STUDIES OF ANOMALOUS DISPERSION IN THE SLC SECOND ORDER ACHROMAT*

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Summary

Certain causes of anomalous dispersion in the second order achromats of the SLC arcs are investigated. For matched dispersion, transverse displacements of combined function magnets do not introduce anomalous dispersion. This is shown by deriving a non-dispersive condition connecting the average of the matched dispersion function with the quadrupole and sextupole components of the field. In the SLC Arcs, however, the achromats are rolled producing a dispersion mismatch. In this case, the horizontal (vertical) dispersion is affected linearly by vertical (horizontal) displacement of magnets. The integral condition connecting the dipole and quadrupole fields and the matched dispersion is also derived. Combining this with the non-dispersive condition and the analytic expression of the matched dispersion gives two simple relationships for the fields of second order achromats constructed of combined function magnets.

The effects of the dispersion *mismatch* in the SLC Arcs is investigated using computer simulations. The results show that this mismatch will increase the sensitivity to transverse errors. We report the effects of certain *systematic* errors.

Introduction

The FODO cells of the SLC Arcs are put together to form second-order achromats.[1] Each cell is composed of two combined function magnets each having superimposed dipole, quadrupole and sextupole fields. The strength of the dipole field is the same for both magnets. The field gradients in both magnets are nearly the same but the signs are opposite. The sextupole components are different both in the signs and in the strengths.

In this lattice the *matched* dispersion function η, defining the deviation of the trajectory for an off-momentum particle, and its derivative η' with respect to the path length s are both periodic with a period equal to the cell length.

During the course of designing the SLC *orbit correction system* for the SLC Arcs one of us (JJM) observed[2] that transverse translations of the combined function magnets were effective in generating a *non-dispersive orbit correction* . By this is meant that the beam direction can be made to change and that this change is independent of momentum in the linear approximation. It is shown below that this effect is the result of a simple relationship between the average value of the matched dispersion function $\overline{\eta}$ and the strength of the quadrupole and sextupole components in each combined function magnet.

It has been shown[3] that the integral relationship in this simple form consists of the most dominant terms contained in a more general integral expression which can be used to define the properties of a second order achromat. In most situations, where the radius of curvature is large, the simpler form can be used to calculate sextupole strengths which agree with those obtained from TRANSPORT to within a few percent.

The demonstration of this non-dispersive effect was a key part of the decision to adopt transverse displacements of combined function magnets as the method[4,5] *of correcting the beam trajectory in the SLC Arcs.*

Even though the *matched* dispersion function is unperturbed in the linear approximation by either random or coherent displacment of magnets, it was later found[6] that in the SLC Arcs certain coherent displacements did indeed linearly perturb the dispersion. This observation led to several studies of systematic[7] and random[8] errors with the conclusion that this effect stems from the fact that the dispersion in the Arcs is not always matched (especially in the vertical plane) due to the necessity of rolling the achromats about the beam axis to follow the site terrain. We will now review these findings in simple form.

Non-Dispersive Condition

Consider a differential slice of an Arc magnet with length ds. The transverse field components can be expressed as follows,

$$B_y = B_o + B_o'x + \frac{1}{2}B_o''(x^2 - y^2), \tag{1}$$

$$B_x = B_o'y + B_o''xy, \tag{2}$$

where the coefficients B_o, B_o' and B_o'' represent the dipole, quadrupole and sextupole strengths of the combined function magnet, respectively. Here the prime indicates differentiation with respect to transverse displacement.

Suppose at this location that there is no dispersion in the y plane and that the horizontal dispersion is equal to η. Suppose further that the elemental magnet is displaced from the reference axis by Δx and Δy. Then, for a momentum $p = p_o(1+\delta)$,

$$x = \eta\delta + \Delta x, \tag{3}$$

$$y = \Delta y. \tag{4}$$

With these equations inserted in Eqs. (1) and (2), the angular "kicks" $d\theta$ in horizontal and $d\phi$ in vertical planes of a ray passing through the elemental magnet may be written as follows,

$$d\theta = \frac{-ds}{p_o(1+\delta)}\Big\{(B_o + B_o'\eta\delta + \frac{1}{2}B_o''\eta^2\delta^2)$$
$$+(B_o' + B_o''\eta\delta)\Delta x + \frac{1}{2}B_o''(\Delta x^2 - \Delta y^2)\Big\}, \tag{5}$$

$$d\phi = \frac{ds}{p_o(1+\delta)}\Big\{(B_o' + B_o''\eta\delta)\Delta y + B_o''\Delta x\Delta y\Big\}. \tag{6}$$

Consider only the terms up to the linear approximation in Δx and Δy and neglect all others, then

$$d\theta \approx \frac{-ds}{p_o(1+\delta)}\Big\{B_o(1 + \frac{B_o'\eta}{B_o}\delta) + B_o'(1 + \frac{B_o''\eta}{B_o'}\delta)\Delta x\Big\}, \tag{7}$$

$$d\phi \approx \frac{ds}{p_o(1+\delta)}B_o'(1 + \frac{B_o''\eta}{B_o'}\delta)\Delta y \quad . \tag{8}$$

Clearly, the momentum dependence in both equations will factor out if the following conditions are satisfied everywhere,

$$\frac{B_o'\eta}{B_o} = 1 \quad \text{and} \quad \frac{B_o''\eta}{B_o'} = 1 \quad . \tag{9}$$

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Eqs. (9) cannot be satisfied at all points since within a magnet η varies while B_o B_o' and B_o'' do not. However, if η is averaged over a single Arc magnet the second equation in Eqs. (9) is satisfied, i.e.,

$$\frac{B_o''\overline{\eta}}{B_o'}\Big|_{single\ magnet} = 1 \quad . \tag{10}$$

Eq. (10) is the *non-dispersive condition* given in Ref. 2. This equation is satisfied by the fields of combined function magnets in second order achromats.

Eqs. (7),(8), and (10) can be used to conclude that in the linear approximation the *matched dispersion function η is immune to effects due to random or purposely induced transverse displacements of single magnets* in an achromat composed of combined function magnets. For the SLC Arcs a transverse displacement of 100 microns for Δx (Δy) steers a beam horizontally (vertically) with a strength equivalent to 1.2% of the dipole bending field B_o making such displacements an effective method of steering. Furthermore, since this non-dispersive property applies to a single magnet such a magnet can be used to provide a non-dispersive "kick" anywhere in any lattice provided that the average value of the dispersion is not zero.

Relationship of Field Components

The first equation in Eqs. (9) is also satisfied when η is averaged over each magnet of an entire cell, i.e.,

$$\frac{B_o'\overline{\eta}}{B_o}\Big|_{entire\ cell} = 1 \quad . \tag{11}$$

This is shown to follow from the definition of the η function and a general integral equation derived in Ref. 9 and which apply to a cell of an *arbitrary lattice.* Eqs. (10) and (11) can be used to derive relationships for the field components correct to a few percent for achromats composed of combined function magnets. These relationships are useful because for such achromats η can be integrated analytically as in Ref. 9 . Using the SLC Arc cell for an example, Eqs. (10) and (11) become

$$\frac{B_{oF}''\overline{\eta}_F}{B_o'} = 1 \quad , \qquad \frac{B_{oD}''\overline{\eta}_D}{B_o'} = 1 \quad , \tag{12}$$

and

$$\frac{|B_o'|}{2B_o}(\overline{\eta}_F - \overline{\eta}_D) = 1 \quad , \tag{13}$$

where the subscripts F and D refer to the focusing and defocusing magnets, respectively. Similar relationships can be written for combined function lattices of second order achromats composed of magnets with differing lengths and field components.

Effect of Mismatched Dispersion

Equations (3) and (4) are now rewritten to allow the position in the differential slice to include deviations from the *matched* dispersion. Let

$$x \;=\; \Delta x + \eta_x\delta \;\equiv\; \Delta x + (\eta_o + \Delta\eta_x)\delta \quad , \tag{14}$$

and

$$y \;=\; \Delta y + \eta_y\delta \;\equiv\; \Delta y + \Delta\eta_y\delta \quad , \tag{15}$$

where $\Delta x, \Delta y$ are transverse displacements, η_x, η_y are the actual dispersion functions, η_o is the matched η-function and $\Delta\eta_x$ and $\Delta\eta_y$ are the differences between the actual and matched functions.

Again, Eqs. (14) and (15) are inserted in Eqs. (1) and (2). Then retaining only terms linear in Δx and Δy, averaging the dispersion over the length of the moved magnet, l, and separating the remaining terms in dispersive and non-dispersive groups, one finds

$$\Delta\theta \approx \frac{-l}{p_o(1+\delta)}\left\{\left(B_o' + B_o''\overline{\eta}_o\delta\right)\Delta x + B_o''\delta\left(\overline{\Delta\eta}_x\Delta x - \overline{\Delta\eta}_y\Delta y\right)\right\} \tag{16}$$

$$\Delta\phi \approx \frac{l}{p_o(1+\delta)}\left\{\left(B_o' + B_o''\overline{\eta}_o\delta\right)\Delta y + B_o''\delta\left(\overline{\Delta\eta}_x\Delta y + \overline{\Delta\eta}_y\Delta x\right)\right\} \tag{17}$$

Inserting Eq. (10) and taking the derivative with respect to δ at the point $\delta = 0$, the anomalous $\Delta\eta'$, for x and y planes, respectively, become

$$\Delta\eta_x' = \frac{-B_o''l}{p_o}\left(\overline{\Delta\eta}_x\Delta x - \overline{\Delta\eta}_y\Delta y\right) \tag{18}$$

$$\Delta\eta_y' = \frac{B_o''l}{p_o}\left(\overline{\Delta\eta}_x\Delta y + \overline{\Delta\eta}_y\Delta x\right) \tag{19}$$

These equations show that in the presence of a magnet misalignment a deviation of the dispersion from that of the *matched* dispersion will generate an anomalous $\eta_{x,y}'$. This is important to the SLC Arcs in that the dispersion is not always *matched* due to the required rolls. The consequence is that the Arcs are more sensitive to misalignment than if they had been constructed flat. For a typical 10^o roll as in the Arcs, $\overline{\Delta\eta}_x$ is small compared to $\overline{\Delta\eta}_y$ so the main contribution to both $\Delta\eta_{x,y}'$ comes from the terms proportional to $\overline{\Delta\eta}_y$. That means that $\Delta\eta_x'$ is produced by a vertical displacement Δy and vice versa. On the other hand, for the whole Arc the accumulated error in $\overline{\Delta\eta}_x$ becomes large and both terms in Eqs. (18) and (19) are significant.

Computer Simulations

The predictions for the behavior of anomalous dispersion given in Eqs. (18) and (19) have been compared with computer simulations of both rolled and not rolled achromats. For the displacement of a single magnet, the results are in good agreement. That is, for an unrolled achromat (matched dispersion) there is *no effect*, whereas for a rolled achromat, the induced anomalous dispersion, as calculated using Eqs. (18) and (19), agree quantitatively with the simulation results. This agreement was confirmed for several magnets located at arbitrary points where the mismatched dispersion was either large or small in magnitude.

Systematic Translations of Magnets

With agreement established for simple cases, computer simulations were now used to investigate the effects of systematic errors in whole achromats and finally for the entire north arc.

Systematic translations of magnets in the arcs can have many causes; here we will examine two particular ones. In the arcs there is a one-to-one correspondence between magnet movers (steering correctors) and beam position monitors (BPM's). In this scheme each focusing magnet is moved horizontally (vertically) to steer the beam through the BPM_x (BPM_y) attached to the next focusing (defocusing) magnet. This scheme utilizes 10 correctors and 10 monitors in the horizontal plane and 10 correctors and 10 monitors in the vertical plane per achromat. The position monitors are placed in the drifts between magnets. Thus the matched dispersion has the same magnitude ($\eta_{BPM} \sim 35$mm) at all BPM's. Therefore, either a) a systematic error in BPM_x alignment or signal processing or b) steering a beam which has the wrong energy with respect to the arc excitation will cause a systematic offset of all horizontally focusing magnets. A systematic BPM_y error will do the same in the vertical plane. Fortunately, these systematics will show up in the harmonic analysis of the magnet mover positions and, in principle, is correctable for magnitudes greater than $\sim$ 30 microns. Steering an off-momentum beam with $\frac{\Delta P}{P} = 10^{-3}$ will induce a systematic error in the magnet positions of 28 microns which is just detectable.

In Figures 1 and 2 $\Delta\eta_x$ is shown versus a systematic misalignment of BPM_x's or relative momentum error. Here, $\Delta\eta_x = \eta_x - \eta_o$ where $\eta_o = 47$mm at the end of the north arc.

vertical dispersion η_y is affected nonlinearly in both of the above cases, changing by $\sim$ 12mm (compared to its nominal value of zero) for a BPM_x offset of 100 microns.

A systematic error in the position of the vertical BPM_y's will also cause dispersion changes but at a level reduced by an order of magnitude.

Conclusion

In the linear approximation the *matched* dispersion function is not affected by random or coherent displacements of magnets in a second order achromat made up of combined function magnets. For the SLC Arcs the necessity of rolling the achromats has caused a dispersion *mismatch* which has increased the sensitivity to transverse errors. In particular, the increased sensitivity to systematic errors will require close attention to these errors in component alignment and signal processing electronics for both the beam position monitors and the orbit correcting magnet movers.

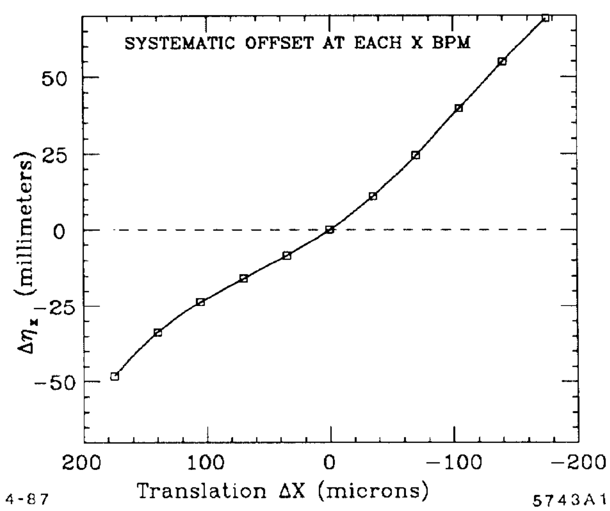

Fig. 1. Error in η_x when beam is steered with a *Horizontal* displacement at entrance of each focusing magnet in north Arc.

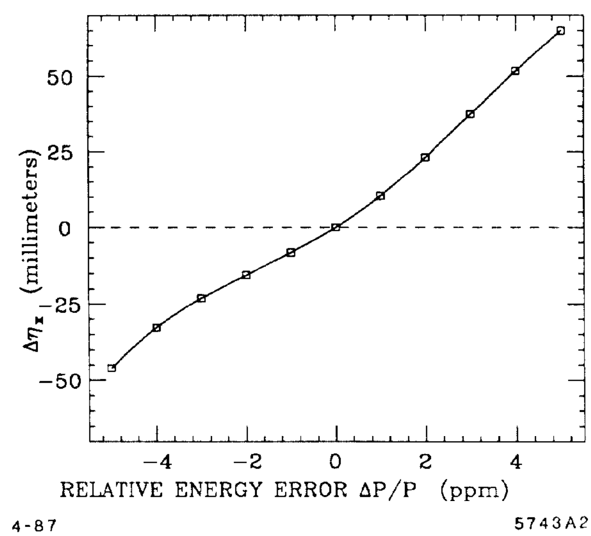

Fig. 2. Error in η_x when off momentum beam is steered through the north Arc.

In Figure 1, each BPM_x is offset by the same amount and the arc steering algorithm applied. Because of the one-to-one correspondence between each BPM_x and the upstream magnet mover a systematic translation is introduced at every horizontally focusing magnet. A systematic offset of 100 microns at the BPM_x's will cause an equivalent offset of $\sim$ 80 microns at each horizontally focusing magnet. This same relationship holds for the vertical plane. It can be seen in Figure 1 that a systematic displacement of this magnitude will cause a $50 - 80\%$ change in the horizontal dispersion function η_x at the end of the north arc.

Next, it can be seen that the results shown in Figure 2 are similar to those shown in Figure 1. Here a beam of the wrong momentum with respect to the arc excitation was steered through the system. In this case, prior to the application of the steering algorithm, the centroid of the beam would have the same offset at each BPM_x. The action of forcing this centroid offset to become zero at each BPM_x again introduces a systematic offset of the horizontally focusing magnets. Thus steering a beam with a relative momentum error of 3×10^{-3} is equivalent to a systematic error of $\sim$ 100 microns at the BPM_x's. The

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