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THE SCALING OF LINEAR SUPERCOLLIDERS USING MODIFIED SLAC STRUCTURES*

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Introduction

The development of a linear supercollider which can reach multi-TeV energies will require the use of high-gradient structures so that the size and cost of the accelerator can be minimized. Using frequencies higher than the SLAC frequency (2856 MHz) allows higher gradients to be achieved in smaller structures, at a cost of larger wakefields. The frequency scaling of rf linacs for a linear supercollider has been discussed in earlier publications.[1] Here we discuss the tradeoffs involved in opening the aperture of a scaled SLAC structure. We find that wake effects can be reduced but the cost in terms of shunt impedance and peak rf power per feed may be prohibitive.

The monopole longitudinal wakefield is a longitudinal electric field that produces energy spread and average energy loss for short bunches. Since the tail of the bunch suffers larger wakefield effects than the head, the energy spread can be compensated by placing the bunch ahead of the crest of the rf wave. The dipole transverse wakefield is driven by a beam off axis, and drives the tail of the bunch further off axis. It primarily leads to a growth of the bunch emittance. Since the betatron wavelength depends on the particle energy, it is possible to use Landau damping to reduce the emittance growth due to transverse wakefields by inducing an energy spread on the bunch.[2,3]

Near the optimal energy spread for Landau damping, an interesting effect has been noted by Bane.[4] During the negative phase, the tail of the bunch will have lower energy than the head, and therefore the tail will be more strongly focused than the head in the external focusing system, since the chromaticity in a linac is always negative. The transverse wakefield will counteract this effect, tending to defocus the tail. For large energy spread, these effects can be comparable in magnitude.

A two-particle model analysis[3] demonstrates this effect. If x_1 and x_2 represent the transverse displacements of the two particles, then

$$(x_2-x_1)/\hat{x} = (1-C/2k\Delta k)\ 2i\ \sin(\Delta ks/2)\ \exp[i(k+\Delta k/2)s], \tag{1}$$

where s measures distance along the accelerator, $\hat{x}$ is a complex constant that depends on the initial conditions, k is the betatron wavenumber of the lead particle, $k+\Delta k$ is the betatron wavenumber for the trailing particle, and C is given by

$$C = eQW_{\perp}(z)/2E. \tag{2}$$

Here, $W_{\perp}(z)$ is the transverse wakefield at the location of the trailing particle, Q is the total charge of the bunch, E and $E+\Delta E$ are the energies of the two particles, and $|\Delta E/E|$ is assumed small. It is clear from Eq. (1) that $x_2-x_1=0$ when $C=2k\Delta k$. This condition may be used to define an "optimal" energy spread for Landau damping. Either increasing or reducing $W_{\perp}$ at the optimal energy spread will increase the emittance growth. In the two particle model the emittance growth is zero at the optimal energy spread.

The wakefields may be reduced in amplitude by enlarging the aperture of the accelerator. Other cold-test properties of the structure are also altered by enlarging the iris, such as the shunt impedance which decreases, the Q which increases, and the group velocity which increases rapidly. These parameters affect the amount of power required to drive the accelerator, and constitute a "cost" which must be paid for reduction in wakefields.

Single Bunch Wakefield Effects

A single bunch wakefield effects simulation code[1] has been employed to analyze the effects on emittance of opening the aperture of a (frequency scaled) SLAC structure. Briefly, the simulation code considers a bunch to be composed of slices, each of which is acted upon by (1) the rf accelerating field, (2) linear, smooth focusing forces, and (3) longitudinal (monopole) and transverse (dipole) wake fields produced by preceding slices. Both the wake fields and the focusing forces are applied continuously in this model. The code integrates the differential equations obeyed by the slice centroids and by the variances of the distribution about the centroids. A normalized pseudo-emittance is defined by

$$\epsilon_n^2 = 16\bar{\gamma}(\sigma_{xx}\sigma_{x'x'} - \sigma_{xx'}^2) \tag{3}$$

where $\bar{\gamma}$ is the average value over the bunch of the relativistic factor and σ_{xx}, etc. are the variances of the transverse position and slope of the particles in the bunch.

The code runs described below use analytic fits to the SLAC delta function wake potentials which are then scaled in aperture and frequency. The longitudinal wake function is assumed to be of the form

$$W_\ell(\xi) = W_\ell(0)\exp-(\xi/\xi_o)^{1/2} \tag{4}$$

where the values of the parameters for a SLAC structure are $W_\ell(0)=2.29\times10^{14}$volts/coul-m, $\xi_o=2.08$mm.

The transverse dipole wake function is fit to

$$W_t(\xi) = \frac{W_t'(0)\xi}{1 + d(\xi/\xi_m)^p} \tag{5}$$

where the values for the SLAC structure are $W_t'(0) = 2.28\times10^{18}$volts/coul-m^3, $\xi_m = 7.5$mm, d = 3.04, and p = (1+d)/d = 1.33. Eq. (4) is reasonably accurate out to distances $\xi \sim 6\xi_o$ while (5) holds out to the first maximum, ξ_m.

The changes of the various parameters in (2) and (3), as the aperture a is opened, have been cited by Wilson[5]. Specifically, if we define $\alpha = a/a_s$, where a subscript s denotes the rf frequency scaled SLAC value, then we have taken[5]:

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$$
\begin{aligned}
W_\ell(0) &= \alpha^{-1.68} W_{\ell s}(0) \\
\xi_o &= \alpha\, \xi_{os} \qquad\qquad (6) \\
W_t'(0) &= \alpha^{-3.48} W_{ts}'(0) \\
\xi_m &= \alpha\, \xi_{ms} \\
W_t(\xi_m) &= \alpha^{-2.25}\, W_{ts}(\xi_m)
\end{aligned}
$$

Once the aperture scaling is carried out the code further scales the wake potentials to the linac operating frequency f. The aperture and frequency scaling laws are applied independently. The frequency scaling laws are, defining ν = f/2856MHz

$$W_\ell(\xi/\nu) = \nu^2 W_{\ell s}(\xi) \qquad (7a)$$

$$W_t(\xi/\nu) = \nu^3 W_{ts}(\xi) \qquad (7b)$$

For a linac operating at 7.79GHz, the case to be considered below, the aperture and frequency scaled wake potentials are plotted in Figure 1.

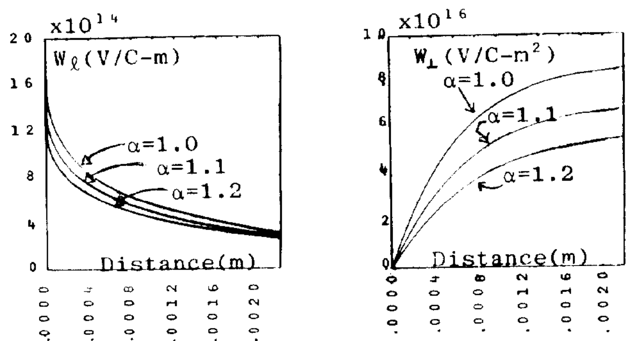

Figure 1: Delta function wake fields at 7.79 GHz, for α = 1.0, 1.1, 1.2.

In the simulations the following quantities are held fixed for all cases: The linac is assumed to operate at 7.79GHz which is close to a minimum in ac power requirements, under certain assumptions.[1] The accelerating gradient is taken to be 136.4MeV/m = 50MeV/m · (7.79/2.856); this frequency scaling of gradient keeps the peak power per feed requirement at 325 MW for the SLAC structure (α=1). The accelerator length is 7374m which gives an average bunch energy of exactly 1 TeV for the case α=1, when the bunch is placed at 3° ahead of the rf crest, to compensate for longitudinal wake effects. The bunch itself is Gaussian with σ_z = 0.367mm = 1mm(2.856/7.79). The number of electrons in the bunch is $1.37\text{x}10^{10}$. It is launched with an initial average energy of 50 GeV and an initial energy spread of 10 MeV. The bunch is divided into 16 slices of equal thickness. The slices are initially matched into the focusing system; the initial normalized emittance of each slice is $1\text{x}10^{-5}$ rad-meters. The transverse size of the bunch is initially $2\sigma_{xx}^{1/2}$ = 40μ. The initial betatron wavelength is 10m and is assumed to increase $\sim$ (energy)$^{1/2}$, in order to keep the optimium energy spread fixed (see below). The chromaticity for all runs is ξ_c = -1.

Results from runs with and without Landau damping are shown in Figure 2. Here X*, the value of initial slice centroid displacement (injection error) which results in a factor of 2 growth in normalized emittance at the end of the linac, is plotted for 4 different cases, versus the scaled aperture size. The different cases are labeled by f, the fraction of the machine during which the bunch is kept at a position θ_1 < 0 behind the rf crest, in order to induce energy spread; the bunch is then abruptly shifted to θ_2 > 0, ahead of the crest, for the remaining fraction, 1-f, of its acceleration. The compensating phase θ_2 is chosen so that the bunch arrives at the end of the machine with approximately the minimum possible energy spread.

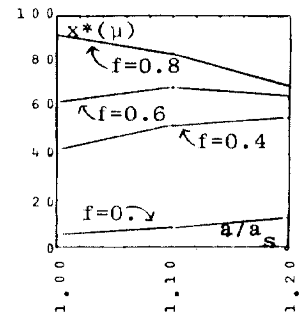

Figure 2: x* versus α, f = 0 [no Landau damping]; f = 0.4, 0.6, 0.8 [θ_1 = -5°].

In Figure 2 data is shown for f=0 (no Landau damping), and θ_1 = -5°, f = 0.4, 0.6, and 0.8. When examining Figure 2 it is useful to think of the effect of increasing f as increasing the average energy spread during the acceleration and of increasing a/a_S = α as decreasing the wake field strength.

Two trends are evident in Figure 2. First, for fixed α, increasing f increases tolerance to injection error. Second, for fixed f, increasing α (opening the aperture) increases tolerance to injection error for small f but decreases that tolerance for large f. These trends may be understood qualitatively by considering the two particle model[3], which illustrates the competition between wake field effects and dispersion. The model suggests that as wake field strengths are increased from zero, emittance growth will first <u>decrease</u>, <u>if</u> the energy of the bunch tail is less than that of the head. This is because the slightly stronger focusing experienced by the tail particle, due to its lower energy, tends to cancel the wake force it feels due to the displacement of the lead particle; as wake strengths are increased further, one reaches a regime where cancellation of dispersive and wake effects is optimum. (In a two particle model it can be "perfect".) The two particle model gives this optimum when[2,3]

$$\left[\frac{\Delta E}{E}\right]_{opt} = \frac{eQW_\perp}{4k^2E\xi_c} \qquad (8)$$

where $W_\perp$ is the wake at the tail particle due to displacement of the head. As wake fields are increased beyond the value given in (7), emittance growth will increase. We will refer to the small wakefield regime, as the dispersion dominated regime and to the large wakefield regime as the wakefield dominated regime.

The f=0 and f=0.4 curves of Figure 2 are examples of the wake field dominated regime; the f = 0.8 curve illustrates a dispersion dominated case. The f = 0.6 curve is transitional; α = 1.1 for this case is seen to be near an optimum value.

It remains to explain why, in all cases in Figure 2, increasing f for fixed wake field strength reduces emittance growth. This appears to be due to the "beating" phenomenon illustrated again by the two particle model, according to which if the wavenumber spread is such that $\Delta kL/2 \simeq \pi, 2\pi, \ldots$ the emittance will minimize at the end of the linac, even if it took large intermediate values. Since increasing f increases Δk, the trend of reduced emittance growth for increasing f may be explained if $\Delta kL/2$ is somewhat less than an integer $\times\pi$ for all values of f. For cases run with large negative θ, (eg. $\theta_1 = -30°$), an optimum f is found; this is because the beating must be adjusted so that a minimum lands at the end of the machine.

Cost of Opening the Aperture

Basic cold-test properties of the accelerating structure are determined by the iris. Table 1 shows the scaling of these quantities with the iris radius, a.

TABLE 1

r	= Shunt Impedance per Length	$\sim a^{-1/2}$
Q	= RF Quality Factor	$\sim a^{1/2}$
v_g	= Group Velocity	$\sim a^4$

These scaling rules imply that the fill time, $t_F \sim Q/\omega \sim a^{1/2}$, the length of an rf feed, $\ell_{rf} = v_g t_F \sim a^{9/2}$, and the number of rf feeds scales as $N_{rf} \sim a^{-9/2}$. The peak power required scales as $P \sim E^2/r \sim a^{1/2}$. The peak rf power required per feed scales as $P_f \sim P/N_{rf} \sim a^5$. The power scaling shown in these relationships is, in fact, an underestimate since it does not include the degradation in the beam efficiency associated with the reduction in the longitudinal wakefield. The wakefield effect is included in the numerical scaling analysis presented below.

An rf linac scaling code has been developed, which uses the parameters required at the interaction point of a linear collider together with a model for power flow in an rf linac to determine the accelerator requirements under various scaling scenarios.[1] That model has been modified to include the wakefield scaling with iris as well as the scaling for the parameters described in Table 1.

A calculation has been carried out for a 1 Tev-1 TeV linear collider with a total luminosity of 10^{33} $cm^{-2}\cdot s^{-1}$, 30% energy loss to beamstrahlung, and 12 bunches per rf pulse. The transverse beam size is assumed to be 0.1 μ (round) at the final focus. The bunch length, accelerating gradient, rf frequency and number of particles per bunch are the same as those used in the simulation presented in the previous section. The disruption parameter for this collider is 0.71, the repetition frequency is 278 Hz, and the average beam power is 7.5 MW per linac.

Figure 3 shows the number of rf feeds per linac and the peak rf power per feed as functions of α. The number of rf feeds per linac decreases rapidly and the peak rf power required per feed increases with α because both the fill time and the group velocity increase with α. The product of the two, the peak rf power required per linac, increases with α since the shunt impedance and the beam efficiency decrease as the iris is enlarged. The wall power, to achieve the required average beam power at the final focus, increases with α, as shown in Figure 3. The average ac power per linac, plotted in Figure 3, increases from 376 MW at α=1 to 491 MW at α=1.2.

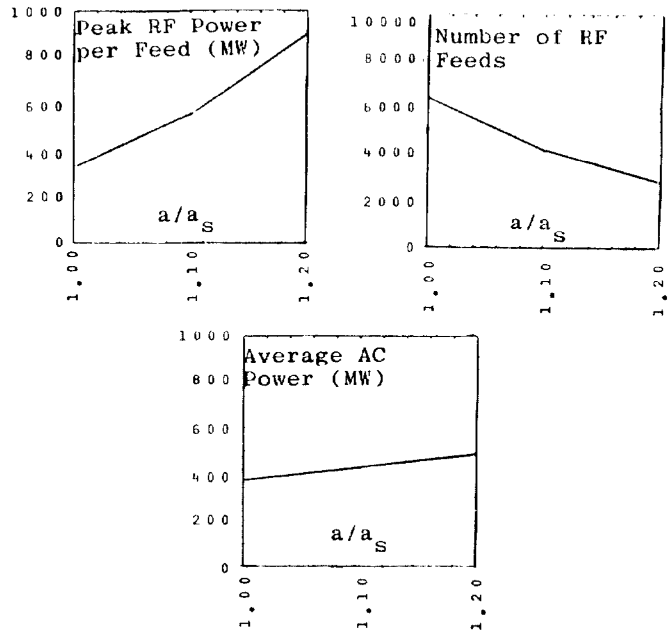

Figure 3: Number of rf feeds per linac, peak rf power per feed, and average ac power per linac versus α.

Conclusion

The possibility of using a modified SLAC structure, ie. a frequency-scaled SLAC structure with an enlarged aperture, to reduce wakefields and control emittance growth has been explored in this paper. This method works well when the energy spread is small. Near the optimal energy spread for Landau damping, however, the transverse wakefield can be brought into competition with the additional focusing of the tail due the chromaticity of the linac. In this situation reducing the wakefield can spoil the cancellation between these effects, leading to emittance growth. In any case, opening the aperture causes the power requirements of the accelerator to increase. The rf power per feed, in particular, rises to a level where a single rf source with pulse compression will be required for each feed.

The advantages of enlarging the aperture for wakefield control appear to be quite limited, compared to Landau damping, at least for the perfectly aligned machine. Consideration of random magnet alignment-errors may lead to the choice of using limited Landau damping with small energy spread together with enlarging the iris to optimize the transverse wakefield.

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