CALCULATION OF SPACE CHARGE EFFECTS IN ISOCHRONOUS CYCLOTRONS

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Abstract

An introduction to the problems related to space charge calculations in isochronous cyclotrons is followed by the presentation of a new computer simulation model for the longitudinal space charge effects. The new model uses an analytical tansformation to reduce from three to two dimensions and a particle-in-cell method to calculate the charge distribution as a function of time. The new computer program enabled us to study the "spiralling instability", which can evoke filamentation in the phase-plane related to energy and RF-phase of the particles. While this instability is most important at low energies, there is also a steady increase of energy spread in the beam pulses during the acceleration. Also this second effect causes an increase of the beam losses at extraction, therefore both can limit the beam current that can be handled.

Introduction

The construction of the SIN Injector II [1], as an alternate first stage for the 590 MeV Ring-cyclotron of the Swiss Institute for Nuclear Research has brought actuality to investigations of space charge effects in isochronous cyclotrons [2]. Simulations using the new computational models presented here and in earlier publications [3,4], confirm the results from Gordon [5] and Chabert et al [6,7].

The future operation of the SIN accelerators at intensities above 1mA relies on very small beam losses at extraction. At low intensities clearly separated turns can be achieved using the flattop-system [8]. Longitudinal space charge forces produce energy variations in the beam pulses. Changes of the RF parameters can only partially compensate these energy variations. In order to understand these effects the "Disks" model was incorporated into a general computer simulation of beams [3]. Simulations at low energies became possible with the particle-in-cell method described in this paper. It allowed a detailed study of the instability caused by space charge forces.

Special Properties of Isochronous Cyclotrons

In isochronous cyclotrons, space charge effects can be divided into two categories, transversal and longitudinal, named after their relevant components of the space charge forces w.r.t. to the direction of the beam pulse. Transverse space charge forces weaken the focussing strength in the cyclotron. Sacherer [9] presented a method to calculate their effects. A standard program for beamlines can also handle a cyclotron orbit as a special case. In ring-cyclotrons the longitudinal components of space charge forces affect the beam at much lower intensities than the transverse ones. In an isochronous cyclotron there exists no focussing in the phase-space dimensions "energy" and "phase". Motions in this plane are metastable: A particle position in a corresponding plane defined by "radius" and "azimuthal distance" always moves at a right angle to the force acting on it. The additional energy gain of e.g. the leading particles in a beam bunch due to the repulsion by the others steadily piles up during acceleration.

The "Disks" Model

An extension of an older program for the simulation of beams in the cyclotron led to the first tool for investigations on space charge effects. It kept the name "Disks" model although the representation of the beam with charged disks was soon changed to charged cylinders. A series of 50 to 200 reference particles is followed through the accelerator. Periodically the positions of those particles are updated, to account for space charge forces. Simulations with this method published in 1981 [3] did not take into account that space charge forces in the longitudinal direction w.r.t. to the beam pulse can have components in the radial direction. Calculations which included these small forces became completely unstable in the low energy region. Careful investigations of these instabilities led to the conclusion that a model which partitions a beam pulse into less than 200 cylinders is inadequate.

The Particle-in-Cell Method

A precise model has to take into account that the particles in an isochronous cyclotron perform vertical as well as radial-longitudinal betatron oscillations, which are rapid compared to their motion due to space charge forces. In a coordinate system moving with the center of the beam pulse, the particle motion due to the radial-longitudinal oscillations is approximately circular. In combination with the vertical oscillation, the particles move around on the surfaces of little cylinders. The slow motion of the particles in this reference frame is governed by the space charge force averaged over the path of the quick motion. It is a good approximation to assume that all particles moving around on the same little cylinder are affected the same way by space charge forces and even that the slow motion of particles on all coaxial cylinders is the same.

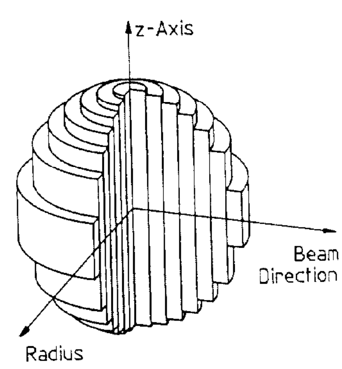

Fig. 1: Schematic view of how the combination of all betatron oscillation cylinders with identical center points form a sphere. For phase space density distributions of usual beams, the majority of particles with high vertical amplitude (long cylinders) do not have large horizontal amplitudes (large diameter of cylinders) as well. The assumption that the resulting charge distribution is a homogeneously charged sphere, is a simplification of this model. In reality it has ellipsoidal shape and a smoothly decreasing charge density with increasing distance from its center.

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The fact that the group of cylinders with betatron motions around the same center-point can be represented by a sphere, gives a substantial simplification of the calculation. Such a "betatron-oscillation-sphere" has to be assumed around each of a large series of points in the midplane of the cyclotron. The combination of all these spheres, which may partially overlap, forms the charged cloud defining the space charge field. This field then has to be averaged over the same kind of sphere yielding the force relevant to the motion of a center point. Simply rearranging the sequence of these multiple integrals separates the three dimensional case into a convolution problem in two dimensions and hence only the need to calculate the force between two charged spheres as a function of their distance.

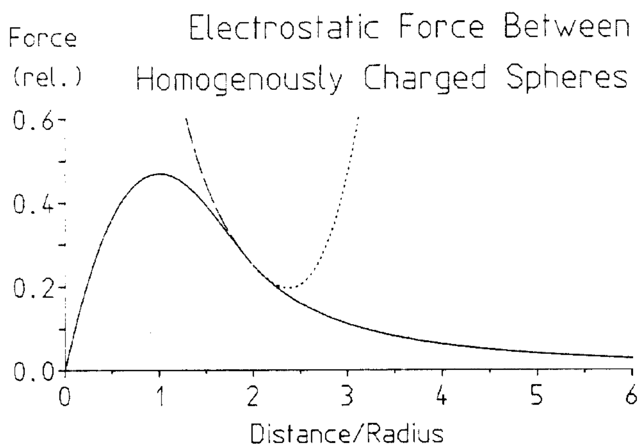

Fig. 2: The force between charged spheres as a function of their distance can be calculated analytically. At a distance of twice the radius, the function switches from the polynomial, relevant to the inner region, to the simple function $1/r^2$. Both functions are drawn beyond their region of definition (dashed resp. dotted), in order to visualize the smooth transition.

Although the three dimensional model for the particle motion with space charge forces has now been transformed into a two dimensional convolution problem, a substantial computational effort is needed for its solution. The remaining problem has some similarities to the hydrodynamics of an incompressible medium [10,11]. In contrast to hydrodynamics, the equations are of first order in time only, but on the other hand the force at a point depends on all other points i.e. there remains an integro-differential equation.
The method used to solve this equation is called particle-in-cell. A beam pulse is represented by a series of points in the midplane of the cyclotron. Each of them represents the center of a betatron oscillation sphere. An intensity value is assigned to every point. The whole space charge calculation is done in a reference frame moving with the center of mass of the beam pulse. The intensity distribution defined by the weighted points is transferred onto a regular grid in the following way: The charge carried by each point is distributed to the four corner points of the cell containing it. Bilinear functions of the particle's relative position in the cell define this distribution. On the grid points, the force relevant to the particle motion can be found by a two dimensional convolution of the intensity distribution with the elementary force function. The velocity at the individual points is then found by two dimensional interpolation of the velocity values at the grid points. Bilinear or bicubic-spline interpolation can be applied, both are consistent with the bilinear assignment of charge to the grid.

The integration cycle for the particle motion ends with changing the coordinates of each point according to the interpolated velocity and to the time step chosen for the integration. The whole method acquires an enhanced stability when it is applied in a so called "leapfrog" scheme: Two of the previously described integration steps are used in an interleaved way - each one of two sets of coordinates is used to define the velocities for the other. The two sets correspond to time values which are half an integration step apart.
The effort to calculate the convolution can be reduced using the folding theorem: Two-dimensional fast fourier transforms of both original functions, followed by a simple product element by element and finally another fourier transform of the resulting array are equivalent to a convolution [12].

Results from the Particle-in-Cell Simulation

The particle-in-cell method revealed the reason for the failure of the simpler model at low energies; there is an instability caused by space charge forces which tends to deform the beam pulse finally into a double spiral, a shape similar to some galaxies. This has been called "spiralling instability". The steps of the deformation of phasespace due to this instability can best be seen when coasting beams are simulated. The present program version uses 12000 "particles" and a grid of 256x128 points. The mesh size is automatically adapted when parts of the beam get near to the border of the grid. This allows to treat also accelerated beams.

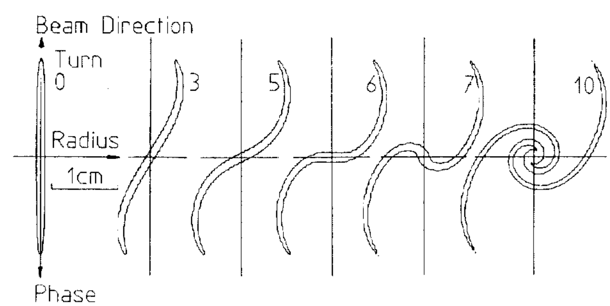

Fig. 3: Some steps from the evolution of the spiralling instability. The figures represent density distributions of betatron oscillation centers in the cyclotron midplane related to different turns of a coasting beam at 3 MeV. As the S-like bend of the beam pulse becomes strong enough at turn 5, the rotation speed of the central part of the beam pulse compared to the rotation of the outer parts starts to increase rapidly.

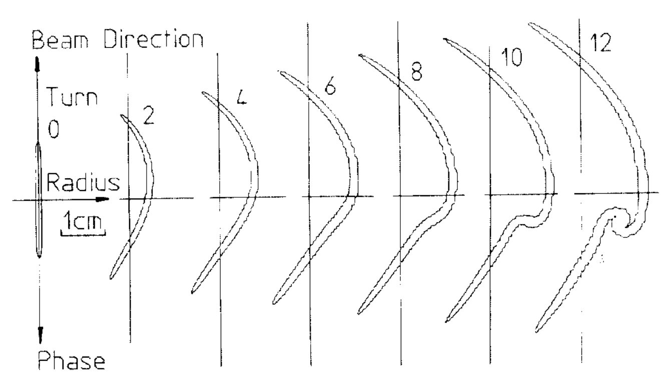

Fig. 4: Simulation of an accelerated beam. The beam pulses, represented by density distributions of oscillation centers, are deformed due to acceleration (without flattop), increase of radius and space charge.

Using a simple modification of the force function and a change of the mesh size in the radial direction, the effect from neighbouring orbits can be studied. The usual pictures from the particle-in-cell simulations show density distributions of oscillation centers. In order to get the shape of the beam pulses, a convolution with the density distribution of the betatron oscillation sphere has to be applied.

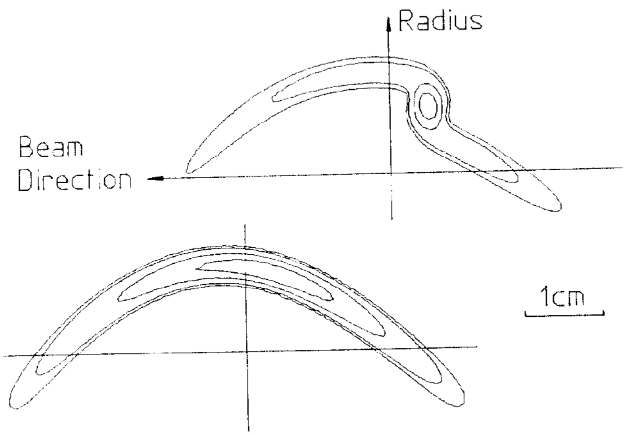

Fig. 5: Two examples of simulated shapes of beam pulses (charge density projected into the cyclotron midplane). The two cases shown are turn 10 in SIN Injector II at .7 and 1.4mA.

A large variety of cases has been calculated with the particle-in-cell method. One, two or more spiralling centers can be formed in the instability. Their amount depends on the length and diameter of the beam pulse and on the longitudinal charge density distribution. Special attention was paid to the early states of the beam deformation. For the case of a short bunch (see fig. 3) the rotation of the center of the beam pulse has been analyzed. This is the part which has the highest rotation speed. During the first part of the simulation its rotation speed is nearly constant.

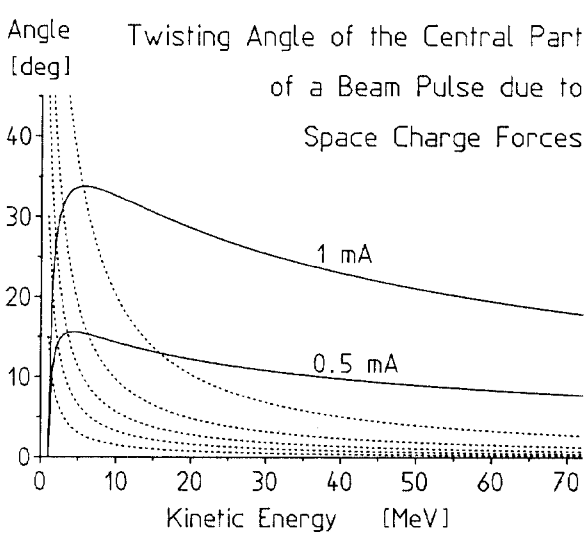

Fig. 6: The combined effect of space charge and acceleration. The parts of a beam bunch rotate in the midplane due to space charge forces. The angle of orientation of the central part should therefore steadily increase. After a short increase at low energies, due to the fact that azimuthal distances grow and radial distances shrink with acceleration, this angle starts to decrease. The dotted lines indicate the effect from acceleration only.

The amount to which the pulse shape of the beam deviates from a straight line is essential to the increase of the rotation speed. As a measure of this deviation we can take the rotation angle, the center part has accumulated during acceleration. The plot of this angle as a function of energy (fig. 6) demonstrates, why the importance of the spiralling instability is restricted to low energies.

Conclusions

The new program based on the particle-in-cell method has improved the possibilities for the simulation of space charge effects in isochronous cyclotrons. This is specially valid at low energies. It allows study of the spiralling-instability in detail. The investigations have shown that the instability is only important in the low energy region of Injector II. For the 590 MeV Ring Cyclotron and for the Injector II at intensities below a critical level the simpler "Disks" model can be used. The simulations predict a maximum current for the Ring of about 2mA (limited by energy spread) and a limit for Injector II between 1mA and 2mA (given by this instability). A substantial uncertainty still remains due to the approximations which have been made in the model; in the near future comparisons between beam measurements [1] and calculations will be possible. The two programs will also be helpful for the planning of beam experiments.

Acknowledgement

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