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A GENERALIZATION OF THE CHILD-LANGMUIR RELATION FOR ONE-DIMENSIONAL TIME-DEPENDENT DIODES

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INTRODUCTION

The steady-state Child-Langmuir relation between current and applied voltage has been a basic principle upon which all modern diode physics has been based. With advances in pulsed power technology and diode design, new devices which operate in vastly different parameter regimes have recently become of interest. Many of these devices cannot be said to satisfy the strict requirements necessary for Child-Langmuir flow. For instance, in a recent pulsed electron device for use in high-current accelerators,[1] the applied voltage is sinusoidal in time. In another case,[2] development of sources for heavy ion fusion necessitates understanding of transient current oscillations when the voltage is applied abruptly.

We derive the time-dependent relationship between the emitted current and time-dependent applied voltage in a nonrelativistic planar diode. The relationship is valid for arbitrary voltage shapes V(t) applied to the diode for times less than the beam-front transit time across the gap. Using this relationship, transient and time-dependent effects in the start-up phase of any nonrelativistic diode can be analyzed.

A RELATION BETWEEN EMITTED CURRENT AND APPLIED VOLTAGE

In this section, we derive an equation which relates global properties of one-dimensional, nonrelativistic charged particle beam flow in a diode to the applied voltage. Our starting point is the set of cold plasma equations for the particle density, n, flow velocity, v, and electric field, E.

$$\partial n/\partial t + \partial(nv)/\partial x = 0 \tag{1}$$

$$\partial v/\partial t + v\partial v/\partial x = qE/m \tag{2}$$

$$\partial E/\partial x = nq/\varepsilon_0 \quad . \tag{3}$$

The particle charge and mass are q and m. MKS units have been employed so that $4\pi\varepsilon_0 \times 10^{-7} = c^2$, where c is the speed of light.

The electric field is the gradient of a potential so that $E = -\partial\phi/\partial x$. Placing the current source at x = 0, and the collector plate of the diode at x = L, the boundary conditions for $\phi(x,t)$ are taken to be

$$\phi(0,t) = 0 \quad , \ \phi(L,t) = V(t) \quad , \tag{4}$$

where V(t) is the applied time-dependent voltage on the collector plate.

If Poisson's equation is solved for the potential with the boundary conditions given in Eq. (4), then Eqs. (1) - (3) imply that the electric field satisfies the Ampere-type equation[3]

$$\varepsilon_0 L(\partial E/\partial t) + Lnqv = -\varepsilon_0(dV/dt) + K(t)$$

$$K(t) = \int_0^L nqv dx \quad . \tag{5}$$

We shall use Eqs. (1), (2), and (5) to obtain a generalization of the Child-Langmuir relation for time-dependent diode problems. In particular, a relation between the line-integrated current, K(t), and the applied potential will be obtained as well as a relation between the potential and the current emitted at the diode source.

Equations (1) and (2) may be used to obtain an equation for j = nqv:

$$\partial j/\partial t + \partial(jv)/\partial x = (nq^2/\varepsilon_0 m)\varepsilon_0 E \quad . \tag{6}$$

As is common in problems of this type, we will assume that particles are emitted with negligible velocity, so v(0,t) = 0. Thus, for nonvanishing current emission, an infinite number density is required at the source. We will also restrict ourselves here to the case of space-charge limited emission so E(0,t) = 0. Considering only times less than the beamfront transit time across the diode, n(L,t) = 0, and we obtain from Eq. (3) and an integration of Eq. (6)

$$dK/dt = Nq^2E(L,t)/2m \quad , \tag{7}$$

where the total particle inventory N is the integral of the density over the diode gap.

$$qN(t) = q\int_0^L n(x,t)dx = \int_0^t j(0,\tau)d\tau = \varepsilon_0 E(L,t) \quad . \tag{8}$$

Equation (5) provides a relationship between the line-integrated current, the applied potential, and the emitted current density via the space-charge limited emission condition E(0,t) = 0,

$$Lj(0,t) = -\varepsilon_0(dV/dt) + K(t) \quad . \tag{9}$$

We note if we require that the initial current vanishes, then dV/dt must also vanish initially. Employing Eqs. (8) and (9) in Eq. (7) yields

$$d[qK(t)/L]/dt = 1/(2\varepsilon_0 mL)\left[\int_0^t d\tau\, qK(\tau)/L + \psi(t)\right]^2 \tag{10}$$

where $\psi = (-\varepsilon_0/L)qV$. The relationship between the current density at the emitting surface and the applied voltage is obtained similarly:

$$d[qj(0,t)]/dt - 1/(2\varepsilon_0 mL)\left[\int_0^t qj(0,\tau)d\tau\right]^2 = d^2\psi/dt^2 \tag{11}$$

Equations (10) or (11) are generalizations of the Child-Langmuir relation for time-dependent diode problems. They describe the current pulse obtained from an applied voltage V(t), or conversely, define a voltage shape V(t) needed to extract a current density j(0,t) at the source. Both relationships are only

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valid for times less than the beam transit time across the diode since they have been obtained from Eq. (7) using the condition $n(L,t)v^2(L,t) = 0$.

CYLINDRICALLY AND SPHERICALLY SYMMETRIC DIODES

In this section, we generalize the treatment for planar diodes to cylindrically and spherically symmetric diodes. In these geometries, the presence of radial moments in integrals of the continuity equation prohibits one from obtaining a simple relationship between emitted current and applied voltage. However, in the special limiting case of constant emitted current, the limiting voltage may be obtained by numerically integrating two ordinary differential equations.

We assume that a potential $V(t)$ is applied at the collector at $r = r_1$. A current, $j_0(t)$, is extracted at $r = r_2 > r_1$. The position $r = r_2$ is held at zero potential. Solving Poisson's equation for the potential determines the radial electric field, E. One finds that in cylindrical geometry

$$\partial rE/\partial t + rj/\varepsilon_0 = (\ln r_2/r_1)^{-1}[dV/dt + K(t)/\varepsilon_0] \quad (12)$$

while in spherical geometry

$$\partial r^2E/\partial t + r^2j/\varepsilon_0 = (1/r_1 - 1/r_2)^{-1}[dV/dt + K(t)/\varepsilon_0] \quad (13)$$

In these equations,

$$K(t) = \int_{r_1}^{r_2} j(t,r')dr' \quad . \quad (14)$$

If we assume constant space-charge limited emission, in the Lagrangian frame of the beam front, the electric fields are then given by $r_bE(t,r_b) = r_2j_0t/\varepsilon_0$ and $r_b^2E(t,r_b) = r_2^2j_0t/\varepsilon_0$ for the cylindrically and spherically symmetric cases, respectively. Consequently, the beam front satisfies the differential equations

$$d^2r_b/dt^2 = (qj_0/\varepsilon_0 m)(r_2/r_b)t \quad (15)$$

in the cylindrically symmetric diode and

$$d^2r_b/dt^2 = (qj_0/\varepsilon_0 m)(r_2/r_b)^2t \quad (16)$$

in the spherically symmetric diode. We note that the beam-front transit time is greatest in planar and least in spherically symmetric diodes.

For steady flow behind the beam front, K(t) may be evaluated using the relationships $rj(t,r) = r_2j_0$ or $r^2j(t,r) = r_2^2j_0$. Substituting the result in the appropriate Ampere-type equations for E, we obtain differential equations relating V(t) and $r_b(t)$. In cylindrical symmetry, we find

$$dV/dt = (r_2j_0/\varepsilon_0)\ \ln\ (r_b/r_1) \quad (17)$$

while in spherical symmetry we obtain

$$dV/dt = (r_2^2j_0/\varepsilon_0)(1/r_1 - 1/r_b) \quad (18)$$

We note that the beam-front positions which appear in Eqs. (17) and (18) are determined by Eqs. (15) and (16). The latter equations do not contain the potential V(t). Consequently, Eqs. (15) and (16) may be integrated directly to provide the source terms in the differential equations for V(t) to yield

$$V(t) = (r_2j_0/\varepsilon_0)\int_0^t \ln r_b(\tau)\,d\tau - (\ln r_1)t \quad (19)$$

and

$$V(t) = (r_2^2j_0/\varepsilon_0)\ t/r_1 - \int_0^t r_b^{-1}(\tau)d\tau \quad (20)$$

in the two geometries.

SUPPRESSION OF CURRENT TRANSIENTS

The analyses in the previous section provided an expression for the applied voltage which will produce a constant current in time. In real diodes, however, the appropriate initial condition is that the current vanish at t = 0 (V = 0) and therefore cannot be constant for all time. Therefore if the voltage predicted by Eqs. (19) or (20) or the equivalent voltage profile for the planar diode [2], is applied to the diode, there will be an initial start up period where the current rises from zero to the predicted constant value. Furthermore, real diodes only approximate the special cases of planar, cylindrical or spherical configurations. Therefore, in order to assess the usefulness of these solutions to real diodes, simulations of a diode design for heavy-ion fusion experiments were performed.

Figure 1 shows the electrode configuration and a snapshot of the ion positions from a simulation of a sodium ion diode designed for experiments at Los Alamos [5]. The calculations were performed with the 2-dimensional, particle-in-cell simulation model ISIS [6]. Figure 2b shows the current measured at the right-hand end of the simulation region shown in Fig. 1, as a function of time for the applied voltage shown in Fig. 2a. The voltage risetime is 10ns. The diagnotic measures zero current until the particles reach the end of the simulation region, then rapidly overshoots the steady-state space-charge limited current and then decays to the steady-state value.

The voltage profile shown in Fig. 3a was obtained by approximating the diode shown in Fig. 1 by a spherical diode and using Eq. (20). The voltage risetime is clearly much slower than the current risetime shown in Fig. 3b. Although the current profile is not exactly a step-function predicted by the simplified models, it has much less overshoots than the calculation in Fig. 2, while maintaining a fast risetime.

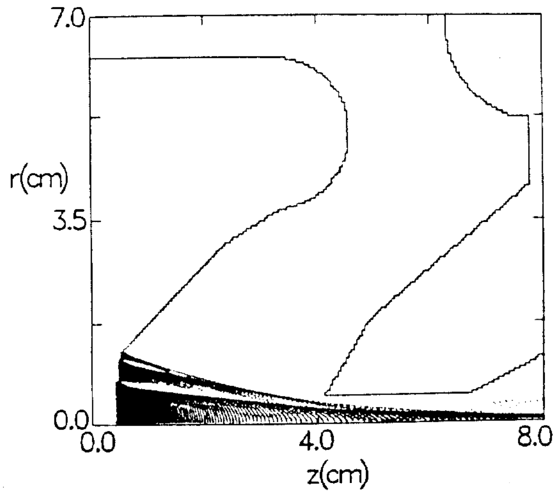

Fig. 1 ISIS simulation snapshot showing diode configuration and particle positions.

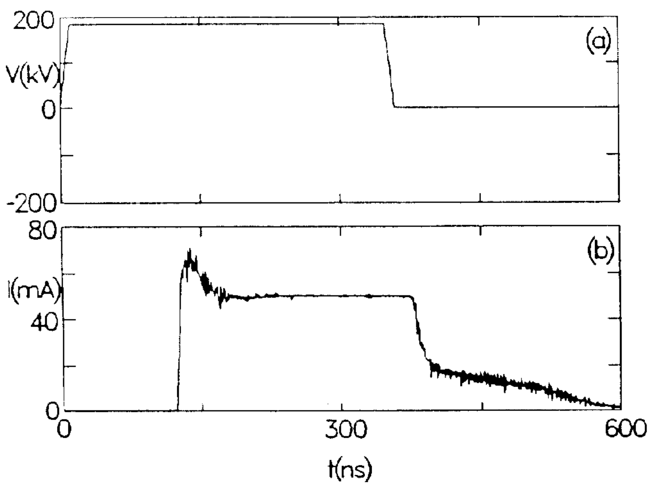

Fig. 2 (a) Voltage profile for 10 ns risetime in ISIS simulation.
(b) Resulting current profile showing overshoot.

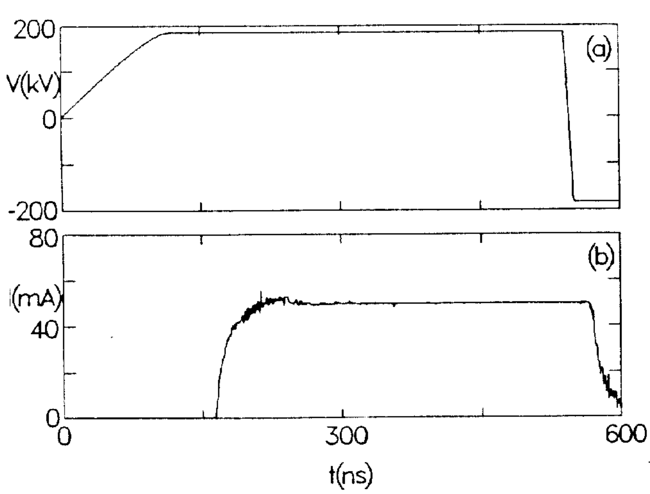

Fig. 3 ISIS simulation showing (a) tailored voltage profile (b) nearly step-function resulting current profile.

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