MACROFILAMENT SIMULATION OF HIGH CURRENT BEAM TRANSPORT

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Summary

Macrofilament simulation of high current beam transport through a series of solenoids has been used to investigate the sensitivity of such calculations to the initial beam distribution and to the number of filaments used in the simulation. The transport line was tuned to approximately 105° phase advance per cell at zero current with a tune depression of 65° due to the space charge. Input distributions with the filaments randomly uniform throughout a four dimensional ellipsoid and K-V input distributions have been studied. The behavior of the emittance is similar to that published for quadrupoles with like tune depression. The emittance demonstrated little growth in the first twelve solenoids, a rapid rate of growth for the next twenty, and a subsequent slow rate of growth. A few hundred filaments were sufficient to show the character of the instability. The number of filaments utilized is an order of magnitude fewer than has been utilized previously for similar instabilities. The previously published curves for simulations with less than a thousand particles show a rather constant emittance growth. If the solenoid transport line magnetic field is increased a few percent, emittance growth curves are obtained not unlike those curves. Collision growth effects are less important than indicated in the previously published results for quadrupoles.

Introduction

The space charge program SCHAR[1] was developed when TRACE failed to reproduce beam transport observed in the LAMPF injector lines. When the H+ line beam profile measured at the first emittance monitor was taken as input for SCHAR, it reproduced the waist locations and beam emittance observed in the remainder of the line. Typically a few hundred filaments were used in the beam simulations. A limited effort was made to determine the minimum number of filaments required. A definitive answer was not obtained due to the complexity of the line and to the limited time available. It was evident a more efficient study of the effects of using too few filaments could be made with fields more symmetric than those of the quadrupoles and bend magnets of the LAMPF injector line.

Solenoid fields were selected. They provide azimuthal symmetry and can be represented analytically with reasonable approximation. Also, POISSON calculated fields were available for solenoids. The solenoid fields utilized are analytical approximations of the University of Maryland electron model periodic solenoid system[2-4]. We have studied the evolution of a high current density electron beam as it passed through the series of individual, uniformly spaced solenoids. To do this we used the space charge program SCHAR which has been modified to include relativistic effects. The results reported here are for 50 - 800 filaments.

Previous studies have examined the stability of the K-V distribution in quadrupole, interrupted solenoid, and continuous solenoid transport systems[5]. Corresponding computer simulations used a particle-in-cell method and typically 10^4 simulation particles[6-8]. Alternate methods have employed over 1000 particles[9].

The conclusion drawn from the results published in reference (8) is that for the tune depression investigated ($\mu_0 = 90°$, $\mu = 30°$), in excess of 1000 particles are necessary to correctly simulate the instability. A particle-in-cell method was utilized for that quadrupole transport line. The authors of reference (9), using a different method subsequently studied a quadrupole transport line with similar tune depression and obtained qualitative agreement for 1500 particles.

Calculational Method

Beam evolution was simulated using N infinitely long uniformly charged filaments which follow quasi-parallel paths along the Z axis and which affect each other according to the simple parallel line-charge expression of electromagnetic theory. Since, in the interaction equation, an individual filament's motion is affected only by its own charge to mass ratio and by the charge on the remaining filaments; each other filament, as it interacts, is assumed to carry a charge/unit length equal to the total charge/unit length divided by N-1. Values of N from 64 to 784 were employed for the solenoid runs.

Input parameters for the beam which affected the initial conditions of motion were: XOMAX and YOMAX, which gave the maximum displacement of the filament from the beam axis at Z=0; VO, the (same) initial speed given to all particles; VXSMAX and VYSMAX, the maximum X and Y components of velocity at Z=0.

The initial values for X, Y, V_x and V_y were chosen so that these values for each filament would lie on the K-V sphere. The axial component of the velocity V_z was set initially by the requirement that the speed of each particle be the same. Thus random selection of V_{xo} and V_{yo} gave V_{zo}.

The input was symmetric in X and Y. The input phase ellipses were selected to be upright and matched. Usually the distribution over the sphere was a random uniform distribution so as to simulate a K-V distribution. However, distorted density distributions were also employed.

A significant increase in the number of solenoids over which a K-V beam could be held at essentially constant emittance was obtained by replacing the initial statistically random distribution on the K-V sphere by a "forced" statistically random distribution. This was done in several ways but the method most used consisted of dividing the X-Y area into N regions of equal area. The filaments were then scattered statistically uniformly over the X-Y circle but once one filament had been placed in one of the regions that region was closed to further filaments. The V_x and V_y initial values were then determined by placing each filament, with one further random number call, on the sphere. Another "force" method which gave similar results was to divide the surface of the hypersphere into N equal areas and then to place the filaments randomly on the sphere, closing each area after it had received one filament. With either method a number of random input "forced" distributions were calculated for a given N. Only those with X-V_x and Y-V_y correlations less than a specified amount were acceptable. For the smaller N, only a few percent of the forced distributions met the criteria. The beam matching was done with the requirement that later X and Y rms values reproduce the input.

Integration of the differential equations of motion was, in general, accomplished by the fourth order Runge-Kutta method. Checks were made by integrating with the Livermore ordinary differential equation routine LSODE, employing the Adams predictor-corrector[10]. Results were, using high precision for both methods, indistinguishable; but for equal precision the Runge-Kutta ran about 30% faster. Since the current is independent of time, t can be eliminated from the equations

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of motion and z taken as the independent variable. This reduces the number of differential equations per filament from six to five.

In order to avoid unrealistically large forces between colliding macrofilaments, the force law between interacting filaments with separation R was changed at a critical distance R_c from $F = \frac{K}{R}$ to $F = \frac{KR}{R_c^2}$.

R_c was defined by $R_c^2 = \frac{C}{2}((XOMAX)^2 + (YOMAX)^2)$

Values for C of .01, .001, .0001 were investigated. This change led to slight quantitative changes in the emittance curves but to no qualitative changes. This lack of dependence on C is in agreement with the results of reference (9).

The magnetic fields used were obtained by assuming, for the solenoid axial magnetic field on the axis, the expression: $B(z') = \frac{B_o}{2}(1+\cos\frac{\pi z'}{L})$ where z' is measured from the solenoid center. The effective solenoid field length was 2L (.10 meters). The drift distance between the solenoids was L (.05 meters). This is an approximation to the axial field given in reference (2) where the cell length was .15 meters.

Taking the equation for B(z') as a boundary condition for the Laplace equation governing the magnetic potential leads to unique off-axis representation of axial and magnetic field components in terms of Bessel and trigonometric functions. This method leads to a nonphysical field discontinuity off the axis at the edges of the effective field; but its magnitude is small and the discontinuity always occurs at the end of an integration step.

Some calculations were made with POISSON calculated fields. Results varied little in changing between the interpolated POISSON field values and the calculated Bessel field values. Emittance growths were somewhat larger for the POISSON calculated data fields.

Statistical analyses of the results of the integration were available at the center and after each solenoid. Outputs were: rms emittance values, number of critical distance collisions, tune angle advance, convex polygon emittance values, mean and rms averages for the filament coordinates, moments (correlations) for the filament coordinates, and the total, kinetic, and potential energies per unit rest mass in the beam. Additional output was the extent to which filaments initially placed on the K-V sphere remained on that sphere. It was obtained by calculating the average four dimensional filament sphere radius and the rms deviation from that average radius.

The energy monitor enables one to place a limit on cumulative round off error in the calculation. Typically, potential energy changes through the run were of the order of .1% of the total energy. The total energy remained constant over 90 solenoids to better than one part in 10^6; potential energy increases being compensated by corresponding kinetic energy decreases. The program has, with slight variations, been run on VAX-11, DEC20, and CRAY computers. When used on the DEC20, input and integration calculations were carried out to single precision but emittance and energy calculations were made by double precision analysis (of the single precision results of the integration).

Normally 36 integration steps per solenoid were taken. Reduction of this number to 18 steps per solenoid resulted in no observable change in the emittance behavior through the emittance growth region but the larger number of steps was necessary to ensure the quoted constancy of the total energy.

Results

Figure 1 shows the Y-V_y distribution at the twenty-second solenoid. The characteristic spiral "arms" indicate the presence of an instability. The beam simulation was for 100 ma of 10 keV electrons with X and Y rms emittances of 1.2π mr cm. The maximum axial field in each solenoid was 165.5 gauss. The phase advance per cell was evaluated as the beam was stepped through the solenoid and its associated drift distance. The phase advance for no current was approximately 105° per cell. The beam decreased the phase advance per cell by 65°. Figures 2A through 2E show the beam emittance behavior as the beam is transported along the line. The average of the normalized X and Y rms emittances at the end of each solenoid is plotted. Curve 2A is for 784 filaments, 2B for 400 filaments and 2C for 196 filaments. The curves are quite similar. The initial emittance growth for the 196 filament curve is slightly larger. Curve 2D, for 121 filaments, and curve 2E for 64 filaments show increasingly greater initial slope and a less pronounced emittance growth but the presence of the instability is apparent, even for 64 particles.

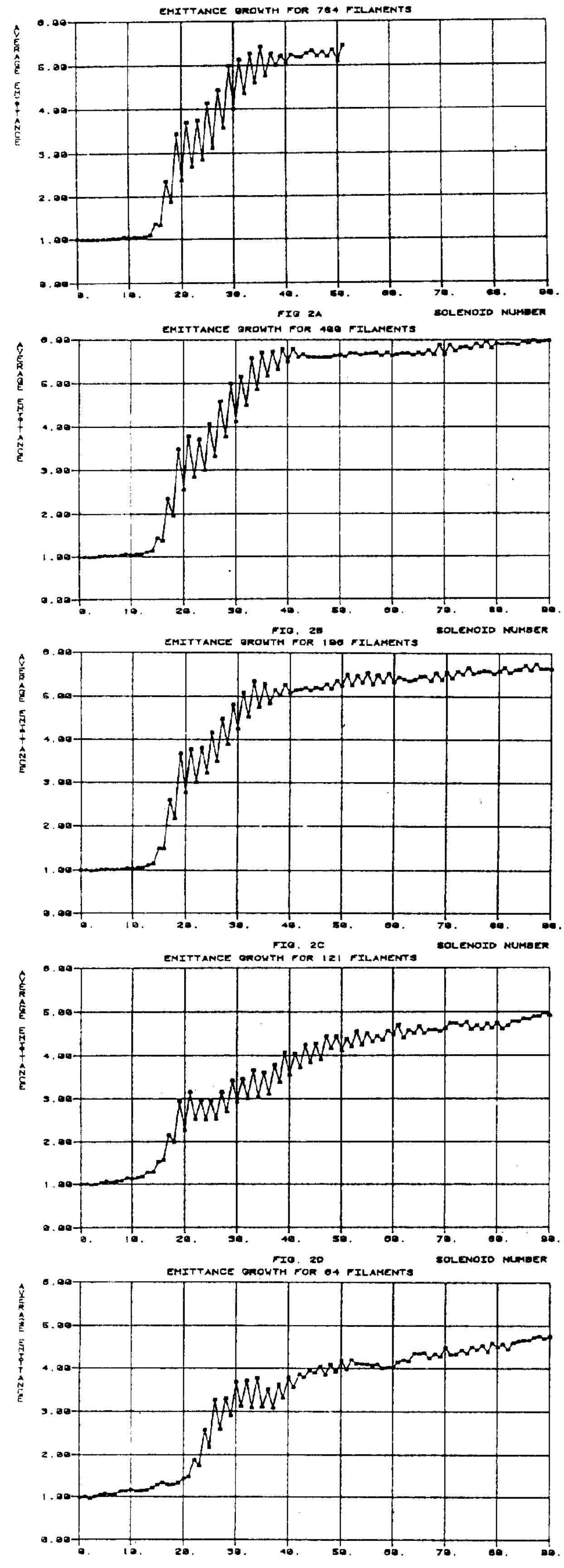
In order to evaluate the initial collision growth, the effect of altering the critical distance for collisions must be carefully examined, the input beam must be precisely matched and the growth due to the non-linear fields must be subtracted. We have not completed this study, but the data indicate about a 1% growth per cell for 100 particles - for the tune depression and transport system utilized.

The sensitivity of the emittance growth to the solenoid tune can be seen by comparing figure 3 with figure 2C. The curves are for an identical input beam; the solenoid fields have been increased from 165.5 gauss to 169.0 gauss. The curve of figure 3 shows a gradual emittance increase, similar to the curves of ref. (8) for less than 1000 particles.

Remarks

The tune depression for these calculations is comparable to the tune depression used for previous collisional growth studies with quadrupoles. An order of magnitude more filaments were used for the particle-in-cell quadrupole calculations by Haber[8]. The collisional effects observed are less than for those calculations. Tuning of the line is critical, a fraction of a percent field change alters the emittance growth pattern. Forcing the filaments of a K-V distribution into equal areas rather than using a statistically random distribution for the entire area provided consistency and repeducibility. A total energy monitor was useful to ensure calculational step size and round off errors were negligible.

Many of the characteristics of the instability due to space charge can be demonstrated with a few hundred particles for the solenoid transport considered here and for comparable quadrupole transport. For problems in which collisional emittance growth effects are of greater significance, more particles may be needed for the simulation.



Figures 2A-2E. Plots of average normalized emittance growth for different numbers of macrofilaments.

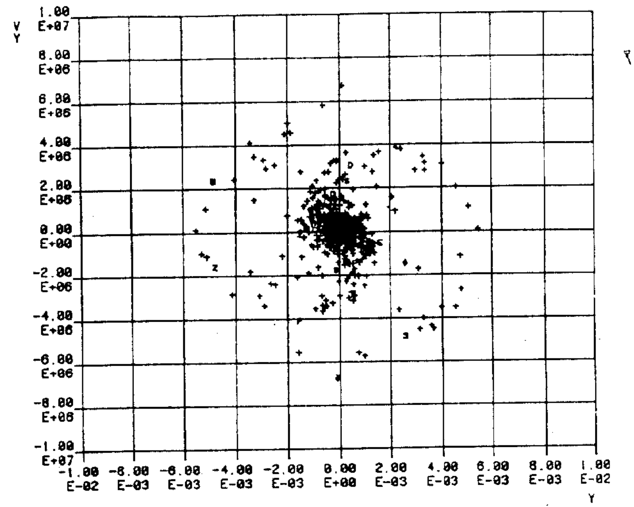


Figure 1. Y-V_y emittance plot at the twenty-second solenoid (MKS units).

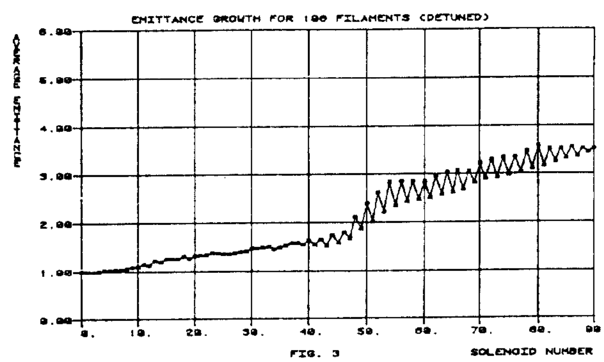


Figure 3. Demonstrates sensitivity of emittance growth to solenoid field increase. Compare with Fig. 2C. (3.5 gauss increase)

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