BEAM DYNAMICS IN MABE*

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Abstract

MABE[1,2] is a multistage linear electron accelerator which accelerates up to nine beams in parallel. Nominal parameters per beam are 25 kA, final energy 7 MeV, and guide field 20 kG. We report recent progress via theory and simulation in understanding the beam dynamics in such a system. In particular, we emphasize our results on the radial oscillations and emittance growth for a beam passing through a series of accelerating gaps.

Single Gap

A typical MABE accelerating gap is shown schematically in Fig. 1. The figure represents the trajectories of electrons in a 25 kA beam with initial kinetic energy of 1 MeV ($\gamma = 3$), final kinetic energy of 3 MeV (gap voltage = 2 MV). The calculation was done with the 2-D electromagnetic particle code MAGIC,[3] using methods described previously for similar problems,[4] including foilless diodes,[5] gaps,[6] extraction from the guide field,[4] and the effects of beam stops.[7,8] In addition to the beam, Fig. 1 shows the parasitic gap leakage of 19 kA. Gap losses such as this have been a serious problem in MABE,[2] because of the coaxial design of the gap. The intention of this design is to shield the beam from the self-magnetic fields of the other eight beams.

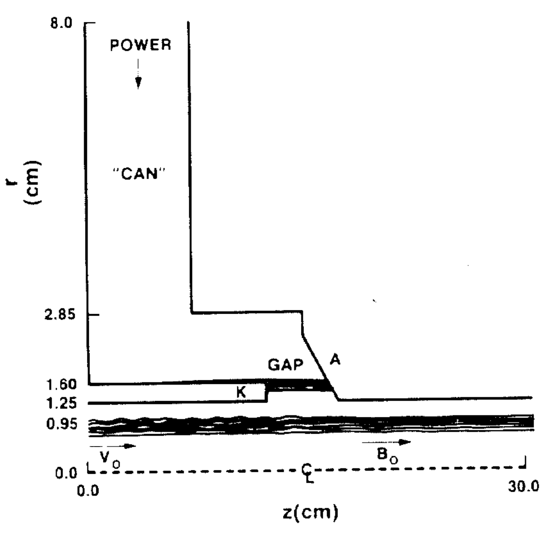

Fig. 1. Particle simulation of a MABE accelerating gap. The lower trajectories represent the beam (25 kA, γ(LHS) = 3, γ(RHS) = 7), the upper trajectories represent the parasitic leakage electrons (19 kA). The gap voltage is 2 MV. B_0 = 20 kG.

The effect of the gap in inducing radial oscillations in the beam is negligible for the case of Fig. 1. The reason is that the cyclotron wavelength λ_c is approximately equal to half the gap spacing d. This is an "antiresonance," as we now show.

The perturbing radial electric field δE_r of an accelerating gap induces radial oscillations (zero frequency cyclotron waves) in an azimuthally symmetric electron beam according to[9]

$$\delta R(z) = \int_{gap} \frac{e}{mc^2\gamma k_c}\,\delta E_r(z_0)\,\sin k_c(z_0 - z)dz_0 \quad , \qquad (1)$$

where the integration is over the interval of width ≈ d where δE_r differs from zero, and where δR is the perturbed beam envelope, γmc^2 is the average beam energy (kinetic and rest mass), e is the electron charge, and $k_c = 2\pi/\lambda_c = eB_0/m\gamma V_0$, with guide field B_0 and beam velocity V_0. Let us idealize the shape of δE_r by the somewhat realistic

$$\delta E_r(z) = \phi_1\delta(z) - \phi_2\delta(z - d) \quad , \qquad (2)$$

where $\phi_1 = \phi_2 > 0$ by Gauss' Law, as shown by Adler.[10] Of course, the main deviation of Eq. (2) from reality is that the peaks in δE_r should have a finite width and amplitude, as in Fig. 2a, but the delta functions are sufficient for our purpose. We obtain

$$K\delta R(z) = -\phi_1 U(z)\sin k_c z + \phi_2 U(z - d)\sin k_c(z - d) \qquad (3)$$

where $K = mc^2\gamma k_c/e$, and U(z) = 1 for z > 0, 0 for z < 0. This result is plotted in Fig. 2b, where the total δR is given by the sum of the two sine waves.

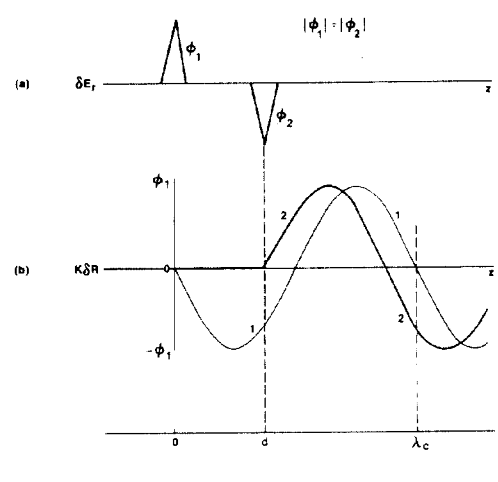

Fig. 2. The analytic calculation. (a) Input δE_r of Eq. (2). (b) Output δR of Eq. (3), where the total δR is to be obtained by adding the 1 and 2 curves. Maximum δR amplitude results for λ_c = 2d.

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Clearly constructive interference between the ϕ_1 and ϕ_2 contributions occurs for $d = \lambda_c/2$, which we term "resonance" in the sense of maximum amplitude. More generally,

$$\lambda_c = 2d,\ 2d/3,\ 2d/5,\ \text{---}\ \underline{\text{resonance}} \tag{4}$$

whereas the two waves are out of phase for

$$\lambda_c = d,\ d/2,\ d/3,\ \text{---}\ \underline{\text{antiresonance}}\ . \tag{5}$$

Returning to Fig. 1 and using the average λ_c, we find $\lambda_c \approx d/2$, explaining why the rather large gap induced no oscillation on the beam.

To test these results for a realistic $\delta E_r(z)$, a series of calculations with a beam-envelope code was performed using the δE_r shown in Fig. 3. The envelope solver includes an effect neglected in Eq. (1); namely, the z-variation of γ. However, we find basic agreement with Eqs. (4) and (5), as shown in Fig. 4. Here, the amplitude of δR is plotted vs. λ_c/d (using the average λ_c). Note maxima in the amplitude at about $\lambda_c/d = 1.9$ and 0.8, corresponding to Eq. (4), and minima at about 1.0 and 0.6, corresponding to Eq. (5). The actual $\delta R(z)$ obtained for $\lambda_c = 2d$ is given in Fig. 5; other points in Fig. 4 were obtained by varying λ_c and measuring the amplitude of the resulting final oscillation in a similar fashion.

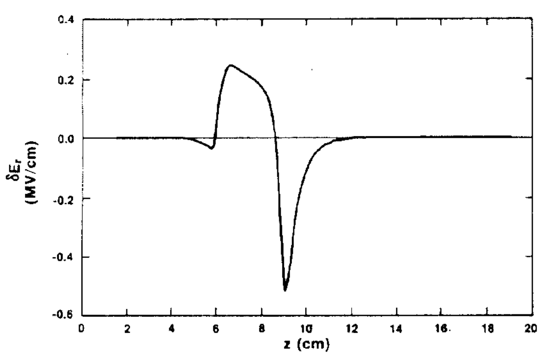

Fig. 3. $\delta E_r(z)$ at beam edge from Poisson solution for the fields of a square MABE-type gap accelerating a 25 kA beam.

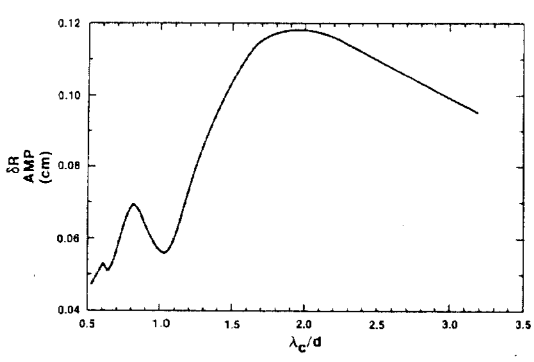

Fig. 4. Amplitude of beam envelope oscillation vs. λ_c/d, from solution to envelope equation in field $B_o = 20$ kG, for a 25 kA beam in gap corresponding to Fig. 3.

We note several features of Fig. 4. First, the main $\lambda_c = 2d$ "resonance" is very broad; values of λ_c/d from, say, 1.5 to 2.5 all give a substantial amplitude, so that if one wishes to design a system to miss this region, one must for typical parameters use a high voltage injector. Second, the relative amplitude in Fig. 5 is about 11%, carrying the beam edge to a radial excursion slightly less than halfway to the drift-tube wall. Thus, in this (and most) examples, being near $\lambda_c = 2d$ for one gap is not in itself catastrophic. The danger comes for electrons which acquire this "large" oscillation and then reach the next gap with the wrong phase. Finally, we may compare the envelope result of Fig. 5 with the analytic Eq. (3). Estimating $\phi_1 = \phi_2$ from Fig. 3, we get the maximum amplitude of 1.6 mm, compared with 1.2 mm in Fig. 5. Finally, we note from Eq. (1) that at very high γ, such that $\lambda_c >> d$, the amplitude of δR decreases as $1/\gamma$.[10]

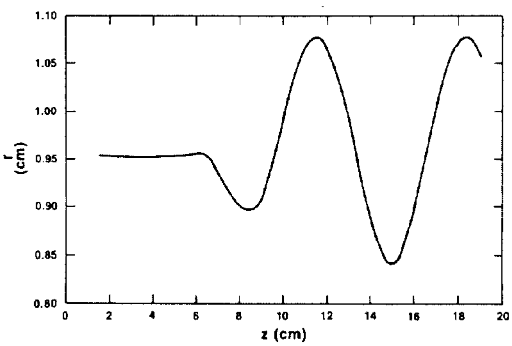

Fig. 5. Beam envelope for "resonance" case $\lambda_c = 2d$ in Fig. 4. The drift-tube wall is at $r = 1.25$ cm.

Multiple Gaps

We consider next the behavior of a beam as it passes through four gaps in series. An electron map from a sample simulation is given in Fig. 6. The

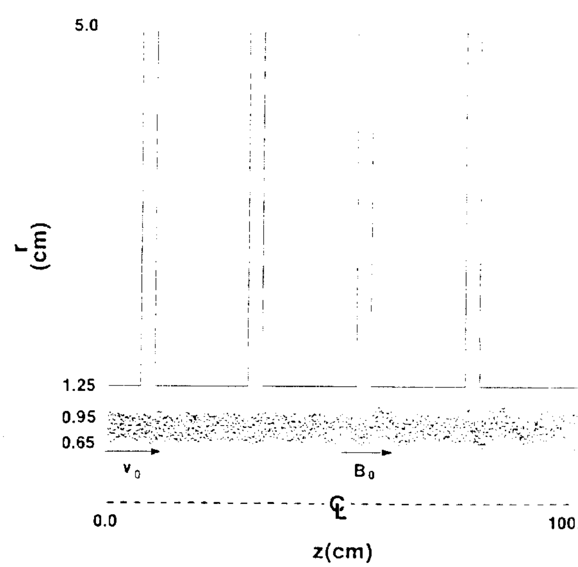

Fig. 6. Four-gap MABE simulation (electron map) using MAGIC. Beam: initially warm, 25 kA, input energy 1 MeV, output energy 9 MeV. Gaps: 2 MV each, $d = 3$ cm. $B_o = 20$ kG. The beam passes through the $\lambda_c = 2d$ resonance in gap 3.

parameters used earlier are again employed, except the input beam is "warm"; specifically, the initial r and θ velocities V_r and V_θ were selected randomly from the interval (-c/10, +c/10). We note the sudden increase in radius after the third gap; this is accounted for by the fact that λ_c = 2d there, a "resonance" from Eq. (4). (Compare Fig. 5.) The beam is accelerated from 1 MeV to 9 MeV by the E_z field seen in Fig. 7; the δE_r of each gap is similar to that in Fig. 3.

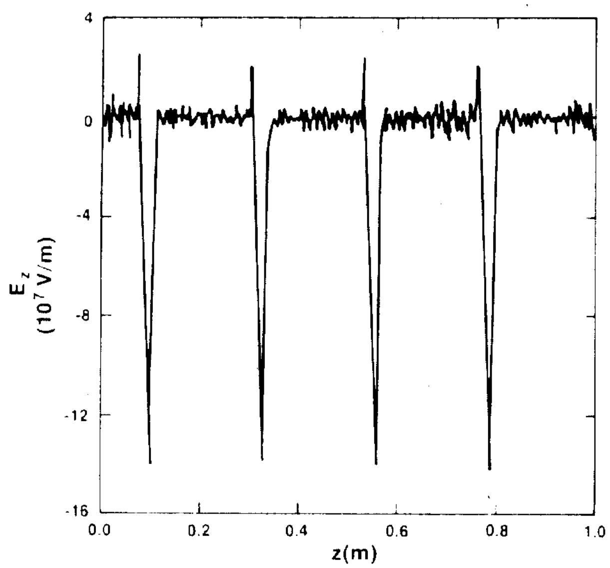

Fig. 7. $E_z(z)$ at r = 1.1 cm corresponding to Fig. 6.

From Fig. 6 we see that one effect of the gaps is to thicken the annulus somewhat; however the cumulative δR is only about 1 mm. A more interesting (but related) effect is the emittance growth. If we define emittance as the radius in transverse $\gamma V_r - \gamma V_\theta$ phase space, the result is shown in Fig. 8. This "emittance" increases by a factor of about 4.5 in passing through four gaps. However, this definition includes the increase in γ of about 4 per gap caused by the accelerating E_z of Fig. 7. If instead we define emittance in terms of $\beta_\perp = (V_r^2 + V_\theta^2)^{1/2}/c$,[11] (i.e., divide by γ in Fig. 8), we find that the emittance actually decreases slightly as the beam traverses four gaps. (This would not be true for an initially "cold" beam, but in that case the emittance grows, levels off, and then decreases after many gaps.)

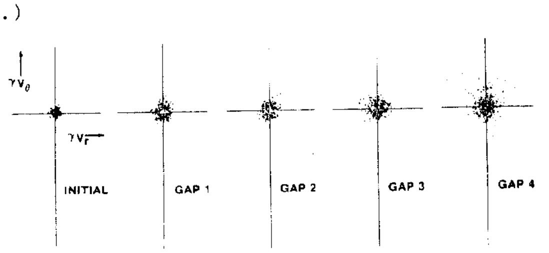

Fig. 8. γV_θ vs. γV_r phase space corresponding to Fig. 6. This illustrates the beam emittance growth as a function of the gap number, including the increases in γ.

To understand why $\beta_\perp$ should decrease after enough gaps that $\lambda_c \gg d$, recall that δR (induced by a single gap) ∝ 1/γ so that the total cumulative radial oscillation amplitude is limited to some value δR_{lim} (we assume no wall loss). Since the final oscillation is still a (zero-frequency) cyclotron wave, it follows that the associated $\beta_\perp$ must be

$$\beta_\perp = \frac{2\pi\delta R_{lim}}{\lambda_c} \propto \frac{1}{\gamma} \quad . \tag{6}$$

Thus the emittance as conventionally defined[11] should decrease with gap number after many gaps.

Conclusion

Based on the calculations presented, there does not seem to be (in principle) an upper limit on the number of gaps which can be traversed by an initially good-quality beam from a carefully designed diode injector. The first few gaps are the most dangerous, especially if the "resonances" of Eq. (4) are not avoided. As γ becomes large enough that $\lambda_c \gg d$, the beam emittance should decrease as 1/γ. These results suggest a considerable advantage in beginning with a high-voltage injector.

We emphasize that all our calculations have been two-dimensional. Perhaps further studies of beam dynamics in high-current linacs should await the advent of efficient 3-D codes which can include phenomena such as "beam-breakup" and "image-displacement" instabilities.[4,12] If these occur in systems such as MABE, they may dominate the azimuthally symmetric radial oscillations.

Acknowledgment

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