WAVE GROWTH TECHNIQUE FOR COLLECTIVE WAVE ACCELERATORS

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Summary

The use of parametric scattering as a means of space charge wave growth for a converging guide, ion accelerator system is discussed. A 400-600 keV, 1.5-3.0 kA electron beam is passed through a self biasing, electrostatic wiggler in order to grow both slow and fast space charge waves. We present measurements of the wave frequency and associated wave fields as a function of the equilibrium. Furthermore, a calculation of the wave kinetic power flow and wave electromagnetic power flow is used to estimate the achievable axial electric field.

Introduction

Wave collective acceleration schemes are essentially four stage devices. The first portion consists of the wavegrowth region, where the drive particle beam is density modulated. The modulation occurs through either the excitation of slow space charge waves or slow cyclotron waves. The second portion of the generic collective device consists of the load particle trapping region which frequently occurs simultaneously with wavegrowth. In this area, load particles (usually protons), are trapped in the wave potential wells by the wave electric fields. The third section of a collective wave accelerator is the bunching and transport section. The fourth and final section of the machine is the wave acceleration region. It is in this region the protons are accelerated synchronously with the wave.

In an effort to realize a practical converging guide collective accelerator, we have examined a number of different methods of space charge wave excitation. Since the slow space charge wave is a negative energy mode, passive schemes appear to be the most sensible. Parallel to many microwave tubes, passive excitation can use a resitive wall, a two-wave interaction or a three-wave interaction. For high power electron beams, the coupled cavity structure has historically been the preferred method. Recently, the use of periodic external fields, as are used in free electron lasers and electron cyclotron masers, have proven efficient in growing large amplitude waves. After employing both methods, we have determined that parametric excitation of the slow space charge wave is the most desirable from an accelerator stand point. The initial work concerning the experimental implementation of this approach was reported previously[1]. In this paper we concern ourselves with the details of the experiment and the practical development of this system as the initial portion of a slow space charge wave accelerator. The parametric excitation is also useful as a means of exciting fast waves for high energy accelerator applications.

Using a self biased, azimuthally symmetric electrostatic wiggler, we have grown both linear and non-linear electrokinetic waves on an electron beam. The method is very similar to the velocity jump amplifier proposed and implemeted experimentally by Field, Tien and Watkins[2] in 1951. Further analytic work on electrostatic wigglers to grow these modes was done by Bekefi[3] (1980). Later a comprehensive non-linear theory and numerical simulation was done on the excitation of the waves by Seyler and Fenstermacher[4] (1987).

The goal of the experiment is to investigate the use of a three wave process to grow suitable space charge waves for accelerator purposes. The advantages of a parametric technique lie in its intrinsic ability to grow waves at any phase velocity over short distances and at currents near to the electron beam space charge limit. With these advantages, the wavegrowth, particle trapping and bunching region of a collective accelerator can be incorporated in a single device. As the wave is grown at the synchronous velocity, the load particle sees the space charge waves grow around them. As the wave grows, the load particles will be pushed towards the bottom of wave potential wells. This traps and bunches the load particles within the beam. After this point, the beam embedded with protons, can move to the converging section where both wave and particles will be synchronously accelerated.

Basic Theory

The linear theory used follows Ramo's[5] analysis for one dimensional electron motion in a bounded, one component plasma. The beam equilibria is given by the relavistic, large magnetic field model for pencil beams ($b/a \geq 4$) in confined flow. Since the linear theory is fully electromagnetic, the fast space charge wave velocity limits are $v_{\text{drift}} < v_{\phi\ fast} < c$ and the slow space charge wave is bounded as $0 < v_{\phi\ slow} < v_{\text{drift}}$. The linear analysis employs the complete set of Maxwell's equations, the continuity equation and the momentum conservation equation. The wave equation is the Helmholtz equation for cylindrical geometry in terms of E_z

$$\nabla_\perp^2 E_z + k_\perp^2 E_z = 0 \tag{1}$$

where

$$k_\perp^2 = \left(\frac{\omega^2}{c^2} - k_z^2\right)\left(1 - \frac{\omega_p^2}{\omega_b^2}\right)$$

and the plasma and the Doppler shifted frequencies are defined by

$$\omega_b = \omega - k_z v_d \quad \text{and} \quad \omega_p^2 = \frac{ne^2}{\gamma^3 m\epsilon_0}$$

Matching the wave admittance at the beam-vacuum boundary we determine the characteristic dispersion relation for azimuthally symmetric waves.

$$\frac{k_\perp J_1(k_\perp a)}{hJ_0(k_\perp a)} = \frac{I_0(hb)K_1(ha) + I_1(ha)K_0(hb)}{I_0(hb)K_0(ha) - I_0(ha)K_0(hb)}$$

where

$$h^2 = k_z^2 - \frac{\omega^2}{c^2} \tag{2}$$

This dispersion relation gives the azimuthally symmetric space charge branches and the transverse magnetic waveguide modes.

Wave Power Flow

Employing the linear wave dispersion relationship, we can calculate the relative magnitude of the axial electric field for the fast and slow waves using the Poynting theorem[6]. The energy conservation equation for the system is

$$-\frac{\partial}{\partial t}\left(\frac{mn\,|\,\mathbf{v}\,|^2}{2} + \frac{\epsilon_o\,|\,\mathbf{E}\,|^2}{2} + \frac{|\,\mathbf{B}\,|^2}{2\mu_o}\right) =$$

$$\nabla\cdot\left(\frac{mn\mathbf{v}\,|\,\mathbf{v}\,|^2}{2} + \frac{\mathbf{E}\times\mathbf{B}}{\mu_o}\right) \tag{3}$$

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which is determined by using the continuity, momemtum conservation and power conservation equations. Using the linear values of density and velocity coupled with the wave fields, we can determine the net amount of power contained in each wave mode. The energy conservation equation can be modified by using the linearized variables to calculate the wave power flow. Taking the power flow term and applying Stokes theorem we can calculate the net power flow for each wave mode across the beam. The net power per unit area, for the linear waves, can now be calculated. The power flow, both kinetic and electromagnetic, for the modes in terms of the wave axial field is given by

$$\nabla \cdot \mathbf{S} = \left(\frac{v_d \omega \omega_p^2 E_z^2}{\omega_b^3} + \frac{\epsilon_o \omega k_z}{h^4} \left(\frac{\partial E_z}{\partial r} \right)^2 \right) \tag{4}$$

Intergration of this power flow per unit area across the guide yields an equation of the following form

$$P_{\text{total}} = \beta_{\text{eff}} \pi a^2 c \epsilon_o E_z^2 \tag{5}$$

where the normalized effective velocity relates the axial field energy to the power in a particular mode. The ratio of this value for different modes allows one to estimate the difference in axial field strength for a given mode. In our experimental case, we estimate the ratio will be about 14 to 1 in favour of the slow space charge wave. This becomes important when devising a wave growth method suitable for collective accelerators. The fast space charge wave might tend to push the load particles out of the potential wells of the slow wave. Fortunately, this debunching by the presence of a non-synchronous wave in the system is usually unimportant due to the large difference in relative phase velocity and amplitude. The analysis reveals that for lower operating frequencies, the fast space charge wave has most of its field energy concentrated in the azimuthal magnetic field and the radial electric field. On the other hand, the slow space charge wave concentrates its field energy in the axial electric field.

Experiment

The relativistic electron beam is generated in an ARA type diode at 400-500 keV followed by a beam skimmer producing a cool 1.7-2.5 kA beam immersed in a 1.1 T axial magnetic field. The beam has a radius of 0.58 cm while the guide radius is 2.23 cm. The particle beam is in confined flow and no large order perpendicular motion of beam particles has been detected. Measurement of the beam parameters before and after the wavegrowth section show no appreciable decrease in beam kinetic energy. This is in contrast with excitation of slow waves in a disc loaded guide where the energy losses to the cavity fields are substantial. The current remains at about 1.7 kA and the beam to wall potential at about 200 kV throughout the experiment. For slow wave lengths of ~3 cm., the nonlinear maximum axial gradient is set at ~40 MV/m. From the beam dispersion relations (Figure 1), we see that below 5.5 GHz, only the slow and fast space charge waves can be excited. Using an electrostatic wiggler with a wavenumber of 1.20/cm, we can predict the frequency and wavelength for the interaction (Figure 2). Using the Manley-Rowe relations, we expect the interaction to occur at about 2.3 GHz for the experimental condition.

$$\omega_{\text{fast}} = \omega_{\text{slow}} \quad \text{and} \quad \mathbf{k}_{\text{slow}} = \mathbf{k}_{\text{fast}} \pm \mathbf{k}_{\text{wiggler}}$$

Using a broadband double balanced mixer, we can examine the signal from a coupling loop located at the guide wall (Figure 3). The signal shows a frequency mismatch has occurred

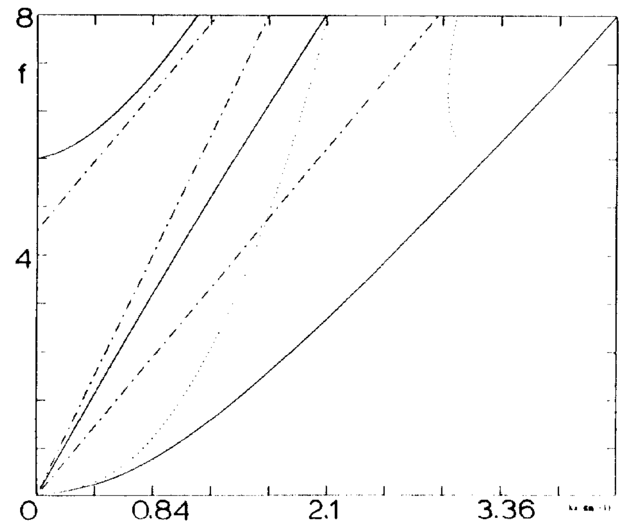

Fig 1. Beam dispersion relation for the experimental parameters. Frequency in GHz and wavenumber in cm^{-1}

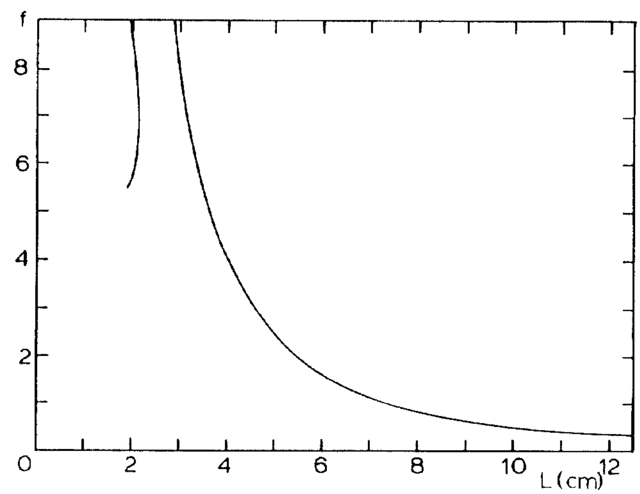

Fig 2. The value of wiggler period as a function of frequency needed for the interaction. Frequency in GHz.

with the waves, occurring at about 2.3 GHz and 1.95 GHz. The difference in frequency appears to be due to the excitation of cavity modes in the wiggler region as evidenced by the fact that the frequency separation is inversely proportional to the length of the wiggler region. The higher frequncy wave is assumed to be the slow space charge wave because of energy considerations. Since the wiggler is self biased, this may prove to be a source of wave energy loss. Using the capacitive probes, and the current monitors, we can estimate the average drift velocity and thus average density of the beam (Figure 4). With this method, we can make a crude estimate of the wave phase velocity. Since two waves exist in the system, exact phase detector measurements of the wave velocity are difficult. Using the simple technique, the slow wave velocity is about 0.18c and the fast wave velocity is about 0.78c. Calculating the normalized effective velocity for the experimental parameters, we can estimate the ratio of slow wave to fast wave axial field strength (Figure 5). Field measurements show the slow space charge wave may have an axial electric field of ~15 MV/m while the fast space charge wave has an axial field of ~ 1.0 MV/m. This ratio is near that predicted by the power flow calculations. Us-

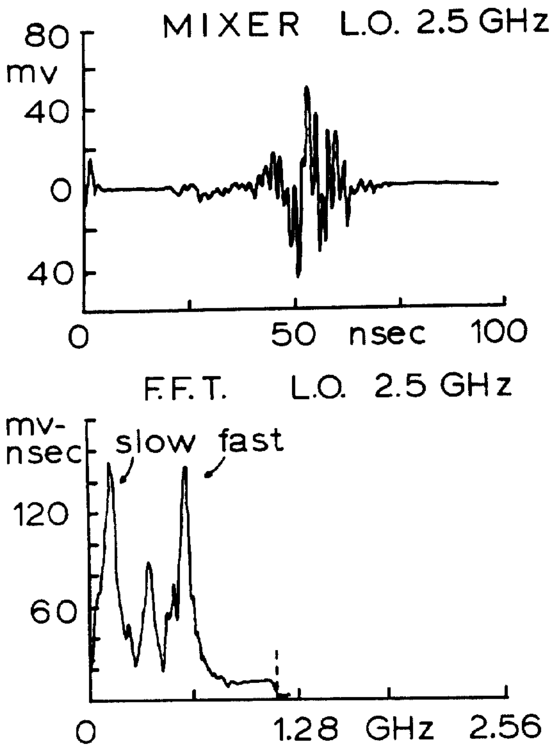

Fig 3. The frequency spectrum for the wave fields showing the two modes of the system.

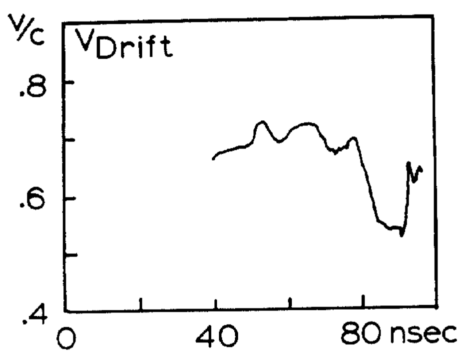

Fig 4. The average drift velocity calcuted from the beam to wall potential and the beam current.

ing the measured parameters for the various fields associated with the beam, we can tabulate the experimental parameters for various conditions (Table 1).

Conclusion

The use of azimuthally symmetric electrostatic wigglers to grow slow space charge waves for collective acceleration is extremely practical for a number of reasons. First, no appreciable power loss to the beam is measured during the process. This aspect is probably further enhanced by the fact that the

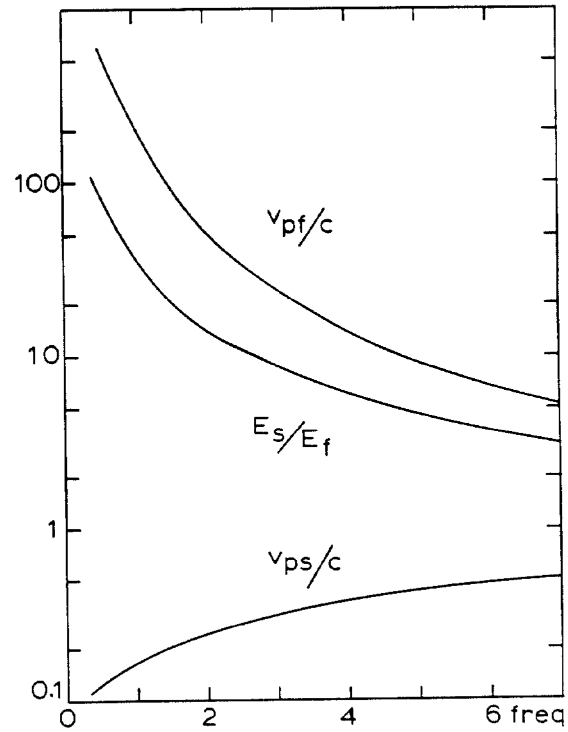

Fig 5. The normalized effective velocities and the ratio of axial electric field for the two modes as a function of frequency. Frequency in GHz.

modes are excited below cutoff, disallowing any coupling to radiative modes. Secondly, the process can grow the wave at the synchronuous velocity so that the device can act as wavegrowth region, particle trapping region and buncher for wave acceleration region. Thirdly, the system is capable of growing large amplitude waves coherently over a reasonable length of guide. This greatly adds to the compactness of a practical device. Fourthly, the power flow relations demonstrate that the fast space charge wave has relatively little axial electric field and will not affect the containment or the acceleration of load particles. The implemetation of this system using a similar beam could result in a collective device that could accelerate at least 5 A ions from 5 MeV to 35 MeV over 2 meters. This far exceeds the limits set on conventional drift tube linear accerators that operate for ions at these energy regimes. More importantly perhaps, for a beam of 100 kV, 50 amps, this technique would allow for the development of a device to accelerate low energy ions (500 keV) to moderate energies at large currents and may be a serious competitor for low energy injectors, e.g. R.F.Q.'s. The inherent stability of the net electrostatic focusing of collective wave accelerators will always provide for greater load particle transport than available in conventional devices.

Parameter	*Slow Wave*	*Fast Wave*
Frequency (GHz)	2.2-2.4	1.8-2.0
Phase Velocity (c)	0.19-0.23	0.80-0.85
Axial Field (MV/m)	10-15	1-2
Effective Velocity (c)	0.27	40.5
Field Ratio	12.3	

Table 1. Tabulated data for various beam conditions.

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