PERMANENT-MAGNET QUADRUPOLES IN RFQ LINACS*

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Summary

We investigated the possibility of increasing the current-carrying capability of radio-frequency quadrupole (RFQ) linear accelerators by adding permanent-magnet quadrupole (PMQ) focusing to the existing transverse focusing provided by the rf electric field. Increased transverse focusing would also allow shortening RFQ linacs by permitting a larger accelerating gradient, which is normally accompanied by an undesirable increased transverse rf defocusing effect. We found that PMQs were not helpful in increasing the transverse focusing strength in an RFQ. This conclusion was reached after some particle tracing simulations and some analytical calculations. In our parameter regime, the addition of the magnets increases the betatron frequency but does not result in improved focusing because the increased flutter more than offsets the gain from the increased betatron frequency.

Introduction

Paul Channell has proposed[1] adding PMQs in the vane tips of RFQ linear accelerators. He has shown that if the rf and magnet periods are incommensurate, the effective force that describes the slow part of the motion is given by the sum of the effective force for the rf and the effective force for the magnets. Therefore, the betatron frequency will be raised by the addition of the magnets. The magnets can be of a convenient length not related to the short distance traveled by the particles in one rf period. The advantage of added focusing is increased current-carrying capability. Also, the accelerator can be shortened by increasing the vane modulations at low energies to increase the acceleration rate. Any additional focusing arising from the magnets can be used to cancel the larger rf defocusing that results from the increased vane-tip modulation.

Focusing in Time-Dependent Systems

We measure focusing strength by the size of a matched beam of a given emittance. For a given emittance, the strongest focusing system will have the smallest matched beam. Because space charge is a defocusing force, strong focusing systems allow accelerators to accelerate high currents with a matched beam size that fits into the machine bore. For simplicity, we will assume in this section that the focusing forces are linear and that space charge is not present. The goal is to understand focusing so that we can apply the results to real accelerators in which we try to maximize the focusing to maximize current-carrying capability.

Consider a linear focusing system and neglect space charge. Let time be the independent variable and let x be the particle displacement from the equilibrium orbit. The single-particle equations of motion are

$$\frac{dx}{dt} = \frac{p}{m\gamma} \quad , \quad \frac{dp}{dt} = -k(t) \, x \quad , \tag{1}$$

where k(t) is the time-dependent force constant. Let us write the solution to this equation in the following form

$$x(t) = A \, f(t) \cos[\psi(t)+a] \quad , \tag{2}$$

where f(t) and $\psi(t)$ are time-dependent functions with f(t) suitably normalized. The amplitude A and phase a are constants that depend on the initial conditions.

We can write an invariant for our equations of motion as follows, where primes are time derivatives.

$$f^2 p^2 - 2m\gamma f f' xp + [(m\gamma f')^2 + (m\gamma f \psi')^2]x^2$$

$$= (m\gamma A f^2 \psi')^2 \quad . \tag{3}$$

This is a time-dependent ellipse in x-p phase space. For our purposes, it is useful to choose the Courant-Snyder normalization for f, which we obtain by setting

$$f^2 \psi' = 1 \quad . \tag{4}$$

A matched beam has a phase-space distribution function that is constant along the invariant curves. Consider a matched beam bounded by an invariant ellipse corresponding to particle motions of amplitude A_{max}. The normalized emittance ε_n is defined as the area in x-p phase space divided by πmc. Therefore, the normalized emittance for a matched beam bounded by particle trajectories of amplitude A_{max} is

$$\varepsilon_n = \frac{\gamma A_{max}^2}{c} \quad . \tag{5}$$

If we substitute this into Eq. (2) and drop the cosine factor, we get the beam envelope for a matched beam as a function of time.

$$x_{en}(t) = \left(\frac{c\varepsilon_n}{\gamma}\right)^{1/2} f(t) \quad . \tag{6}$$

Define f_{max} as the maximum of f over time. For a given emittance, the smallest beam occurs for the focusing force constant that corresponds to the smallest value of f_{max}.

Focusing in Periodic Systems

If k(t) is a periodic function, we know that there exists a number ν, called the betatron frequency, such that

$$\psi = \nu t + \phi(t) \quad , \tag{7}$$

where ϕ and f are periodic with the period of k. Averaging the normalization condition Eq. (4) over time, remembering that the average of the derivative of a periodic function is zero, gives

$$\nu = \left\langle \frac{1}{f^2} \right\rangle \quad . \tag{8}$$

Equation (8) shows that, roughly speaking, a large betatron frequency corresponds to a small f, but that the time dependence of f may be important. If k is time independent, $f = \nu^{-1/2}$ and therefore a larger betatron frequency always means stronger focusing.

Doubly Periodic Forces

If we neglect space charge, the transverse single-particle equation of motion for a doubly periodic focusing system is

$$\gamma m \frac{d^2 x}{dt^2} = [k_1 \sin \omega_1 t + k_2 \sin \omega_2 t]x \quad , \qquad (9)$$

where k_1 and k_2 are the force constants for the RFQ and PMQ, respectively. Define

$$K_1 = \frac{k_1}{\gamma m \omega_1^2} \quad , \quad \text{and} \quad K_2 = \frac{k_2}{\gamma m \omega_2^2} \quad . \qquad (10)$$

The quantity $2\pi K_1$ is the phase advance per period $2\pi/\omega_1$ caused by the RFQ force in the absence of the PMQ force. A similar interpretation holds for K_2. The regime we consider is specified by

$$K_1 \ll 1 \quad , \quad K_2 \ll 1 \quad , \quad \text{and} \quad \omega_1 \neq n\omega_2 \quad , \qquad (11)$$

where the last condition is a nonresonance condition.

We found an approximate solution to Eq. (9) in the form given by Eq. (2), that is, in terms of f and ψ, as follows.

$$f(t) = \frac{e^{U(t)}}{[\frac{1}{2}(K_1^2 \omega_1^2 + K_2^2 \omega_2^2)]^{1/4}} \quad , \qquad (12)$$

where

$$U(t) = -K_1 \sin \omega_1 t - K_2 \sin \omega_2 t + \frac{K_1^2}{8} \cos 2\omega_1 t$$

$$+ \frac{K_2^2}{8} \cos 2\omega_2 t$$

$$+ K_1 K_2 \omega_1 \omega_2 \left[\frac{\cos(\omega_1 + \omega_2)t}{(\omega_1 + \omega_2)^2} + \frac{\cos(\omega_1 - \omega_2)t}{(\omega_1 - \omega_2)^2} \right] + \ldots \qquad (13)$$

$$\psi(t) = [\frac{1}{2}(K_1^2 \omega_1^2 + K_2^2 \omega_2^2)]^{1/2}$$

$$\times [t - \frac{2K_1}{\omega_1} \cos \omega_1 t - \frac{2K_2}{\omega_2} \cos \omega_2 t + \ldots] \quad . \qquad (14)$$

Using the above solution, we can use Eq. (6) to write down the matched beam envelope for a beam of emittance ϵ_n.

$$x_{en}(t) = \left(\frac{2^{1/2} c \epsilon_n}{\gamma} \right)^{1/2} \frac{1}{(K_1^2 \omega_1^2 + K_2^2 \omega_2^2)^{1/4}}$$

$$\times [1 - K_1 \sin \omega_1 t - K_2 \sin \omega_2 t + \ldots] \quad . \qquad (15)$$

The envelope has a maximum of

$$x_{max} = x_{mo} g \quad , \qquad (16)$$

where

$$x_{mo} = \left(\frac{2^{1/2} c \epsilon_n}{\gamma K_1 \omega_1} \right)^{1/2} (1 + K_1) \qquad (17)$$

is the maximum of the envelope when there are no PMQ forces ($K_2 = 0$) and

$$g = \frac{1 + \frac{K_2}{(1 + K_1)}}{\left[1 + \left(\frac{K_2 \omega_2}{K_1 \omega_1} \right)^2 \right]^{1/4}} \quad . \qquad (18)$$

The quantity g represents the modification of the envelope's maximum caused by the PMQ forces. If $g < 1$, then a smaller beam is achieved by adding the PMQ focusing. A necessary condition for $g \leq 1$ is

$$K_2 \geq \frac{4K_1^2}{1 + K_1} \left(\frac{\omega_1}{\omega_2} \right)^2 \quad . \qquad (19)$$

We considered an example in which we added PMQ focusing to the first part of the Los Alamos accelerator test stand (ATS) RFQ. We took $\omega_2/\omega_1 = 0.22$, which corresponds to a magnet period of 57.1 mm. For our case, $K_1 = 0.096$; therefore, the condition of Eq. (19) gives

$$K_2 \geq 7.2 K_1 \quad . \qquad (20)$$

To get $g \leq 1$, we need an unreasonably strong PMQ focusing system (479 T/m PMQ gradients in our example). Weaker PMQ focusing just makes things worse.

Numerical Simulations

For our numerical simulations, we used a long section of RFQ structure whose RFQ parameters were fixed at the values corresponding to those at a point 1 m from the entrance of the Los Alamos ATS RFQ linac. To this structure with fixed RFQ parameters, we added a PMQ force whose gradient varied linearly from zero to 137.6 T/m in 100 rf periods. The magnet period was 57.1 mm. A beam, matched to the entrance of this structure by the method of adiabatic deformation, was traced through the structure. This procedure ensures a matched beam throughout and eliminates confusion

from mismatch effects. Figure 1 shows the rms beam sizes in the transverse directions as a function of rf period number. The plots correspond to the time in the rf phase when the rf force is maximum focusing in the y-direction (note the size is slightly larger in the y-direction than in the x-direction). The matched beam envelope contains both the rf and the PMQ frequencies, but not the betatron frequency.
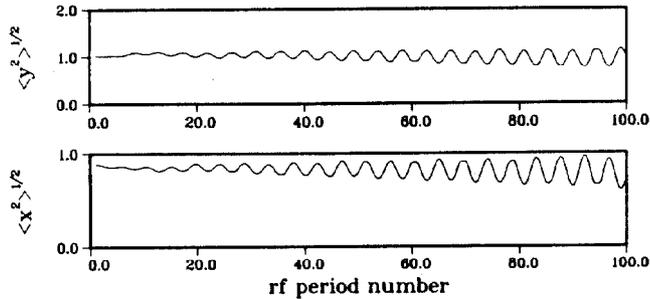

Fig. 1. The rms beam sizes, at the rf phase at which the forces are maximum focusing in the y-direction, as a function of rf period number. The PMQ strength is linearly increasing throughout, starting at a value of zero. Although the average beam size decreases, the peak beam sizes do not decrease with increasing magnet gradients.

Because the points plotted are integral numbers of rf periods apart, only the PMQ frequency is evident in the graph. The important feature of our result is that, although the average transverse beam size decreases, the flutter amplitude (at the PMQ frequency) is so large that the peak amplitude does not decrease from the initial value when there was no PMQ force present. Consequently, in this parameter regime, the addition of the permanent magnets does not increase the focusing strength.

Conclusion

We found that permanent-magnet quadrupoles are not helpful in increasing the transverse focusing strength in an RFQ linac. The regime studied was the low-energy section of a high-brightness hydrogen ion RFQ with PMQs of the shortest practical period. Applications that would allow shorter (in relation to the rf cell length) magnet periods may be more favorable. There is also the possibility that permanent magnets may be helpful in an RFQ structure to help in matching between the time-dependent focusing of the RFQ and the space-dependent focusing of adjacent structures.

Reference

1. P. J. Channell, "The Ponderomotive Force Due to a Doubly Periodic Force," Los Alamos National Laboratory Accelerator Theory Note, AT-6:ATN-83-20 (August 1983).