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ACHROMATIC DISSIPATIVE FOCUSING

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ABSTRACT

The energy loss due to synchrotron radiation during beam-beam collisions can be exploited to obtain an achromatic focusing lens. We consider the successive collisions between a relativistic electron and two short bunches with the same number of particles: the first is a bunch of electrons and the second of positrons. Owing to the energy loss during the first collision, the two successive radial kicks do not compensate each other and, in thin lens approximation, there is a residual focusing effect which is independent of the initial electron energy. A proper choice of the radial density profile of the two bunches leads to a linear achromatic focusing. This principle could be applied to reduce the final spot size in future electron-positron colliders with very large energy spread.

1. INTRODUCTION

There is a close analogy between light-ray optics and particle trajectories in high energy accelerators and storage rings. Magnetic quadrupoles in high energy beam optics play the role of glass lenses in light optics, while particle energy is the counterpart of light colour (or frequency). The dependence of the focal length of a magnetic quadrupole on particle energy gives rise to chromatic aberrations similar to those affecting conventional optical systems as a consequence of the dispersive properties of glass lenses. However, by a proper choice of different glasses (e.g. crown and flint glasses [1], whose refractive indices have different variations with the colour of light), one can design focusing lenses which are achromatic over a broad spectrum of light frequencies. In beam optics, on the contrary, we could say that "only one type of glass is available", since the focusing strength of all magnetic lenses decreases with the same law for increasing particle energy. As a consequence, one can show that any "strictly achromatic" system made of magnetic quadrupoles is also "strictly defocusing" [2] and, therefore, "strictly useless" for the confinement of particles in accelerators.

In circular machines, this negative result is usually circumvented by means of nonlinear elements (magnetic sextupoles), which allow partial compensation of the effects with chromatic associated quadrupoles. However, the motion of particles with large betatron amplitudes is strongly affected by such nonlinear elements and becomes unstable beyond a limit known as dynamic aperture of the machine [3]. Moreover, the sextupole correction scheme is based on the inherent momentum dispersion in the bending arcs. For future electron-positron colliders in the TeV energy range, a dispersive system of acceptable length may not be practicable, because of the emittance growth associated with quantum fluctuations of the synchrotron radiation in the bending magnets. On the other hand, a final energy spread of the order of several per cent seems unavoidable in such future linear colliders [4], since even larger spreads are required during acceleration in order to provide Landau damping of the betatron oscillations and thus to ensure beam stability against transverse wakefields. As a consequence, the spot size at the interaction point and then the maximum achievable luminosity will be seriously limited by chromatic effects

 $\left[5\right] ,$ unless new schemes for the final focus can be devised.

The negative result reported before [2], applies to the Hamiltonian motion of charged particle beams in the field of quadrupole magnets: it does not take into account energy losses due to synchrotron radiation. In terms of the previous analogy with conventional optics, this would correspond to neglecting light absorption in the medium and the associated phenomenon of anomalous dispersion [6]. In order to discuss the focusing properties of dissipative systems, let us consider two successive, short magnetic lenses of equal strength and opposite polarity. They could be quadrupole magnets, in which case our discussion is restricted to a single betatron plane, plasma lenses, or bunches of relativistic particles with opposite charge. Neglecting synchrotron radiation, the two successive radial kicks experienced by an electron travelling through the system at a speed close to that of light would compensate each other. However, owing to synchrotron radiation energy loss in the first lens (or to further radiation in the region between the two lenses, which could consist of a short magnetic wiggler), the electron is lighter when it reaches the second lens and the angular deflection produced by the second kick is stronger than the deflection in the first lens. We will show that the residual angular deflection is independent of the electron energy. Therefore, if the second lens is focusing and the radial dependence of the synchrotron energy loss is properly combined with the dependence of the two kicks, one can obtain a linear, achromatic focusing system.

To illustrate this principle, in Section 2 we discuss the case where the two lenses consist of relativistic bunches of particles with opposite charge, e.g. electrons and positrons. However, owing to quantum fluctuations of the synchrotron radiation, the residual focusing strength of the system has a random component which, though still independent of particle energy, induces a broadening of the beam spot size. Section 3 contains a numerical discussion of this effect, leading to the conclusion that our achromatic dissipative focusing scheme could become of practical interest only for beams with an energy spread considerably larger than ten per cent.

2. BEAM-BEAM COLLISIONS AND RESIDUAL FOCUSING

We consider the successive collisions between a relativistic electron with radial displacement r and two short bunches with the same number of particles N: the first has a negative charge -Ne and the second a positive charge Ne. Both these bunches travel at relativistic speed in the direction opposite to that of the incoming electron (see Fig. 1). They are assumed to





be cylindrical bunches of length |L| and linear charge density $\lambda(r)$

$$\lambda(\mathbf{r}) = \pm (Ne/L) f(\mathbf{r}). \tag{1}$$

Here f(r) is a dimensionless function expressing the fraction of particles contained within a radius r. From Gauss law, the radial electric field E(r) is given by

$$E(r) = 2 \lambda(r)/r.$$
 (2)

Since the bunches are relativistic, the azimuthal magnetic field B(r) is approximately equal to E(r) and the radial Lorentz force F(r) experienced by the incoming electron during the two successive collisions can be written

$$F(r) = -2eE(r) = \pm 4 (Ne^2/L) f(r)/r.$$
 (3)

If the electron velocity remains close to c and the radial slope $r' \equiv dr/ds$ of its trajectory is small compared to unity, the equation of the trajectory reads

$$\gamma mc^2 d^2 r/ds^2 = F(r), \qquad (4)$$

where γ is the relativistic Lorentz factor of the electron. The electron energy loss per unit distance, due to synchrotron radiation, can be written [6]

$$d(\gamma mc^2)/ds = -2/3 r_e mc^2 \gamma^4/\rho^2$$
, (5)

where $r_e = e^2/mc^2$ is the classical electron radius and ρ the instantaneous radius of curvature of the trajectory. For small radial slopes r', we take

$$1/\rho \approx |d^2r/ds^2| = |F(r)|/\gamma mc^2.$$
 (6)

Therefore, from Eq. (3), the equations for the electron trajectory and energy loss can be written

$$d^{2}r/ds^{2} = \pm 4 (N/\gamma) (r_{e}/L) f(r)/r,$$
 (7)

$$d(1/\gamma)/ds = 32/3 N^2 (r_e^3/L^2) [f(r)/r]^2.$$
 (8)

The interesting fact is that the instantaneous rate of variation of the slope r' depends on $1/\gamma$, but the rate of variation of $1/\gamma$ is independent of γ itself (i.e. independent of the electron energy).

We will now solve Eqs. (7) and (8) in thin lens approximation. This means that we assume the radial position r of the incoming electron to be constant during the two successive collisions: the only effect of the collisions is therefore to change the slope r' of the trajectory and the energy γmc^2 . From Eq. (8), we see that the inverse of the Lorentz factor $\gamma(s)$ increases linearly with the longitudinal coordinate s of the electron

$$1/\gamma(s) = 1/\gamma_0^{-4} 32/3 N^2 (r_e^{-3}/L^2) [f(r)/r]^2 s,$$
 (9)

where γ_0 is the Lorentz factor corresponding to the initial electron energy. Therefore, from Eq. (7), the variation of slope $\Delta r'(s)$ of the electron trajectory during the first collision is given by

$$\Delta r'(s) = 4N (r_e/L) [f(r)/r] \{s/\gamma_0 + 16/3 N^2 (r_e^3/L^2) [f(r)/r]^2 s^2\}. (10)$$

Since both the incoming electron and the two opposite bunches travel at near the speed of light, each collision takes place over a length L/2, i.e. half the bunch length. Thus the total variation of slope $\Delta_1 r' \equiv \Delta r'(L/2)$ of the electron trajectory after the first collision is

$$\Delta_{1}r' = 2N (r_{e}/L) [f(r)/r] \{L/\gamma_{0} + 8/3 N^{2} r_{e}^{3} [f(r)/r]^{2}\}.$$
(11)

In a similar way, the variation of slope $\ \Delta_2 r'$ due to the second collision is

$$\Delta_2 r' = -2N (r_e/L) [f(r)/r] \{L/\gamma_1 + 8/3 N^2 r_e^3 [f(r)/r]^2\}$$
(12)

where $\gamma_1 \equiv \gamma(s=L/2)$ is the Lorentz factor of the electron after the first collision (see Fig. 2). Owing to synchrotron radiation, the effect of the first, defocusing kick $\Delta_1 r'$ is weaker than that of the second, focusing kick $\Delta_2 r'$, because after the first collision the electron is lighter. The total variation of slope $\Delta r'$ after the two successive collisions is the algebraic sum of $\Delta_1 r'$ and $\Delta_2 r'$; from Eqs. (9), (11) and (12) we obtain

$$\Delta r' = \Delta_1 r' + \Delta_2 r' = 2N r_e [f(r)/r] (1/\gamma_0 - 1/\gamma_1) =$$

= - 32/3 N³ (r_e⁴/L) [f(r)/r]³. (13)

Since it depends on the variation of the reciprocal Lorentz factor $1/\gamma$, this residual focusing effect is independent of the electron energy. Let us remark that, contrary to the alternating gradient scheme (where there can be a focusing effect for both the directions of motion), by reversing the order of the two successive bunches in our achromatic dissipative focusing scheme, the sign of the residual angular deflection is opposite.



Fig. 2 Electron trajectory during the two collisions.

From Eq. (13), we see that the total variation of slope $\Delta r'$ depends on the radial displacement r of the incoming electron through the factor $[f(r)/r]^3$. Therefore, in order to obtain a linear focusing effect, the dimensionless function f(r) must be of the form

$$f(r) = (r/R)^{4/3},$$
 (14)

where R is the characteristic radius of the two bunches. Recalling that f(r) expresses the fraction of particles contained within a radius r [see Eq. (1)], this corresponds to a spatial charge density p(r)given by

$$p(r) = 2/3\pi (Ne/LR^2) (R/r)^{2/3}$$
. (15)

In Fig. 3 we have plotted this theoretical charge density, which varies as $r^{-2/3}$, and a more realistic density profile, reaching a finite, maximum value on the bunch axes and dropping to zero in the vicinity of the bunch radius R. We will neither discuss the technical difficulties connected with the production of bunches



Fig. 3 Radial density profile for the bunches.

having such radial profiles nor the tolerances allowed. However, the deviation from linearity of the residual focusing effect (13), resulting from a charge density slightly different from the theoretical density (15), would presumably lead to a negligible broadening of the beam spot size compared to that caused by chromatic or geometric aberrations in a conventional final focus. Using expression (14) for f(r), the residual angular deflection (13) can be written

$$\Delta r' = -32/3 (r_{p}/L) (Nr_{p}/R)^{3} (r/R).$$
(16)

We are interested in the case where the synchrotron energy loss of the electron is only a small fraction of its initial energy. Therefore, using Eq. (9) and denoting by $\epsilon = |\Delta \gamma / \gamma_0| \equiv |\gamma(L) - \gamma_0| / \gamma_0$ the relative energy loss, we assume

$$\epsilon \simeq 32/3 \gamma_0 (r_e/L) (Nr_e/R)^2 (r/R)^{2/3} << 1.$$
 (17)

Finally, to be consistent with our thin lens approximation, we shall require that the radial deviation δr , experienced after the first collision by an incoming electron with initial slope of the trajectory $r' \approx 0$, be much smaller than the initial displacement r. For a typical displacement of the order of the bunch radius R, the relative deviation is

 $\delta r/R \simeq (L/\gamma_0 R) (Nr_e/R) \ll 1.$ (18)

3. QUANTUM MECHANICAL LIMITATIONS

The results obtained in the previous section do not take into account the quantum nature of synchrotron radiation. In particular, expression (5) for the electron energy loss is valid only in the so-called classical regime [7], i.e. when the ratio $\xi = u_c / \gamma_o mc^2$ between the critical energy of the emitted photons and the initial particle energy is much smaller than unity. Therefore, from Eqs. (3), (6) and (14), we shall require

$$\xi = 3/2 \gamma_0^2 \chi_e / \rho = 6 \gamma_0 (\chi_e / L) (Nr_e / R) << 1, (19)$$

where $x_e = \hbar/mc$ is the reduced Compton wavelength of electron and we have assumed a typical radius $r \approx$ R. Furthermore, owing to the finite number of radiated photons N_{ph}, the residual achromatic focusing strength (13) is affected by statistical fluctuations: since the emission process obeys a Poisson distribution, the resulting r.m.s. relative variation of the residual focusing strength is proportional to $1/\sqrt{N_{ph}}$. Therefore we shall require that N_{ph} be a large number [8]

$$N_{ph} = 5/2\sqrt{3} \alpha \perp \gamma/\rho \approx 10/\sqrt{3} \alpha (Nr_e/R) >> 1$$
, (20)

where $\alpha = e^{2}/\hbar c$ is the fine structure constant and, again, we have assumed $r \approx R$. This is by far the most important limitation of our focusing scheme. Indeed, as a consequence of quantum fluctuations, the broadening of the beam spot size is comparable to that produced by chromatic effects for a relative energy spread of the order of $1/\sqrt{N_{ph}}$. Since the average number of photons radiated by an electron during beam-beam collisions can hardly approach a hundred, our achromatic focusing scheme becomes of practical interest only for beams with an energy spread considerably larger than ten per cent. It is interesting to remark that Eqs. (16) to (20) all contain the same dimensionless factor (Nr_{a}/R) .

To conclude the discussion, we can summarize the main results as follows:

$$f = (5 \cdot 10^{14}/N)^3 (R/1mm)^4 (L/10cm) 119 cm,$$

$$\varepsilon_{max} = (\gamma_0/10^6) (N/5 \cdot 10^{14})^2 (1mm/R)^2 (10cm/L) 0.59,$$

$$\delta r/R = (10^6/\gamma_0) (N/5 \cdot 10^{14}) (1mm/R)^2 (L/10cm) 0.14,$$

$$\xi = (\gamma_0/10^6) (N/5 \cdot 10^{14}) (1mm/R) (10cm/L) 0.03,$$

$$N_{ph} = (N/5 \cdot 10^{14}) (1mm/R) 60,$$

f denotes the achromatic focal length where corresponding to Eq. (16) and we have assumed suitable numerical values for the various parameters. The constraint on $~\delta r/R,$ which follows from the thin lens approximation, together with the required large number of emitted photons $N_{\mbox{ph}\prime}$ leads to a rather extreme value of 240 kA for the peak current in the two Nevertheless, the implementation of the bunches. principle of achromatic dissipative focusing would become much more feasible if one could devise a different scheme, avoiding the thin lens approximation. For example a 1 TeV electron, going through a wiggler 100 m long with a magnetic field of 1 T, would radiate 600 photons by losing only 13 per cent of its initial energy. By a proper arrangement of focusing and defocusing quadrupoles or plasma lenses, one could then obtain a thick achromatic system of practical interest already for beam energy spreads of a few per cent.

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