# PROPOSED USE OF THE RADIO-FREQUENCY QUADRUPOLE STRUCTURE TO FUNNEL HIGH-CURRENT ION BEAMS*

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## Introduction

Several new linear accelerator applications require output ion beams with very high current and small transverse emittance. The transverse beam brightness (beam current divided by the product of the x- and y-emittances) is often used as the figure of merit, and the design objective is to obtain a very high brightness output beam. When linear accelerator channels operate near their space-charge limit, one method to increase brightness is to combine the beams from two (or more) space-charge-limited channels. The resultant beam would have interlaced microstructure bunches and would be suitable for further acceleration in a linac operating with twice the frequency. This operation has been called funneling and was first proposed as a necessary part of the rf linac approach to heavy-ion fusion. If the funneling of two beams is accomplished with no transverse emittance growth or beam loss, the resulting beam will have twice the brightness of the beams that are combined.

In some applications, to meet brightness requirements, it would be possible to use arrays having multiple beams traversing the whole accelerator system with the final beams all focused to a common target spot. If the required brightness is not too high, funneling to combine multiple beams as early as possible would be especially advantageous in large accelerators where most of the acceleration could then be provided in a minimum-cost single-beam accelerator. In practice, funneling to increase beam brightness is a difficult design problem. Initial work using discrete optical elements has been done by Bongardt[1] and by Guy and Wangler.**

In this paper, we describe a new approach to funneling beams that are initially accelerated in two radio-frequency quadrupole (RFQ) accelerators. Instead of discrete optical elements, we propose to funnel within an RFQ structure, so that during the funneling process the beam is always confined by periodic transverse focusing. Beams with high space charge experience irreversible emittance growth when they emerge from a periodic focusing system. To alleviate this problem, in the proposed funneling system it should be possible to maintain the same focusing periodicity as that of the accelerators preceding the funnel. Also, instead of conventional deflection systems, we propose to use the properties of a modified RFQ structure to deflect two parallel beams toward each other and to merge them into a single final beam.

## Beam Deflection in Periodic Focusing Channels

Our first objective was to devise a method to produce transverse deflections of a beam traveling in a periodic focusing system such as an RFQ. We found that introducing periodic transverse displacements of the focusing and defocusing lenses was very effective in producing a deflecting force. To develop this idea, we used the thin-lens array shown in Fig. 1. The input beam first traversed a focus lens whose center was displaced in the plus x-direction by an amount $\alpha$. Next was a defocus lens displaced in the minus x-direction by the same amount, etc. The lenses

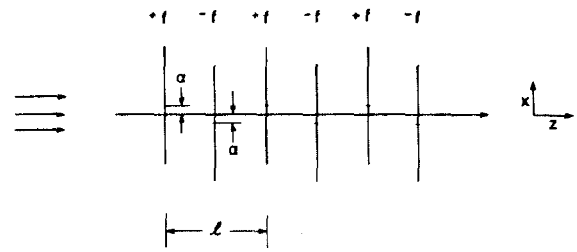


Fig. 1. Periodic thin-lens array used for simulation.

had focal length f and were equally spaced with focusing period $\ell$. This arrangement produces a net force on the beam in the plus x-direction. To determine the beam optical properties, we performed single-particle simulations. We learned that such alternating lens displacements produce a displaced neutral axis about which sinusoidal betatron motion takes place. The betatron frequency is independent of $\alpha$; thus, in the spirit of the smooth approximation, there exists a constant average transverse force, and the betatron motion corresponds to that of a biased, simple-harmonic oscillator.

One of many simulated trajectories is shown in Fig. 2. The particle started at the origin with zero displacement and slope, and the parameters were $\alpha = 0.3$, $f = 16$, and $\ell = 10$, where these three parameters as well as the Fig. 2 displacements are all in the same units of length. The top and bottom curves, respectively, are drawn through the maxima and the minima of the trajectory flutter. The betatron period

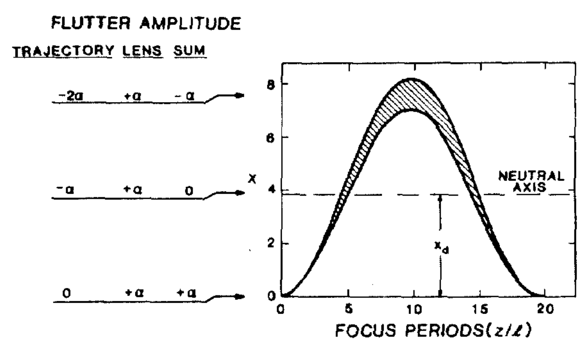


Fig. 2. Results of a thin-lens simulation.

is 20 focus periods, and the average of the two curves is symmetric about the dashed neutral axis that is displaced from the z-axis an amount $x_d$. Note that the trajectory flutter is zero on the z-axis, increases to $\pm\alpha$ at $x = x_d$, and to $\pm 2\alpha$ at $x = 2x_d$. The left part of the figure explains these effects. In the smooth approximation the transverse forces are proportional to the amplitude of the flutter motion. The trajectory flutter produces a restoring force proportional to the displacement from the z-axis. The lens flutter produces a constant force of magnitude proportional to $\alpha$, and these forces balance at $x = x_d$, producing the displaced neutral axis. The algebraic sum of forces from the two types of flutter then produces a restoring force that is linear relative to displacements from the neutral axis. Further, the betatron

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frequency is independent of $\alpha$, so that the focusing properties are preserved.

Next, we formulated a smooth-approximation calculation to describe these effects. The calculations correspond to an rf quadrupole focusing system that has successive lenses displaced transversely from the z-axis. We used a sine-wave displacement function of amplitude $\alpha$ and space period $\beta\lambda = 2\pi v/\omega$, where v is the particle velocity and $\omega$ is the rf quadrupole operating frequency. The appropriate differential equation is

$$M \frac{d^2x}{dt^2} + K(x - \alpha \sin \omega t) \sin \omega t = 0 \quad , \tag{1}$$

where K is the spring constant and M the particle mass. If $\alpha = 0$, Eq. (1) reduces to one form of the Mathieu equation used to describe periodic focusing. We assume a solution of the form

$$x = R \sin \Omega t + S \sin \omega t + x_d \; , \tag{2}$$

where the first term represents the betatron motion, the second term the flutter, and $x_d$ is the displacement of the neutral axis. If we take R>>S, and $\omega \gg \Omega$ (the smooth approximation), we obtain

$$S = \left(\frac{\omega_0}{\omega}\right)^2 (R \sin \Omega t + x_d) \quad , \quad \text{where} \quad \omega_0^2 = K/M \quad .$$

After substituting this into Eq. (2), Eq. (2) into Eq. (1), and then averaging the terms over one rf period, we find the equation satisfied if both

$$\Omega = \frac{\sqrt{2}}{2} \frac{\omega_0^2}{\omega} \quad , \quad \text{and} \quad x_d = \alpha\left(\frac{\omega}{\omega_0}\right)^2 = \frac{\sqrt{2}}{2} \alpha\left(\frac{\omega}{\Omega}\right) \quad . \tag{3}$$

First we see that the betatron frequency $\Omega$ is independent of $\alpha$. Next we observe that $x_d$ is equal to $\alpha$ multiplied by the factors $(\sqrt{2}/2)(\omega/\Omega)$. Because $\omega \gg \Omega$, then $x_d \gg \alpha$, which demonstrates that small transverse displacements are very effective in producing large displacements of the neutral axis. Although we have used the smooth approximation to obtain simple expressions for $\Omega$ and $x_d$, neither the beam manipulations discussed below nor the funneling scheme discussed later are necessarily limited to the region where the smooth approximation is valid.

Now we discuss the use of this deflection force to sidestep a beam from the z-axis to a displaced parallel axis. In doing this, we must use a method that does not induce additional betatron motion that would increase the beam emittance. One possible method is to start with a particle moving along the z-axis and, at the origin turn on an initially constant value of $\alpha$, to induce a half-period betatron motion. When this excursion reaches its full amplitude, we then suddenly double the value of $\alpha$ to move the neutral axis to a point equal to the betatron amplitude. This procedure results in the displaced trajectory shown in Fig. 3, Curve a, where the curve is the mean of the flutter motion. After $\alpha$ has reached its final value of 0.6, Curve a has a residual betatron amplitude of only 0.05% for the particular manner in which $\alpha$ was programmed. Alternatively, the particle could be injected along the displaced neutral axis. Then by reducing $\alpha$ in a programmed manner, the particle could be brought to the z-axis with zero slope. This sidestep maneuver forms one-half of the funneling scheme that in the next section will be extended so that two beams can be brought to a common z-axis. Also shown in Fig. 3 are Curves b and c for which the motion was started with initial ± slopes. The amplitude of the resulting betatron motion is equal to the case with $\alpha = 0$, showing that the sidestep maneuver produces no increase in the transverse emittance.

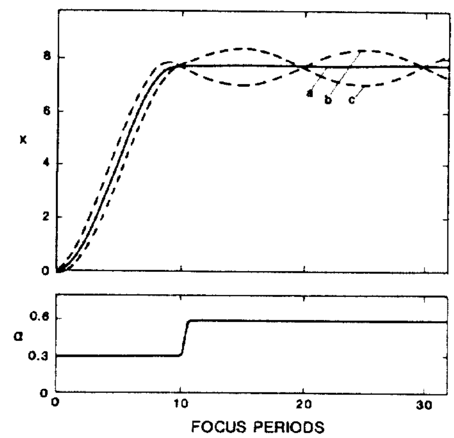


Fig. 3. Sidestep maneuver to displace a beam.

## RFQ Potential Functions

The standard RFQ accelerator potential function can be written in cylindrical coordinates (r, $\psi$, z) as

$$U = \frac{V}{2} \left[X \left(\frac{r}{a}\right)^2 \cos 2\psi + A I_0(kr) \cos kz\right] \sin (\omega t + \phi), \tag{4}$$

where V is the maximum potential difference between adjacent pole tips and $k = 2\pi/\beta\lambda$. Note that the $I_0$ Bessel function is even and symmetric in x and y. To provide a deflection force in the x-direction, we require $I_0$ to be replaced with a function that is odd in x and independent of y. Also, the potential function chosen must satisfy Laplace's equation. One example meeting these requirements is

$$U = \frac{V}{2} \left[C\left(\frac{x^2 - y^2}{a^2}\right) + D \sinh kx \cos kz\right] \sin (\omega t + \phi), \tag{5}$$

where the quadrupole focusing term is the same as in Eq. (4) but written in x-y coordinates.

The electric field components calculated from (5) are

$$E_x = -\frac{CV}{a^2} x - \frac{kDV}{2} \cosh kx \cos kz \quad , \qquad E_y = \frac{CV}{a^2} y \quad ,$$

$$E_z = \frac{kDV}{2} \sinh kx \sin kz \quad ,$$

with each component multiplied by $\sin (\omega t + \phi)$. The second term in the expression for $E_x$ produces a deflection force. The field averages taken over a $\beta\lambda$ period, for a synchronous particle with x and y held constant, are

$$\bar{E}_x = -\frac{kDV}{4} \cosh kx \sin \phi \quad , \qquad \bar{E}_y = 0 \quad ,$$

$$\bar{E}_z = \frac{kDV}{4} \sinh kx \cos \phi \quad .$$

In the Appendix, we discuss the electrode geometry for this type of RFQ. Figure 4 is a plot of the pole-tip shapes at three values of z along the unit cell. As expected, the poles have a periodic transverse displacement in the x-direction with period $\beta\lambda$.

If we use the force corresponding to the above expression for $E_x$, the Mathieu equation becomes

$$\frac{d^2x}{dt^2} + \frac{qCV}{Ma^2} \left[x + \frac{ka^2D}{2C} \cosh kx \cos kz\right] \sin (\omega t + \phi) = 0 \; .$$

From a smooth-approximation calculation for a particle with synchronous velocity ($kz = \omega t$) and for

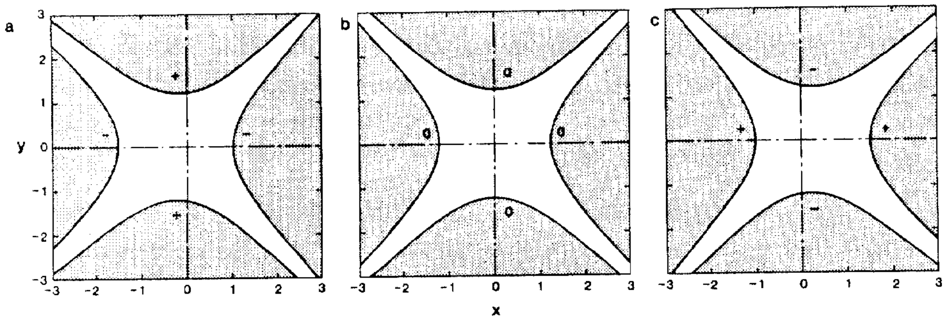


Fig. 4. RFQ deflector electrode cross section at the beginning (a), middle (b), and end (c) of a unit cell of length $\beta\lambda/2$. The curves are from a numerical solution of (A-1) for a = 1, m = 1.5, and $\beta\lambda$ = 10. The polarities shown correspond to a bunched beam with $\phi$ = -90°. In addition to the usual quadrupole focusing forces, the beam will experience a deflecting force in the positive x-direction.

$kx \ll 1$, so that $\cosh kx \approx 1$, we obtain expressions for the betatron frequency $\Omega$ and for the displacement of the neutral axis $x_d$:

$$\Omega = \frac{\sqrt{2}}{2}\frac{\omega_0^2}{\omega} , \qquad x_d = \alpha\left(\frac{\omega}{\omega_0}\right)^2 = \frac{\sqrt{2}}{2}\alpha\left(\frac{\omega}{\Omega}\right) ,$$

where now

$$\omega_0^2 = \frac{qCV}{Ma^2} , \qquad \text{and} \qquad \alpha = -\frac{ka^2D}{2C}\sin\phi \quad .$$

These expressions show that the position of the neutral axis is proportional to $-\sin\phi$. If we introduce a bunched ion beam with $\phi$ = -90°, $x_d$ is positive and these ions can travel along their positively displaced neutral axis parallel to the z-axis. If another ion beam contains bunches with $\phi$ = +90°, $x_d$ is negative and these ions can travel along their negatively displaced neutral axis parallel to the z-axis. Therefore, we can inject two bunched beams having their microstructure bunches displaced by $\Delta z = \beta\lambda/2$ and have them travel along separate parallel trajectories displaced $\pm x_d$ from the z-axis. Both displaced beams will be transversely focused with the same strength about their respective neutral axes. We can then funnel by reducing the deflection parameter D to zero (by decreasing m, the pole modulation parameter, to unity.) This enables us to bring the two beams to the z-axis in a programmed manner that induces no additional betatron amplitude. The transverse focusing is maintained for both beams throughout this process. The expression for $\bar{E}_z$ is similar to that of a conventional rf linac for which the expression would be $E_0T\cos\phi$, where $E_0$ is the space average accelerating field and T is the transit-time factor. Because $\sinh kx$ is positive for positive x-displacement, such a beam will be bunched in a manner similar to a conventional linac beam with the added feature that the bunching forces are proportional to $\sinh kx$. For $\phi$ = -90°, the beam with positive x-displacement will experience both maximum positive-displacement forces and maximum bunching. Similarly, for $\phi$ = +90°, the beam with negative x-displacement will experience both maximum negative-displacement forces and maximum bunching. As the two beams approach the funneling vertex, the bunching forces are reduced to zero. If simulation studies show this to be a serious problem, it may be possible to devise an auxiliary method to maintain adequate bunching in the vertex region.

## Funneling Schemes

The previous discussion suggests a method of funneling that is shown schematically in Fig. 5. With the initial beams parallel and closely spaced, they can enter an RFQ funnel having a minimum aperture. The beams could originate in a conventional RFQ structure that has a single rf resonator with multiple RFQ channels. Also, the two preceding accelerator channels could be designed with a sidestep near the output

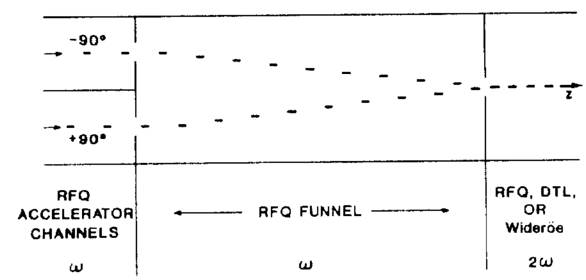


Fig. 5. Scheme for funneling two beams.

to bring their beams closer to the final z-axis of the funnel. If the initial beams are not closely spaced, a periodic focusing system of static electric or magnetic quadrupoles with their centers having programmed periodic transverse displacements could be used to prepare both beams for the final funneling operation. A septum magnet just before the funnel may also be necessary.

## Appendix

### Funneling RFQ Pole-Tip Shapes

To find the shape of the isopotential surfaces of the RFQ poles, we use Eq. (5) and set U(max) = V/2 to obtain

$$\frac{C}{a^2}(x^2 - y^2) + D\sinh kx\cos kz = 1 \quad . \qquad \text{(A-1)}$$

Let y = 0 and set x = a for z = 0, and x = ma for $z = \beta\lambda/2$. We then obtain expressions for the coefficients C and D:

$$C = 1 - D\sinh ka \quad , \qquad D = \frac{m^2 - 1}{m^2\sinh ka + \sinh mka} \quad .$$

Numerical solutions of Eq. (A-1) were used to calculate the electrode shapes shown in Fig. 4. Although Fig. 4 for illustration uses a large value of the deflection parameter m, often much smaller values will be sufficient to produce the desired deflection force. For example, if the phase advance per focusing period is 36°, ka = 0.1, and $\phi$ = -90°, the neutral axis is displaced by a/2 if m = 1.14.

## Reference

1. K. Bongardt, "Study of a High Current Proton Beam Funneling Line," Proc. 1984 Linac Conf. Gesellschaft für Schwerionenforschung, Darmstadt, report GSI-84-11 (September 1984) 389.