Stability of the Driving Bunch in the Plasma Wakefield Accelerator

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Abstract

We investigate the stability of the driving electron or positron beam in the plasma wakefield accelerator. Although the beam is subject to self-focusing, filamentation and two stream instability, we find that all of these can be annihilated by introducing thermal energy and an axial magnetic field.

I. Introduction

In the scheme known as the Plasma Wakefield Accelerator (PWFA) an electron bunch traversing a plasma excites large plasma wave which can accelerate a trailing bunch of electrons or positrons. Chen et al.[1] [2] first studied the PWFA scheme using a single particle model. Later Ruth et al.[3] recognized the similarity between PWFA and wake field accelerator scheme using EM cavities. Once this is seen, the "fundamental theorem of beam loading"[4] known in accelerator physics can be applied to the PWFA. This theorem states that for a symetric driving bunch the maximum energy gain for the driven electrons cannot exceed $2\gamma_0 mc^2$, where $\gamma_0 mc^2$ is the energy of driving beam electron. (i.e. the transformer ratio $R \leq 2$). This limitation can be overcome by introducing asymmetric charge distributions [5] [6] for the driver, in which case the energy gain can be up to $k_p \varsigma_0 \gamma_0 mc^2$, where ς_0 is the length of a properly shaped driving bunch and $k_p = \frac{\omega_p}{c}$.

For a driving bunch with a finite extent, longitudinal instability such as the two stream instability and transverse effects such as self-focusing [7] and filamentation [8] are of concern. The longitudinal instability has been examined previously [9] and can be avoided. The transverse instabilities can cause distortion of the bunch shape.

In this paper we examine the transverse instabilities of the driving beam in the plasma wakefield accelerator analytically and with 2-D computer simulation. The type of transverse instability that dominates depends strongly on the beam's radius. For a narrow beam of radius "a" of order c/ω_p, the beam is strongly self-focused by its own transverse wake . Wide beams ($a \gg c/\omega_p$) are subject to the Weibel instability In section II we study the self-focusing of beams by their own wakefields. In section III we investigate the Weibel instability of a broad beam and discuss the suppression of such an instability. We show that the driving beam can be stablized by the introduction of one or both of the following: a) transverse thermal energy spread in the driving bunch; b) an axial magnetic field. The transverse energy required for stabilization is independent of beam energy (γ) and so is modest for high energy beams. The magnetic field required is proportional to $\gamma^{1/2}$.

II. Self-focusing

The self-focusing of the driving beam in a plasma is a result of the transverse wake. Physically it arises beause the plasma electrons respond to the beam's space charge by moving away from the beam. The remaining plasma ions thus neutralize the space charge of the beam. This enables the beam current generated azimuthal magnetic field B_θ to pinch the beam by the $v_z \times B_\theta$ force (current shielding is less effective than charge shielding).

The strength of the self-focusing depends on the radius and the density of the beam. For a wide beam the pinching is not severe since the plasma sets up a cancelling return current. For a narrow beam ($a \approx c/\omega_p$) most of the return current is on the outside of the beam. Thus within the beam, B_θ remains and strongly pinches a narrow beam.

To quantify our discussion of self-focusing we consider the wakefields produced by first driving beam with parabolic transverse the density distribution $\rho(r,\varsigma) = f(\varsigma)\cdot\sigma(r)$

$$\sigma(r) = \begin{cases} \sigma_0(1 - r^2/a^2) & ,r \leq a \\ 0 & ,r > a \end{cases} \tag{1}$$

a is the beam radius. The longitudinal and transverse wakefields are given by [3] [10]

$$\begin{aligned} W_\parallel &= 8\pi\sigma_0 \left\{ K_2(k_p a) I_0(k_p r) + \frac{1}{2}(1 - \frac{r^2}{a^2}) - \frac{2}{(k_p a)^2} \right\} \cdot \\ & \int_0^\varsigma d\varsigma' f(\varsigma') cos k_p(\varsigma' - \varsigma) \end{aligned} \tag{2}$$

$$\begin{aligned} W_\perp &= 8\pi\sigma_0 \left\{ K_2(k_p a) I_1(k_p r) - \frac{r}{k_p a^2}) \right\} \cdot \\ & \int_0^\varsigma d\varsigma' f(\varsigma') sin k_p(\varsigma' - \varsigma) \end{aligned}$$

where $k_p = c/\omega_p$, $W_\parallel = eE_z$ and $W_\perp = e(E_r + \beta_z \theta) \approx (E_r + B_\theta)$ The radial dependencies of $W_\perp$ and $W_\parallel$ are plotted in Fig. 1 for beams of radius $k_p a = 1$ and $k_p a = 10$.
Explain that the transverse wake within the driving bunch is always self-focusing [7] [8].

Ruth and Chen [11] showed how a flat rather than a parabolic driving beam profile improves both the emittance and energy spread of the trailing bunch. Such a profile also reduces the self- focusing of the driving bunch. This can be seen by considering the wakefields for a driving profile of the form

$$\sigma(r) = \begin{cases} \sigma_0 & ,r \leq a \\ 0 & ,r > a \end{cases} \tag{3}$$

The wakefields are then

$$\begin{aligned} W_\parallel &= -2\pi\sigma_0 \{1 - k_p a \cdot K_1(k_p a) \cdot I_0(k_p r)\} \cdot \\ & \int_0^\varsigma d\varsigma' f(\varsigma') cos k_p(\varsigma' - \varsigma) \end{aligned} \tag{4}$$

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$$W_\perp = 2\pi\sigma_0 K_1(k_p a) I_1(k_p r) \cdot \int_0^{\varsigma} d\varsigma' f(\varsigma') sin k_p(\varsigma' - \varsigma)$$

$W_\perp$ and $W_\parallel$ versus $k_p r$ are plotted in Fig. 2 for $k_p a = 1$ and $k_p a = 10$. Note that the similarity between Fig. 1-a and 2-a suggests that the exact form of the radial profile is not important for narrow beams. For wide beams Figs. 1-b and 2-b show that uniform radial profiles give much smaller $W_\perp$ and more uniform $W_\parallel$ than do parabolic profiles. Physically the plasma waves excited by a flat beam are quasi-one-dimensional. Most of the plasma oscillation is in the longitudinal direction and $W_\perp$ is nearly zero except near the edges.

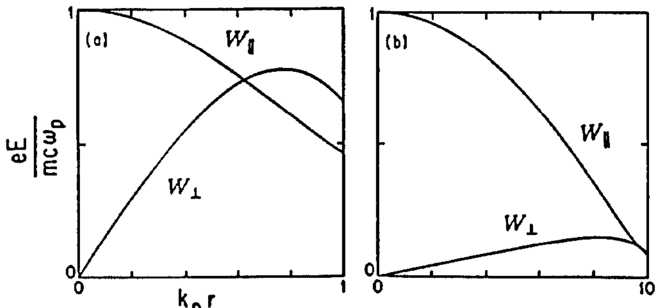

Figure 1 Wakefields vs. r for a parabolic beam profile: (a) beam radius $a = 1c/\omega_p$ (b) $a = 10c/\omega_p$.

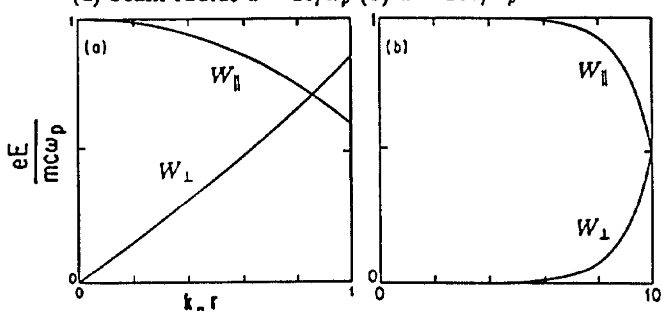

Figure 2 Wakefields vs. r for a uniform beam profile: (a) beam radius $a = 10c/\omega_p$ (b) $a = 10c/\omega_p$.

The self-focusing and betatron oscillation of a narrow beam is illustrated by the 2-D simulation depicted in Fig. 3. For this and the other simulations in this paper the particle-in-cell code ISIS[12] was used. This model solves Maxwell's equations in cylindrical coordinates (r, z) subject to the current and charge densities of the plasma and beam particles. In order to model a high transformer ratio example[5], we shaped the current such that $\rho(\varsigma) = \rho_0$ for $0 \leq k_p\varsigma \leq \pi/2$, and $\rho(\varsigma) = f(\varsigma) = \rho_0(2/\pi)(k_p\varsigma - 2/\pi + 1)$ for $\pi/2 \leq k_p\varsigma \leq k + pl$, where l is the bunch length and $\varsigma = z - ct$. This bunch shape is used in simulations throughout the paper. The beam has a uniform distribution in the transverse direction out to radius 1 c/ω_p. The driving beam parameters are the following: $\gamma = 20, k_p l = 14.14$ and the peak density is 0.2 times the background plasma density n_0. Note that the pinching is strongest towards the tail of the bunch where the beam current is highest.

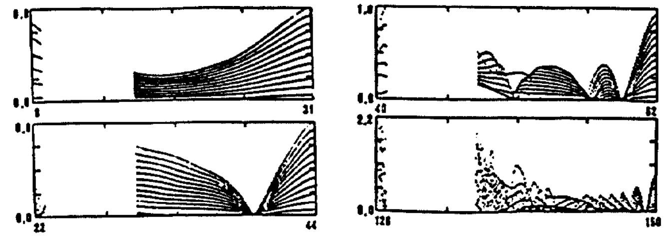

Figure 3 Real space plots R vs. Z ,at $\omega_p t$=32,44,62 and 150 repectively.

For the parameters of Fig. 3, Eq. 4 gives $W_\perp \approx 0.034 mc\omega_p$ at $r = c/\omega_p$ at the tail of the beam. In Fig. 3 b-d the betatron oscillations of the beam are apparent. At late times, phase mixing of the various oscillations causes an effective transverse energy spread of the beam. This leads to stabilization of the majority of the beam at a new small radius, where the Bennett pinch equilibrium[13] has been approach. The tail of the beam, however, has spread.

III. Suppression of Weibel Instability

We have seen in section II that a wide beam can minimize the tendency of self-focusing. However, such beams may still be subject to the well known Weibel instability[14]. Physically this instability arises because like currents attract and this results in local pinched structures. The Weibel instability is a purely transverse and purely growing mode (i.e., $k_\parallel = 0$, Re$\omega = 0$). The maximum growth rate is

$$Im\omega = \left(\frac{n_b}{n_0\gamma}\right)^{\frac{1}{2}} \omega_p \qquad (5)$$

The wavelength of the maximum growth mode is on the order of the plasma skin depth c/ω_p. In this section we study the stabilization of this instability by introducing a transverse thermal spread on the beam and by applying an axial magnetic field. The transverse thermal energy required to stablize filamentation reflects the Landau damping condition that a particle move through an instability wave length during an e-folding time of the instability. Then the instability can be suppressed by phase mixing. Thus if the thermal velocity satisfies

$$\beta_\perp > \left(\frac{n_b}{n_p\gamma}\right)^{\frac{1}{2}} \qquad (\beta_\perp = v_\perp/c) \qquad (6)$$

then the instability can be stabilized. This gives a threshold transverse beam energy of

$$T_\perp = \frac{1}{2}\gamma m v_\perp^2 = \frac{1}{2}\frac{n_b}{n_p}mc^2 \qquad (7)$$

This transverse energy is independent of the beam energy γ. The thermal energy one needs to stabilize Weibel instability depends only on the ratio of beam density to plasma density. Another method to prevent the beam from breaking up into small clusters is to apply an axial magnetic field to the system, if the beam electron cyclotron frequency is greater than the beam plasma frequency, then the system is always stable.
Cary et al.[15] indicates that the Weibel instability can be completely suppressed when the inequality

$$\frac{\omega_p^2 T_{b\perp}}{mc^2} + \frac{\Omega^2}{\gamma_b} > \omega_b^2\beta_b^2 + \frac{\omega_p^2 T_p}{\gamma mc^2} \qquad (8)$$

is satisfied for the cold plasma case. It is clear that suppressing filamentation with an axial B field is difficult. In a cold plasma the B field needed to stabilize filamentation is proportional to $\sqrt{\gamma n_b}$. Stablization by Landau damping appears to be easier. As stated before, the thermal energy required for the driving beam is independent of the beam's energy. For instance, if the plasma density is $10^{17} cm^{-3}$, beam radius is $170\mu m (k_p a = 12)$ with beam density $10^{16} cm^{-3}$, the transverse thermal energy required for stabilization is $k_B T_\perp \geq 50KeV$. For $\gamma = 1000$, this number corresponds to normalized emittance of $\epsilon_N \approx 10^{-6}mm - mrad$. This appears to be quite easy to achieve.

Lee et al.[16] indicated that the existence of a cutoff wave number k_c implies that filamentation instability is absent in a beam with radius $a \leq \pi/k_c$. Our simulations agree with this theoretical prediction.

To study the filamentation instability, we consider a monoenergetic beam $k_p a = 20$, $k_p l = 14.14$, $\gamma = 10$ and $n_b/n_p = 0.2$; the transformer ratio for this case is 4π. A test traling bunch was added with parameters $n_b/n_p = 0.01$, $\gamma_b = 10$, $k_p a = 10$, $k_p l = 1$. Figure 4. shows the time development in real space. Small perturbations enhance the Lorentz force which attracts nearby beam electrons and repels plasma electrons. As time goes on the filaments tend to coalsece into larger filaments (See. Fig. 4). Fig. 5 is the phase space plots (p_r vs.r) corresponding to Fig. 4. Simulations show that the growth of filamentation is proportional to $1/\sqrt{\gamma}$, consistent with the linear theory eq. 5.

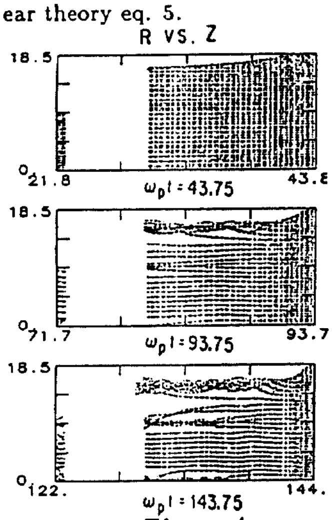

Figure 4

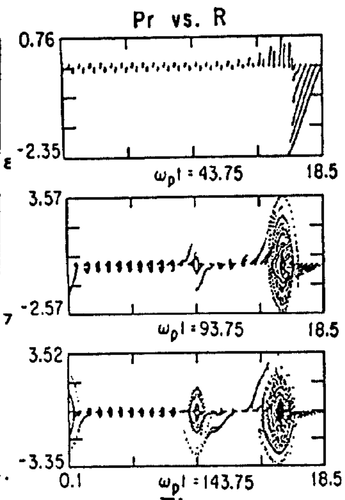

Figure 5

To stabilize the beam a transverse thermal energy $\frac{1}{2}\gamma mc^2\beta_\perp^2 = 25Kev$ is given to the driving beam and an axial B field having $\Omega = \omega_p$ into the system. As a comparson, the other parameters are the same as the first trial. Putting these numbers into eq. 8, the LHS is still slightly less than the RHS. Simulation results show that filamentation is clearly suppressed. The driving beam doesn't display any filamentary structure up to time $\omega_p = 200$ when some trailing particles have already slipped into the decceleration region. Figure 6 shows the time development of the driving and accelerated beams in configuration space. The phase space plots (p_r vs. r) of the driving beam are shown in Fig. 7. Note how different Fig 6 and 7 are from Fig 4 and 5. It's interesting and important to note that the two stream instability has also been observed to be suppressed by the introduction of thermal spread in the beam. For similar parameter but a cold beam the two stream instability is observed in 1-D simulations. Thus it appears that Landau damping may suppress both Weibel and two stream instabilities simulataneously.

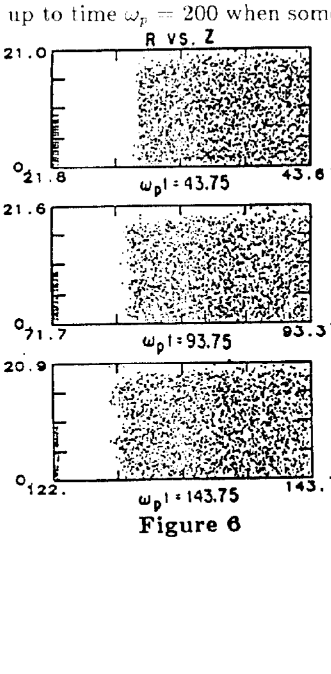

Figure 6

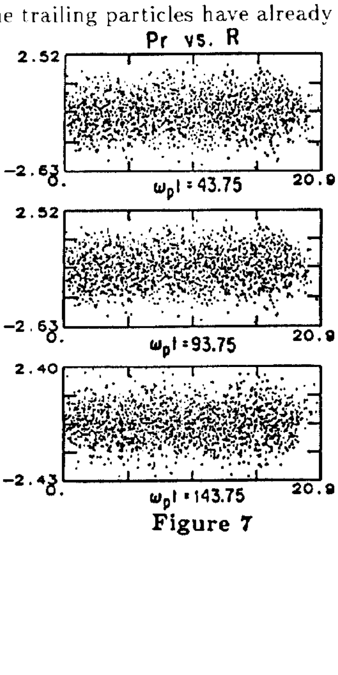

Figure 7

IV. Conclusion

We have reported results from two-dimensional simulations of beam plasma instabilities in the PWFA. We find that beams with radius $k_p a \leq 1$ are strongly self-focused. The self-focusing can be avoided by using uniform broad beams . Such broad beams are subject to Weibel (filamentation) instability. This instability can be stabilized by introducing an axial B field and/or transverse thermal energy in the beam. The thermal energy required is of order $\frac{1}{2}(n_b/n_p)mc^2$, and is independent of γ. For a typical 1 Gev driving beam this corresponds to an emittance of $10^{-6}mm - mrad$. With the introduction of B_z and $T_\perp$ we find that both two stream and Weibel instabilities are suppressed.

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