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EMITTANCE GROWTH CAUSED BY CURRENT VARIATION IN A BEAM-TRANSPORT CHANNEL*

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Summary

Time variation of space-charge forces in a beam-transport channel will lead to a time-averaged emittance growth of the beam. The Kapchinskii-Vladimirskii (K-V) equations have been used to follow the envelope of a round beam with effective beam current fluctuation i through a transport channel. The area of the ellipse that encloses the varying ellipses at the end has been used as the criterion for emittance growth. Simple formulae give the relation between i, initial emittance ϵ, allowable fractional emittance growth $\Delta\eta$, effective average current I, average beam radius R, and transport length. For example, for a long transport channel a nominally compensated beam must have $i < (\beta\gamma\epsilon^2 I_0/R^2)\Delta\eta$, where $I_0 = 4\pi\epsilon_0 Mc^3/e$, and $\beta\gamma$ is the usual relativistic factor. Results for other conditions are presented. A comparison with a numerical calculation from the TRACE code for transport of a 100-keV, 100-mA beam is made.

Statement of the Problem

As brighter ion beams are required for accelerator applications, there may be more stringent requirements on quiescence of the current. Time variation of the space-charge forces in a beam-transport channel will lead to a time-averaged emittance growth of the beam. The residual space-charge force in a neutralized (that is, space-charge compensated) beam is not fully understood, particularly if the current fluctuates, and these questions are not addressed in the present calculation. We assume that the residual space-charge forces can be described by effective average current I and effective fluctuation in current i. No attempt is made to relate I and i to the actual values, except that in many cases we expect that $I \simeq 0$ because of ionization of the background gas. If the frequency of the fluctuation is higher than the background-plasma production frequency (typically 1-1000 kHz for low-energy ion beams), the effective and actual values of fluctuation i may be about equal.

We assume that a matched round beam propagates in a nonaccelerating channel with radius R, normalized emittance ϵ, and average focusing force k_0^2. The beam dynamics are calculated from the K-V equation,[1] following the analysis of Struckmeier and Reiser.[2]

$$X'' + k_0^2 X - 2K(I)/(X+Y) - \epsilon^2/X^3(\beta\gamma)^2 = 0 \quad \text{, and}$$

$$Y'' + k_0^2 Y - 2K(I)/(X+Y) - \epsilon^2/Y^3(\beta\gamma)^2 = 0 \quad ,$$

where $K(I) = 2I/I_0(\beta\gamma)^3$, $I_0 = 4\pi\epsilon_0 Mc^3/e$, and

$$k_0^2 = K(I)/R^2 + \epsilon^2/(\beta\gamma R^2)^2$$

$$= (\epsilon/\beta\gamma R^2)^2(1 + 2IR^2/I_0\epsilon^2\beta\gamma) \quad . \qquad (1)$$

Defining r(I) as the ratio of space-charge force to emittance "force," we get

$$r(I) = K(I)(\beta\gamma R/\epsilon)^2 = 2IR^2/I_0\epsilon^2\beta\gamma \quad . \qquad (2)$$

For example, for a 20-keV, 0.1-A, 0.5-cm-radius H^- beam with $\epsilon = 0.08$ π·cm·mrad, r(I) = 38.2, making space charge dominant in the absence of neutralization. Here we take ϵ to be the emittance of about 90% of the beam. The radial potential drop from the beam edge to the center is $\Delta\Phi = I/4\pi\epsilon_0 c\beta$, or 460 eV for this case. According to the various theories of neutralization,[3] $\Delta\Phi$ will be reduced to approximately kT_e/e (about 5 V) in the presence of a compensating plasma. The sign is such that positive beams would be defocused, negative beams would be focused, and (for the 20-keV beam) the effective current would be about 1 mA, making the emittance dominant according to Eq. (2).

Solution of the Problem

We assume that the matched condition is perturbed by replacing I with I + i, all other factors being held constant at the channel's entrance. Letting X = R + x, where x is a small perturbation and similarly in y, we get to first order[2]

$$x'' + x\left[3K(I)/2R^2 + 4\epsilon^2/(\beta\gamma R^2)^2\right] + yK(I)/2R^2 = K(i)/R \quad \text{, and} \qquad (3)$$

$$y'' + y\left[3K(I)/2R^2 + 4\epsilon^2/(\beta\gamma R^2)^2\right] + xK(I)/2R^2 = K(i)/R \quad .$$

We define $z_1 = x - y$, and $z_2 = x + y$, and obtain the equations

$$z_1'' + k_1^2 z_1 = 0 \quad \text{, and} \qquad (4)$$

$$z_2'' + k_2^2 z_2 = 2K(i)/R \quad ,$$

where

$$k_1^2 = 4\epsilon^2/(\beta\gamma R^2)^2 + K(I)/R^2 \quad \text{, and} \qquad (5)$$

$$k_2^2 = 4\epsilon^2/(\beta\gamma R^2)^2 + 2K(I)/R^2 \quad .$$

The initial conditions for z_1 and z'_1 are zero, and hence z_1 remains zero, so that x and y are in phase. The solution of Eq. (4) then gives

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$$x = R(1 - \cos k_2 z) r(i)/4[1 + r(I)/2]$$
$$= R \sin^2(k_2 z/2) r(i)/2[1 + r(I)/2] \quad . \qquad (6)$$

The wavelength λ of the oscillation is $2\pi/k_2$.

The ellipse parameters for the matched beam are

$$\beta_o = \beta\gamma R^2/\epsilon \quad , \alpha_o = 0 \quad , \text{ and } \gamma_o = \epsilon/\beta\gamma R^2 \quad , \qquad (7)$$

whereas those for the perturbed beam are

$$\beta_i = \beta\gamma X^2/\epsilon = \beta\gamma(R + x)^2/\epsilon = \beta_o(1 + x/R)^2 \quad ,$$
$$\alpha_i = -XX'(\beta\gamma)/\epsilon = -(\beta_o x'/R)(1 + x/R) \quad , \text{ and} \qquad (8)$$
$$\gamma_i = \gamma_o/(1 + x/R)^2 + \beta_o(x'/R)^2 \quad .$$

The analysis of Guyard and Weiss[4] shows that the ellipse just enclosing these two ellipses has an area increased by the factor η, where

$$\eta = \sqrt{1 + \Delta/2 + \sqrt{\Delta^2/4 + \Delta}} \quad , \text{ and}$$
$$\Delta = \beta_i\gamma_o + \beta_o\gamma_i - 2\alpha_i\alpha_o - 2 \quad . \qquad (9)$$

We note that η is identical with $\Delta R/R + 1$ in their notation. The factor $\Delta\eta \equiv \eta - 1$ will be used here to characterize the effective emittance growth, and for small Δ, $\Delta\eta \simeq \sqrt{\Delta}/2$. Substituting from Eqs. (6), (7), and (8),

$$\Delta = (1 + x/R)^2 + 1/(1 + x/R)^2 + \beta_o^2(x'/R)^2 - 2$$
$$\simeq 4(x/R)^2 + \beta_o^2(x'/R)^2$$
$$= \left[\frac{r(i)\sin(k_2 z/2)}{1 + r(I)/2}\right]^2 \left[1 + 1/2\ r(I)\cos^2(k_2 z/2)\right] \quad .$$

Substitution of Δ in Eq. (9) leads to

$$\Delta\eta \simeq \frac{r(i)\left|\sin\frac{k_2 z}{2}\right|}{2[1 + r(I)/2]}\sqrt{1 + r(I)\cos^2(k_2 z/2)/2} \quad . \qquad (10)$$

First, consider the limiting case $I = 0$, corresponding to a fully neutralized beam. Using Eqs. (2) and (5),

$$i = i_a/\sin(k_2 z/2) \text{ , where } i_a = (\beta\gamma\epsilon^2 I_0/R^2)\Delta\eta \quad . \qquad (11)$$

The allowable fluctuation i increases with increasing energy, decreasing radius, and short transport. Allowing 10% emittance growth, i_a must be less than 0.52 mA for the 20-keV beam. For a 100-keV beam with the other parameters the same, i_a must be less than 1.2 mA.

For beams that are space-charge dominated, $r(I)$ becomes large and

$$i = i_a(I)/\sin k_2 z \quad ,$$
$$\text{where } i_a(I) = (2\epsilon\sqrt{II_0\beta\gamma}/R)\Delta\eta \quad . \qquad (12)$$

For the 20- and 100-keV beams, i_a may be 4.6 and 6.8 mA, respectively. For a space-charge-dominated beam, the fluctuations may be greater than those for a neutralized beam by a factor of about $\sqrt{2r(I)}$. This result is true because the channel restoring force is much larger for the unneutralized beam.

Comparison with TRACE Calculations

A test case consisting of a beam propagating through 24 consecutive quadrupoles was devised for comparing the results of this calculation with those from TRACE, a K-V code.[5] The test case parameters are listed below.

z_o = quad length = 10 cm
g = quad gradient = 0.2238 kG/cm
ϕ = beam voltage = 100 kV
ϵ = 0.08 π·cm·mrad
$I = 0$
i = 1 mA

The approximate single-particle-trajectory solution [Ref. 6, Eqs. (15-1) to (15-12)] is

$$x = \cos(\mu z/2z_o)(1 + \delta \sin \pi z/z_o) \quad ,$$

where $\cos\mu = \cosh\theta\cos\theta$,

$$\theta = \sqrt{g/B\rho}\ z_o \quad , \quad \text{and} \quad \delta \simeq \pi\mu^2/8\theta^2 \quad .$$

Using the test case parameters gives $\mu = 0.2838$, from which k_o^2 for Eq. (1) can be calculated:

$$k_o^2 \simeq (\mu/2z_o)^2 = 2.01 \times 10^{-4}\ \text{cm}^2 \quad .$$

TRACE calculates the exact value of $\Delta\eta$, and this is compared in Fig. 1 with the value predicted in Eq. (11) as a function of transport length. The mismatch factor was calculated at the end of Quadrupole 12 as a function of the current perturbation i. This location corresponds to a half wavelength and, therefore, maximum amplitude for $\Delta\eta$. The comparison between TRACE and Eq. (11) is summarized in Table I. Better agreement could be obtained by keeping higher order terms in the development of Eq. (10), but rather than high accuracy, the objective here is to develop the approximate scaling laws in their simplest forms.

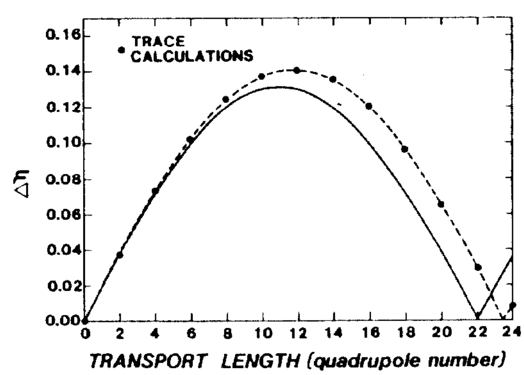

Fig. 1. Comparison of emittance growth factors versus transport length for the test case, using TRACE (points and dashed line) and Eq. (11) (solid line).

TABLE I

COMPARISON OF TRACE AND MODEL MISMATCH FACTORS

	$\Delta\eta$	
i(mA)	Model	TRACE
0.5	0.065	0.068
2.0	0.261	0.295
8.0	1.05	1.29

Thus, the model presented here agrees well with TRACE for $\Delta\eta \leq 1$ for a carefully constructed test case in which the beam envelope has small oscillations. How good is the agreement with a more realistic case? The calculated beam-transport envelopes for a 100-keV, fully neutralized H^- beam ($I = 0$) for our radio-frequency quadrupole (RFQ) experiment have a variation in beam radii of more than a factor of 10 between the accelerating-column exit and the RFQ entrance (Fig. 2). The factor $\Delta\eta$ was calculated for currents i between 0.5 and 5 mA, and the results are shown in Fig. 3. In the X-plane, the values of $\Delta\eta$ calculated by TRACE are about 25% larger than the model predicts for a 5-mm average-radius beam, and no value of R_x will exactly fit the TRACE calculations. For the Y-plane, the TRACE calculations are fit closely with an average-radius of 4.6 mm, somewhat smaller than might have been guessed from the trajectories shown in Fig. 2. The dependence of $\Delta\eta$ on i is very close to that predicted by the model, however, and the model clearly provides a good estimate of the mismatch.

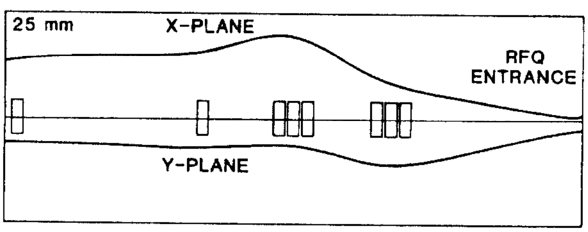

Fig. 2. Matched trajectories for the 100-keV, 100-mA beam transport to the RFQ. Total transport length = 61.8 cm. The X- and Y-plane profiles are shown in the upper and lower curves, respectively.

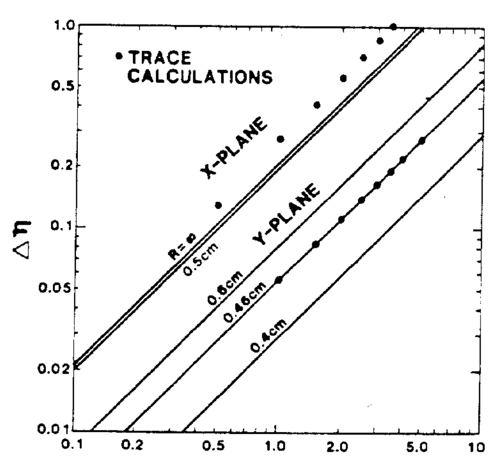

Fig. 3. Emittance growth factor $\Delta\eta$ versus current fluctuation i for the conditions of Fig. 2. TRACE predictions for the two planes are shown by points, and the lines are calculated from Eq. (11) with $I = 0$ and several values of R in the X- and Y-planes.

Conclusion

The model presented here agrees reasonably well with exact calculations by TRACE, and the scaling laws derived are simple and can give insight into this emittance-growth mechanism. For high-current beams, the sensitivity of the emittance growth to current fluctuations can be greatly reduced by preventing neutralization from occurring by, for example, using a transport of electrostatic quadrupoles. However, much stronger focusing strength would be required without neutralization. In the RFQ, there should be reduced sensitivity to fluctuations in current because the beam is unneutralized there. Also, the sensitivity to fluctuations can be reduced by making the transport length short compared with the wavelength, Eq. (11), approaching the limit as z goes to zero,

$$i = [(\beta\gamma)^2 \epsilon I_0 / z] \, \Delta\eta \quad . \tag{13}$$

An interesting, but probably impractical, possibility is to make the transport distance equal to a half wavelength, making the match independent of i, as shown in Fig. 1 at Quadrupole 22. Finally, the tolerance for fluctuations becomes progressively tighter at lower energies, as shown by Eqs. (11) and (12).

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