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Progress in the Development of a 3D Finite Element Computer Program to Calculate Space and Image Charge Effects in RF Quadrupoles

N J Diserens
Rutherford Appleton Laboratory
Chilton, Didcot, Oxon OX11 OQX (UK)*

Abstract: This paper describes the further development of the RFQTRAK computer program. Two RFQ cells, with combined length $\beta\lambda$, are modelled with finite elements and a 'waterbag' distribution of particles is tracked through them. Each particle makes about 20 time steps per cell. At each time step the space charge solution is obtained and the field at the particle positions is calculated. These fields are added to the parameterised RF fields for use in the tracking routine. When one complete RF cycle of particles has emerged from the double cell the next pair of cells is modelled in a similar way. This is repeated until the beam has been tracked through the whole RFQ. Estimates of beam loss for the Chalk River RFQ1-B design are compared with results from the PARMTEQ program.

Introduction

A preliminary report on development of the RFQTRAK computer program was published in the proceedings of the 1984 Linear Accelerator Conference[1]. That paper described a two cell finite element model (length $\beta\lambda$) of an RFQ through which particles were tracked taking into account full three dimensional space charge and image charge effects.

The program has now been further developed. The two cell model is still used but the beam may be tracked through many pairs of cells in tandem to represent a complete RFQ.

A special version of the program uses a mesh generator which models the outer boundary of the problem in such a way as to remove the poles and therefore to remove the image charge effects. The two versions have been run starting with the same input beam parameters to show whether image charges had any significant effect on beam loss and emittance. Results for the low energy stages of an RFQ are given.

The remaining sections of this report give computer timings and suggestions for further development of the program.

Program Arrangement

Figure 1 shows a schematic arrangement of the program.

The mesh generator discretises the double cell model of the RFQ into an assembly of 'brick' elements. Only one quadrant of the RFQ is represented.

Figure 2A shows a typical cross-section through the mesh, which comprises 3-dimensional 20-node quadratic elements.

Figure 2B shows the mesh arrangement for the 'no image charge' version, which has a polygonal outer boundary at a mean radius from the beam axis of about twice the minimum pole tip distance.

*Work done partly at Chalk River Nuclear Laboratories, Ontario, Canada.

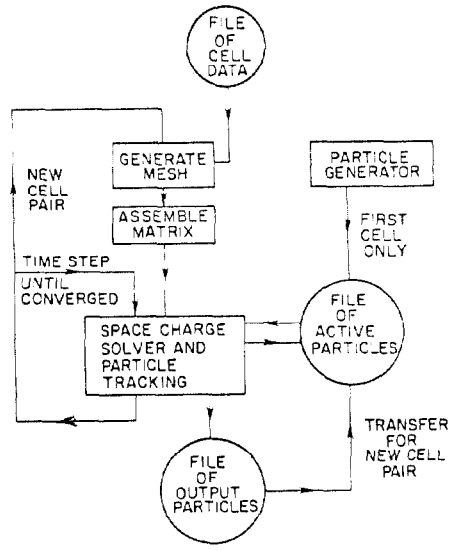

Fig. 1 Schematic Arrangement of Program

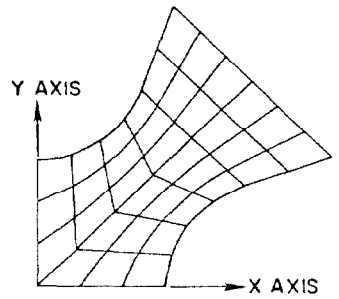

Fig. 2a Typical Cross Section Through Mesh

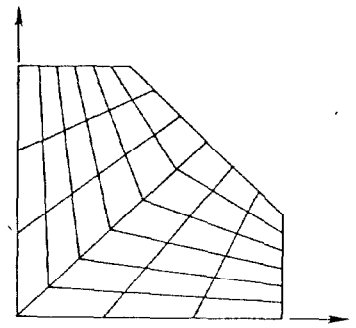

Fig. 2b Special Mesh for 'No Image Charge' Runs

The arrangement of the mesh in the longitudinal direction is such that all elements have the same length within a cell pair. (See Figure 3). This gives a slight displacement of the centre plane of each cell pair from the true position but facilitates the location of particles within a cell.

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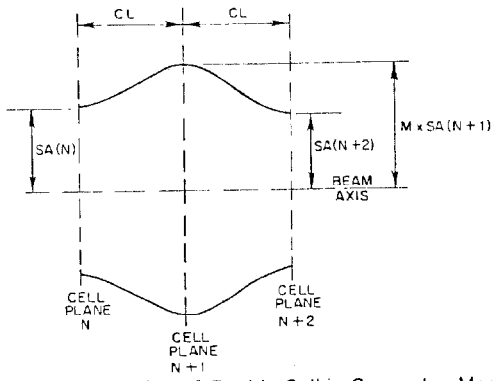

Fig.3 Representation of Double Cell in Computer Model

Macro particles injected into the RFQ model are generated using a random distribution within the estimated acceptance of the acclerator. The initial parameters of these particles are stored in a file which can be used to reproduce these in a closed loop over several RF cycles.

The RF fields for tracking are obtained using the analytic expression, with 8 coefficients derived with programs RFQCOEF[2] or CHARG3D[3]. The space charge fields are obtained by solution of the Poisson Equation:

$$\nabla^2 V = - \rho$$

using the finite element method and applying zero potential to the pole surfaces in the model. In the 'no image charge' version the entire outer surface is set at zero potential. This means that the image charges will in fact be fairly evenly distributed around the surface. This representation will not exactly simulate an infinite boundary, but will be a sufficiently good approximation.

During the third RF cycle the emergent particles are stored in a file for use in the next cell pair.

This process is repeated for each successive cell pair.

Effect of Image Charges

The model used for investigating the effect of image charges was the Chalk River RFQ1-B design[4]. The input beam was 50 keV protons. Cells 5 to 50 were used in these tests, being regarded as the part of the acclerator where image charges are most significant. The matching section, cells 1-4, was not represented in the model but beam output at cell 4 was taken from the PARMTEQ program[5]. Cell dimensions are:

Minimum distance of pole tip from axis	SA = 0.4130 cm at cell 5 = 0.3915 cm at cell 50
Vane modulation	M = 1.0 at cell 5 = 1.069 at cell 50
Cell length ($\frac{3\lambda}{2}$)	CL = 0.575 cm at cell 5 = 0.588 cm at cell 50
Vane voltage	= 76 kV peak.
Frequency	= 269 MHz

The design beam current was 90 milliamperes but it was felt that it would be useful to demonstrate the effect of image charges at considerably higher currents, so computer runs were done at both 90 and 400 mA.

In the computer model 360 macroparticles per RF cycle were input, being one per degree of phase at the entry plane to the RFQ. The number of time steps was 20 per RF cycle.

The 90 mA beam showed no loss of particles between cells 4 and 50 in these runs.

Figure 4 shows the number of active particles remaining plotted against cell number, for computer runs both with and without image charges with 40 mA.

The criterion for a particle being 'lost' was, for both cases, when it had passed outside the radius r = SA.

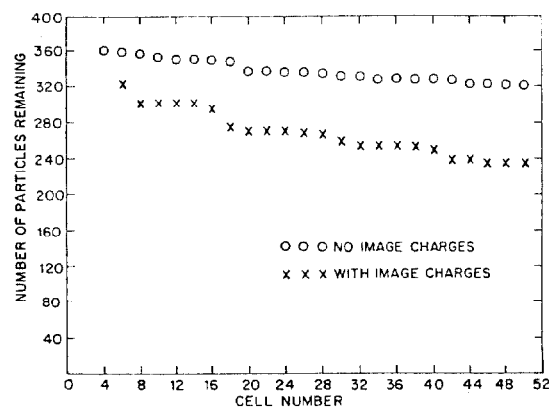

Fig.4 RFQ1-B With 400 MA to Illustrate Particle Loss With and Without Image Charges.

Figures 5(A to D) show the beam profiles at cell 50 for the two cases, and Figures 6(A to D) show the emittances. The above were obtained for particles crossing the output plane of the cell, taken over the whole RF cycle.

The apparent effect of representing the image charges is to show that high current beams will have their outer particles skimmed off leaving a narrower, weaker beam, unless sufficient clearance is allowed for, between beam and poles, in the design of the RFQ.

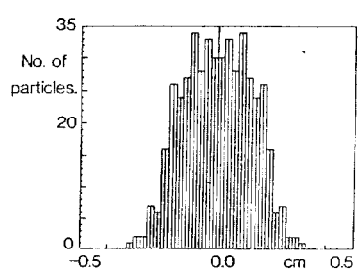

Fig. 5a. Horizontal profile. Cell 50. with image charges.

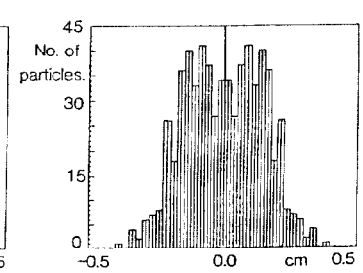

Fig. 5b. Horizontal profile. Cell 50. no image charges.

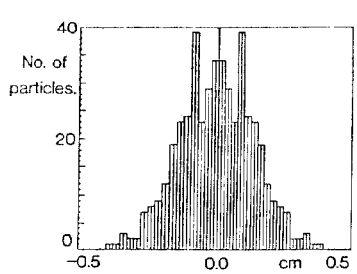

Fig. 5c. Vertical profile. Cell 50. with image charges.

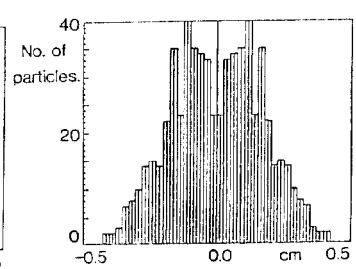

Fig. 5d. Vertical profile. Cell 50. no image charges.

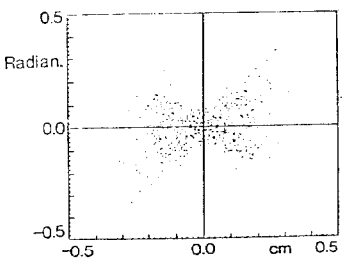

Fig. 6a. X – XP. Cell 50.
with image charges.

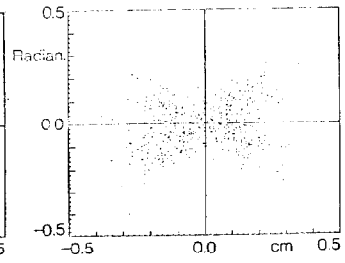

Fig. 6b. X – XP. Cell 50.
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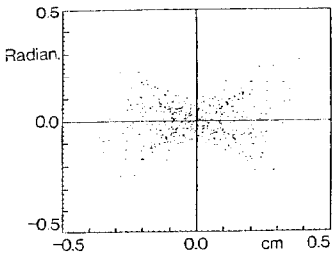

Fig. 6c. Y – YP. Cell 50.
with image charges.

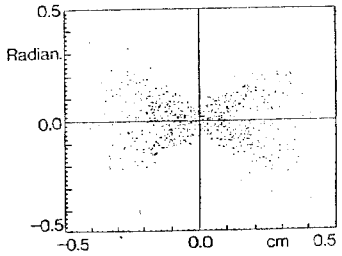

Fig. 6d. Y – YP. Cell 50.
no image charges.

Computer Timings

The above results were obtained on the (FUJITSU) ATLAS 10 computer. The number of elements for each double cell was 288 quadratic bricks. The total solution and tracking time for each double cell was about 44 seconds when the tolerance for convergence of the conjugate gradient matrix solver was set at the usual value of 10^{-7}.

One of the advantages of the conjugate gradient method and similar iterative algorithms is that when an approximate solution is used as a starting point for a further run the number of iterations required should be considerably less. At the above value of tolerance this did not seem to be the case. However, when a figure of 10^{-4} was put in it was found that the tracking parameters for the particles were the same to four figures but the running time per double cell was reduced to 28 seconds. This would mean a total of about 42 minutes, with the present mesh, to run a complete 180 cell RFQ.

Conclusions

The results obtained show that image charges may have a significant effect on beam loss at the low energy end of the RFQ, for a distance which depends on the amount of current in the beam.

At the high energy end, where the bunching is tight, it will probably be more accurate to neglect the modelling of the electrode surfaces and to concentrate the mesh subdivision around the bunches. Either an integral or differential formulation may be used for this region.

It is therefore proposed that the program should be modified so that the meshing be changed at a point in the RFQ where image charges are no longer significant, because the bunches are of considerably smaller width than the bore of the device. At the same time the spacing between individual bunches will be sufficiently large so that the interaction between them is small, and therefore the discretisation need only cover the length of the bunches.

Further experimentation will be required to determine how many mesh elements and how many macroparticles are required to give reliable results.

The program is still rather slow, and further efforts are required to speed up both the solver and the routine which locates the particle within an element. The use of an array processor would help in this task.

Acknowledgement

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