# IMPROVING THE ENERGY RESOLUTION OF LEP EXPERIMENTS

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## Summary

The energy resolution of a LEP experiment could be considerably enhanced by means of a monochromator insertion. These schemes work by introducing opposite correlations between energy and vertical position of particles at the interaction point (IP) in each of the colliding beams. In consequence, and despite the limitations on our ability to reduce the natural energy spread in the beams, the spread in centre-of-mass energies of the interactions would be considerably reduced. We explain why LEP appears to be the first electron-positron storage ring at which most of the obstacles to the realisation of such a scheme can be overcome. Moreover, the size of the machine gives us the option of replacing the electrostatic quadrupoles considered in previous monochromator designs by RF-magnetic quadrupoles. Our detailed design requires a doublet of either kind of device on either side of the IP, entails minimal disturbance to the layout of other hardware and allows the normal LEP optics to be restored with ease. We evaluate the limits to its performance, giving special attention to the generation of vertical emittance by quantum excitation of coupled betatron oscillations in the monochromator insertions.

## 1 Introduction

The LEP machine under construction at CERN will provide $e^+e^-$ collisions with high luminosity focussed on a narrow region of the particle mass spectrum. At the $Z_0$ pole ($2\times 46.5\,\mathrm{GeV}$), for example, the machine can be operated in such a way [1] that the r.m.s. spread in centre-of-mass energies of the interactions is $\sigma_w \simeq 52\,\mathrm{MeV}$ and practically the full luminosity is obtained. Nevertheless, if this energy resolution could be improved still further, the answers to more physics questions [2] could be within reach. As a further bonus, the running time needed to obtain other results could be reduced.

With a normal low-$\beta$ insertion, such as in the nominal LEP optics [3], $\sigma_w$ is determined by the energy spread, $\sigma_\epsilon$ in the beams—about which little can be done beyond changing the damping partition number, $J_\epsilon$, in a range limited by the concomitant increase of emittance. The principles of various interrelated schemes [4,5,6] for improving the situation have been known for many years but have not yet been put into practice. In most previous studies ([6] is an exception) it was found that constraints of space, machine optics and hardware made the potential reductions of $\sigma_w$ seem barely worthwhile. We shall show that this is not the case for LEP.

## 2 Energy resolution and beam parameters

Denote the phase space coordinates of particles in each beam by $\mathbf{X}=(x,p_x,y,p_y,z,\varepsilon)$ where $x=x_\beta+\eta_x\varepsilon$ and $y=y_\beta+\eta_y\varepsilon$ are the radial and vertical displacements from the closed orbit, $\varepsilon=(E-E_0)/E_0$ is the energy deviation and $p_x$, $p_y$ and $z$ are appropriate conjugate variables; $\eta$ denotes dispersion functions.

Even under the influence of the beam-beam effect the particles in a bunch are distributed according to a phase space distribution which is well-approximated by a gaussian [8] in the normal mode variables. At the IP, it may be written

$$f_\pm(\mathbf{X})=\frac{\hat f_\pm(p_x,p_y,z)}{(2\pi)^{3/2}\sigma^*_{x\beta}\sigma^*_{y\beta}\sigma_\epsilon}\exp\left\{-\frac{(x-\eta^*_x\varepsilon)^2}{2{\sigma^*_{x\beta}}^2}-\frac{(y-\eta^*_y\varepsilon)^2}{2{\sigma^*_{y\beta}}^2}-\frac{\varepsilon^2}{2\sigma^2_\epsilon}\right\}$$

where quantities referring to the IP are "starred" and positron and electron bunches are distinguished with subscripts + or −. The details of $\hat f_\pm$ are of no concern here; note that $f_\pm$ is normalised to unity.

For any function $\mathcal{A}(\mathbf{X}_+,\mathbf{X}_-)$ (possibly depending on variables of particles from both bunches), define averages over the distribution functions of the bunches at the IPs as

$$\langle\mathcal{A}\rangle^\pm=\int f_\pm(\mathbf{X}_\pm)\mathcal{A}(\mathbf{X}_+,\mathbf{X}_-)\,d\mathbf{X}_\pm,\qquad \langle\mathcal{A}\rangle^*=\left\langle\langle\mathcal{A}\rangle^+\right\rangle^-. \quad (1)$$

Then one may evaluate the correlation functions of position and energy deviation

$$\langle x\varepsilon\rangle^\pm=\eta^*_{x\pm}\sigma^2_\epsilon,\qquad \langle y\varepsilon\rangle^\pm=\eta^*_{y\pm}\sigma^2_\epsilon. \quad (2)$$

In terms of the numbers of particles in each bunch, $N_\pm$, the number of bunches, $k_b$, and the revolution frequency $f_0$, the *luminosity* is

$$L=k_bf_0N_+N_-\left\langle\delta(x_+-x_-)\delta(y_+-y_-)\right\rangle^*. \quad (3)$$

When a positron with energy deviation $\varepsilon_+$ collides with an electron of energy deviation $\varepsilon_-$, the centre-of-mass energy is

$$w=\hat w(\varepsilon_+,\varepsilon_-)\stackrel{\mathrm{def}}{=}2E_0\sqrt{1+\varepsilon_+}\sqrt{1+\varepsilon_-}\simeq E_0(2+\varepsilon_++\varepsilon_-)$$

and the *differential luminosity* [5,6,9] or luminosity per unit centre-of-mass energy is

$$\Lambda(w)=k_bf_0N_+N_-\left\langle\delta(x_+-x_-)\delta(y_+-y_-)\delta\left(w-\hat w(\varepsilon_+,\varepsilon_-)\right)\right\rangle^*. \quad (4)$$

Clearly, $L=\int_0^\infty\Lambda(w)\,dw$, and $w$ has a gaussian distribution about the mean $2E_0$ with standard deviation $\sigma_w$ given by

$$\sigma^2_w=L^{-1}\int_0^\infty w^2\Lambda(w)\,dw-4E_0^2. \quad (5)$$

### Normal optics

In this case, which we treat for reference, both dispersion functions vanish at the IP: $\eta^*_{x\pm}=\eta^*_{y\pm}=0$ and (2) shows that there is no correlation between the energy of a particle and its position in space when bunches collide; (3) and (4) take the canonical values

$$L=L_0=\frac{k_bf_0N_+N_-}{4\pi\sigma^*_{x\beta}\sigma^*_{y\beta}}. \quad (6)$$

$$\Lambda(w)=\Lambda_0(w)=\frac{L_0}{\sqrt{2\pi}\sigma_w}\exp\left\{\frac{-(w-2E_0)^2}{2\sigma^2_w}\right\} \quad (7)$$

where $\sigma_w=\sqrt{2}\,\sigma_\epsilon E_0$.

### Monochromator optics

A monochromator scheme reduces $\sigma_w$ without reducing $\sigma_\epsilon$. To achieve this, opposite correlations between spatial position and energy are induced in the colliding beams. In beam-optics terms, this requires vertical dispersion functions, of opposite signs for

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## 4 Vertical emittance

The luminosity and $\lambda$ obtainable depend critically upon the vertical emittance. An important contribution to this is generated by quantum excitation in the dipoles of the dispersion suppressor (between SQ1 and SQ2) and this *cannot* be calculated in simple analogy with the familiar results for the radial plane.

Consider the emission of a single photon of energy $u$ at a point $s$ (Fig. 1) in one of these dipoles, exciting a radial betatron oscillation with components $\mathbf{x}_\beta(s) = (\eta_x u, \zeta_x u) = \boldsymbol{\eta} u$, ($\zeta_x \simeq \eta'_x$). Treating the region between SQ1 and SQ2 as an uncoupled transfer line, this oscillation propagates to the entrance of SQ1 (labelled $i$) according to a $2 \times 2$ transfer matrix $M_x(i,s)$. (We are of course free to express $M_x$ in terms of the optical functions of the *uncoupled* machine.) At the exit of SQ1 the oscillation has acquired a vertical component

$$\mathbf{y}_\beta(o) = Q_{yx}\mathbf{x}_\beta(i) = uQ_{yx}M_x\boldsymbol{\eta}, \tag{15}$$

where $Q_{yx}$ is the appropriate off-diagonal $2\times 2$ block of the $4 \times 4$ transfer matrix of SQ1: in thin-lens approximation

$$Q_{yx} = \begin{pmatrix} 0 & 0 \\ K_1 L_1 & 0 \end{pmatrix}. \tag{16}$$

In terms of the uncoupled optics, the betatron invariant is then

$$\mathcal{H}_y = \mathbf{y}_\beta^T(o)\Gamma_y(o)\mathbf{y}_\beta(o), \qquad \Gamma_y(o) = \begin{pmatrix} \gamma_y(o) & \alpha_y(o) \\ \alpha_y(o) & \beta_y(o) \end{pmatrix}. \tag{17}$$

Combining (15) and (17) and then averaging over all possible photon emissions, we find the expectation value of the emittance growth in a single passage of the monochromator

$$\Delta\mathcal{H}_y = \int_a^i N(s)\left\langle u^2\right\rangle \boldsymbol{\eta}^T M_x^T Q_{yx}^T \Gamma_y(o) Q_{xy} M_x \boldsymbol{\eta}\, ds/c, \tag{18}$$

where $N(s)$ is the local photon emission rate.

Well-known arguments [11] can be invoked to incorporate the counter-effect of radiation damping and it follows that the vertical emittance due to quantum excitation in the monochromator insertions is

$$\epsilon_{yq} = \frac{55}{32\sqrt{3}}\frac{\hbar}{mc}\left(\frac{E_0}{mc^2}\right)^2 \frac{I_{5yx}}{I_2}, \tag{19}$$

where the notations are all standard [12] except that $I_{5yx}$ is given by replacing $N(s)\langle u^2\rangle$ by $|G(s)^3|$ in (18); here $G = 1/\rho_0$ is the scaled dipole strength. One must also multiply by $2N_M$ where $N_M$ is the number of monochromators installed. Explicitly, one finds

$$\begin{aligned} I_{5yx} = \; & 2N_M(K_1L_1)^2\beta_y(o)\beta_x(i)\int_a^i |G|^3\left\{\frac{\eta_x^2}{\beta_x}(\cos\psi + \alpha_x\sin\psi)^2\right. \\ & \left. +\eta_x\zeta_x\sin\psi(\cos\psi + \alpha_x\sin\psi) + \zeta_x^2\beta_x\sin^2\psi\right\} ds \end{aligned} \tag{20}$$

where all quantities in the integrand are to be evaluated at the point $s$ and $\psi$ is the horizontal phase advance from $s$ to SQ1.

At first sight one might also expect a contribution from the diagonal block $Q_{yy}$ of the skew quadrupole matrix because a betatron oscillation is generated in the vertical plane. This is at the root of an estimate by false analogy with the radial plane. In fact this betatron oscillation makes no contribution to $\epsilon_{yq}$. Since there are no vertical bends the *absolute* vertical trajectory of the particle is unchanged (neglecting small chromatic effects) and the dispersive phase-mixing required to produce emittance does not take place!

Evaluating (19) and (20) for the monochromator insertion by numerical integration over the 6 dipoles of the dispersion suppressor and using Table 2, we find

$$\epsilon_{yq} = 5.32 \times 10^{-11} N_M \quad \pi\mathrm{m}, \tag{21}$$

independently of $E_0$. For the total vertical emittance, we include a contribution from residual coupling (due to errors *etc.*)

$$\epsilon_{yc} = \epsilon_{yq} + \kappa^2\epsilon_{xc}, \qquad \kappa = 0.1. \tag{22}$$

Since $\epsilon_{xc} \propto E_0^2$, the coupling contribution will dominate at high enough energy.

## 5 Performance

To avoid a complicated optimisation of parameters, we assume that *the beams are separated in those IPs without monochromators*. Then the contribution of $\eta_y^*$ to the beam height $h_y$ dominates over the betatron size and (5) shows that $\sigma_w$ is essentially independent of $\sigma_\epsilon$ and is minimised by minimising $\epsilon_{yc}$. Accordingly, we use the high tune (90° phase advance per arc cell) of the LEP lattice and set the damping partition number $J_\epsilon = 0.5$. *Since $h_y$ is increased the severity of vertical beam-beam effects is reduced*—*as they must be* [13] *if the monochromaticity is not to be lost*. One may run in a regime where normal luminosity is reduced by a factor $\sim\lambda$ and there is little beam-beam blow-up.

| $E$/GeV | 40 | 46.5 | 55 | 70 | 90 |
|---|---|---|---|---|---|
| $\epsilon_{xc}$/nm | 3.5 | 4.8 | 6.6 | 11. | 18. |
| $\sigma_\epsilon/10^{-3}$ | 1.6 | 1.6 | 1.9 | 2.4 | 3.1 |
| $\eta_y^*$/cm | 3.0 | 2.6 | 2.2 | 1.7 | 1.4 |
| $\epsilon_{yc}$/nm | 0.25 | 0.30 | 0.37 | 0.53 | 0.81 |
| $h_y$(IP)/$\mu$m | 43. | 42. | 43. | 43. | 43. |
| $h_y$(QS4)/mm | 1.4 | 1.4 | 1.6 | 2.1 | 2.7 |
| $\lambda$ | 10. | 9.3 | 8.3 | 7.0 | 5.7 |
| $\sigma_w$/MeV | 7.7 | 11. | 18. | 35. | 70. |

Table 4: Performance of LEP with 2 monochromators

Final results for LEP with 2 monochromators installed are given in Table 4. The vertical aperture limitation [9] would occur in the quadrupole QS4 where the acceptance is 33 mm. The values of $h_y$ at QS4 (with full coupling) show that the beam is still comfortably accommodated.

If the beams are allowed to collide at the other interaction points, we expect a degradation of $\lambda$ and $\sigma_w$ by at least a factor of 2.

**Acknowledgement**: We thank E. Keil for helpful discussion.

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the two beams, to be created at the IP. Because $\sigma^*_{y\beta} = \sqrt{\epsilon_{y}\beta^*_y} \ll \sigma^*_{x\beta}$ it is more efficient to do this in the vertical plane. In addition we treat a horizontal dispersion (possibly due to errors) which is the same for both beams:

$$\eta^*_{x+} = +\eta^*_{x-} = \eta^*_x, \qquad \eta^*_{y+} = -\eta^*_{y-} = \eta^*_y. \tag{8}$$

Evaluating (3) and (6) gives

$$L = \frac{L_0}{\sqrt{1+\frac{\eta_x^{*2}\sigma_\epsilon^2}{\sigma^{*2}_{x\beta}}}\sqrt{1+\frac{\eta_y^{*2}\sigma_\epsilon^2}{\sigma^{*2}_{y\beta}}}}. \tag{9}$$

while (4) and (7) give

$$\Lambda(w) = \frac{\Lambda_0(2E_0)}{\sqrt{1+\frac{\eta_x^{*2}\sigma_\epsilon^2}{\sigma_{x\beta}^{*2}}}} \exp\left\{-\left(\frac{\eta_y^{*2}}{\sigma_{y\beta}^{*2}}+\frac{1}{\sigma_\epsilon^2}\right)\frac{(w-2E_0)^2}{2(\sqrt{2}E_0)^2}\right\}, \tag{10}$$

$$\Lambda(2E_0) = \frac{\Lambda_0(2E_0)}{\sqrt{1+\frac{\eta_x^{*2}\sigma_\epsilon^2}{\sigma_{x\beta}^{*2}}}}, \tag{11}$$

*i.e.* the differential luminosity at the centre of the distribution in centre-of-mass energies is reduced by the radial dispersion while the standard deviation of $w$ is reduced by virtue of the vertical dispersion only:

$$\sigma_w = \frac{\sqrt{2}E_0\sigma_\epsilon}{\sqrt{1+\frac{\eta_y^{*2}\sigma_\epsilon^2}{\sigma_{y\beta}^{*2}}}}. \tag{12}$$

This effect is quantified by defining the *enhancement of energy resolution*

$$\lambda = \frac{\sqrt{2}\sigma_\epsilon E_0}{\sigma_w} = \sqrt{1+\frac{\eta_y^{*2}\sigma_\epsilon^2}{\sigma_{y\beta}^{*2}}}. \tag{13}$$

There is no correlation between $w$ and the *vertical* position of the interaction vertex. The *horizontal* dispersion may help experimenters by correlating $w$ with the position of the interaction vertex [4].

Dispersions which are opposite at the IP enhance energy resolution without detriment to $\Lambda$ while dispersions which have *the same sign degrade both differential and total luminosities.* In either case, total luminosity for the same beam intensities is somewhat reduced.

## 3 Monochromator optics

To produce opposite dispersions for the two beams, devices exerting opposite forces on the electrons and positrons are required. We follow the principle [6,7] of using a skew electrostatic quadrupole (SQ2) in the dispersion suppressor to generate $\eta_y$ from $\eta_x$; see Fig. 1. A second such quadrupole (SQ1) is used to

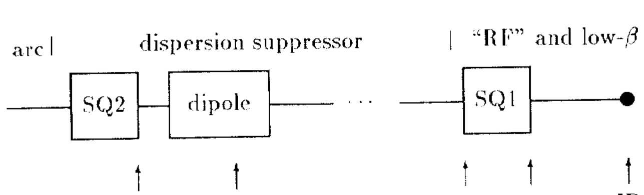


Figure 1: Schematic layout of monochromator

undo the betatron coupling created by the first. The scheme imposes very stringent conditions on the betatron phase advances. In thin-lens approximation, the integrated strengths of the skew quadrupoles are related by

$$K_1L_1\sqrt{\beta_{x1}\beta_{y1}} = -K_2L_2\sqrt{\beta_{x2}\beta_{y2}} \tag{14}$$

and $\eta^*_y$ depends on the initial $\eta_x$

$$\eta^*_y = \sqrt{\beta^*_y\beta_{y2}}\,\eta'_{y2} = \sqrt{\beta^*_y\beta_{y2}}\,K_2L_2\eta_{x2}.$$

A vertical dispersion exists only in the insertion and there is *no coupling between vertical and radial betatron* modes in the rest of the machine.

The authors have made a variety of complete rematchings of the LEP insertions which satisfy all the monochromator conditions *and* the most important constraints normally imposed on the dispersion suppressor and low-$\beta$ insertions. One such solution [9] is summarised in Table 1. Disruption of the normal lattice layout has been minimised in that there are no changes of the geometry and no magnet need be moved or have its powering changed. There is therefore no difficulty in reverting to the normal optics.

| Element | $\mu_x$ $/2\pi$ | $\mu_y$ $/2\pi$ | $\beta_x$ /m | $\beta_y$ /m | $\eta_x$ /m | $\eta_y$ /m |
|---|---|---|---|---|---|---|
| IP | 0 | 0 | 1.75 | 0.07 | 0.00 | 0.027 |
| SQ1 | 0.85 | 5/4 | 69.5 | 44.6 | 0.00 | 0.00 |
| SQ2 | 1.85 | 9/4 | 57.3 | 109. | 1.49 | 0.01 |

Table 1: Monochromator optics for LEP at 45 GeV

It proved possible to match the monochromator very well to the rest of the machine. This is a consequence of the modular nature of the LEP insertions (where the functions of *e.g.* the low-$\beta$ and dispersion suppression sections are achieved quite independently) the relatively large number of independent quadrupole families and the availability of space for the electrostatic quads. Smaller machines [5,6,7] do not enjoy these advantages.

### Installation and hardware

Realistic parameters for maximum effect (limited by SQ2) with electrostatic quadrupoles are given in Table 2.

| | $L$ /m | pole radius | $\pm$potential /kV | $KE_0$ $/\mathrm{m}^{-2}\mathrm{GeV}$ |
|---|---|---|---|---|
| SQ1 | $2\times5$ | 5 cm | 53.6 | $4.28\times10^{-2}$ |
| SQ2 | 5 | 5 cm | 75 | $5.99\times10^{-2}$ |

Table 2: Limiting parameters of electrostatic quadrupoles

Since a vertical dispersion in the RF cavities would lead to severe synchro-betatron resonance effects, monochromator insertions cannot be installed in the 2 straight sections which contain RF in Phase I [3]. However LEP might be brought up to $w \simeq 180$ GeV with superconducting RF these insertions [10], leaving the other two free for monochromators.

Another hardware option which is open to LEP is to use RF magnetic quadrupoles; the frequencies are chosen so that the fields have opposite signs when bunches of opposing beams go by. The ring is so large that the frequencies needed are low enough

| $h_1$ | $h_2$ | $f_1$ /kHz | $f_2$ /kHz | $K_1$ $/K_{\max}$ | $K_2$ $/K_{\max}$ |
|---|---|---|---|---|---|
| 32 | 16 | 359.85 | 179.93 | 1.000 | 0.998 |
| 16 | 16 | 179.93 | 179.93 | 0.697 | 0.998 |
| 32 | 32 | 359.85 | 359.85 | 1.000 | 0.133 |

Table 3: Solutions with RF magnetic quadrupoles

for there to be little difficulty in constructing suitable hardware. Table 3 gives some possible harmonic numbers (preserving the possibility of 4-bunch operation), frequencies and fractions of the peak gradient $K_{\max}$ available to act on the particles as they pass [9].

Such quadrupoles could be built and produce fields of the order of 500 gauss although a detailed engineering study is still required to compare them with the electrostatic devices.