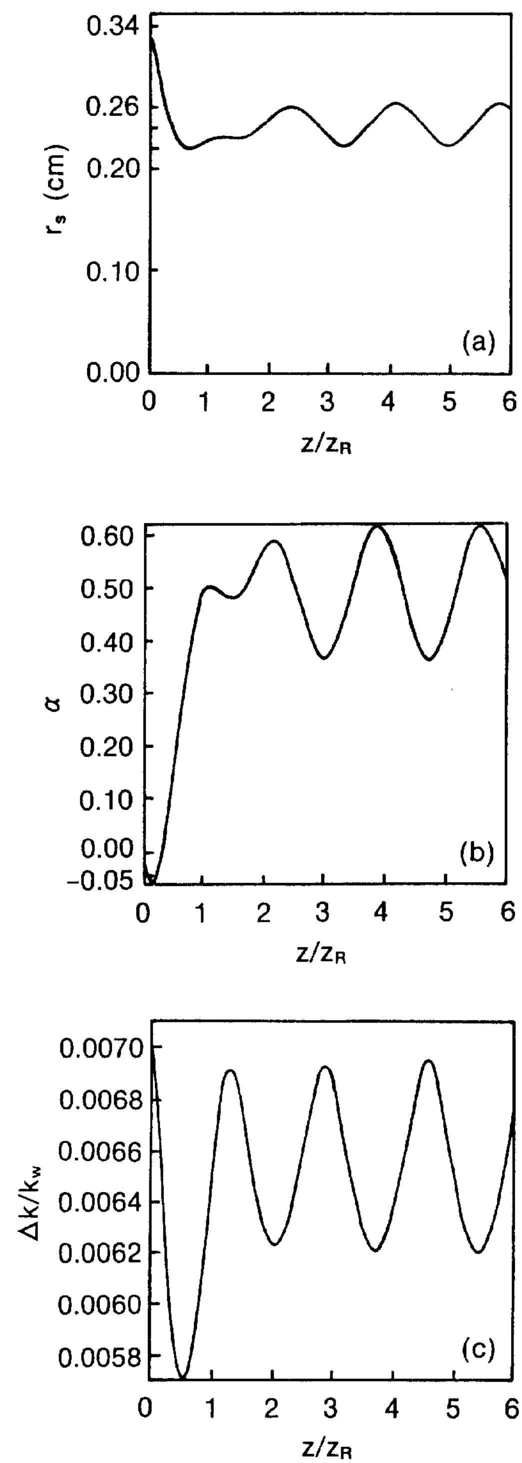
Numerical Results

Figure 1 presents the results for a case where the electron beam is not matched; i.e., the envelope of the electron beam is modulated:

$$r_b(z) = r_{bo} + \delta r_b \sin(k_\beta z), \tag{12}$$

where δr_b is the amplitude of the modulation and for simplicity k_β is chosen to be equal to the betatron wave number [9] $k_w a_w/(\sqrt{2}\gamma\beta_z)$, neglecting self-fields [10]. β_z is the beam speed along the wiggler axis normalized to c. The parameters, typical of the Advanced Test Accelerator experiment at LLNL, are $I_b = 2$ kA, $r_{bo} = 0.3$ cm, $\gamma = 100$, $2\pi/k_w = 8$ cm, $a_w = 1.72$, $r_s(z=0) = 0.35$ cm. In Fig. 1, where $\delta r_b/r_{bo} = 0.1$, it is observed that the optical spot size follows the modulations in the electron envelope apparently identically. Specifically, a number of cases were examined with $\delta r_b/r_{bo}$ up to 0.4. In all cases the electron and optical beams oscillate with almost identical wavelength, although the radiation beam appears to lag behind in phase. However, defining the modulation depth $\Delta = [(r)_{max} - (r)_{min}]/[(r)_{max} + (r)_{min}]$, it is found from Fig. 1(a) that $\Delta_s = 0.087$ whereas, from Eq. (12), $\Delta_b = \delta r_b/r_{bo} = 0.1$. Although the modulation depth of the electron beam differs from that of the radiation beam, it is found that Δ_s increases in proportion to δr_b.

More generally, allowing for the defocusing effect of self-fields, there is always the possibility of a small amplitude ripple on the electron beam envelope and hence on the radiation beam envelope. It is possible to examine the generation of sidebands due to the ripple in a simplified model and it is found that they have, for typical cases, an insignificant effect on the linear dispersion characteristics of the primary optical wave, as implicitly assumed by employing the source term in Eq. (10) in the present case.



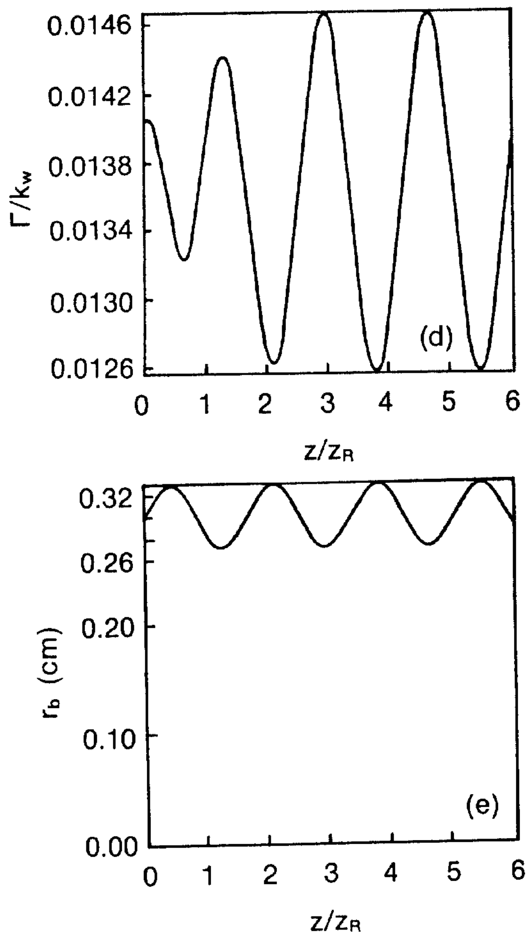


Fig. 1. Spot size (r_s), α, phase shift (Δk), gain (Γ), and radius of electron beam (r_b) vs. distance along wiggler.

Conclusion

Based on the results presented herein, it is found that when the electron beam envelope is modulated in space, the optical beam oscillates on the same spatial scale.

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