

INVESTIGATION OF THE TOUSCHEK EFFECT IN VUV ELECTRON STORAGE RING

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Summary

The lifetime of stored electron beam current due to the Touschek effect was studied experimentally in the SOR-RING, a 400 MeV electron storage ring. From the measurement of the beam size and the lifetime, it is found that the theoretical expression of the lifetime is quite accurate for a flat beam, but some corrections are necessary for a round beam.

Introduction

The Touschek effect was first found and studied experimentally in a low energy electron storage ring AdA<sup>1)</sup>, and several theoretical investigations on the effect have been made<sup>2,3,4)</sup>. But experimental studies reported so far is only one for AdA. And in that case the beam size was not measured, and it was necessary to assume a bunch volume about 40 times bigger than the natural volume to explain the measured lifetime. Meanwhile the theoretical and experimental studies of the effect have been made on a flat or ribbon beam. Concerning the recent storage rings for the radiation, one wants in many cases to have a round beam for a longer lifetime. Therefore it is quite interesting to make a detailed investigation of a flat and a round beam including the measurement of the beam size.

We have made an experimental study of the Touschek effect in the SOR-RING, a 400 MeV electron storage ring. The ring is very convenient to evaluate the Touschek lifetime since the beam size is almost the same along the circumference of the ring because of its basically weak focusing magnet system. The experiment was made in single bunch mode operation where we have the simplisity of constant beam size, independent of the beam current, very low pressure because of low beam current, and no ion trapping effect. The Touschek effect was also studied in a round beam by inducing a coupling between the horizontal and vertical betatron oscillations with a skew quadrupole magnet.

Experimental

The beam lifetime in the experiment  $\tau_b$  is defined as that of exponential decay at a given beam current in a few minutes. The bunch length was measured by displaying in a multi-channel analyser the time difference between an RF frequency trigger and photon signals of the radiation detected with a fast photomultiplier. The transverse beam size was measured by focusing the radiation with an optical lens on a pin hole which was moved horizontally and vertically. A skew quadrupole magnet with the core length of 10 cm was constructed and installed in a straight section of the ring.

In the SOR-RING the single bunch mode operation is performed by the RF knock-out method. The knock out frequency is modified

with a gate pulse synchronized to one seventh of the acceleration frequency, and we can kick six bunches out of seven or full bunches of the stored beam current. The kicking out becomes harder at a higher beam current because of an amplitude dependent tune shift due to ion trapping effect. The maximum beam current in this operation is about 20 mA, and beam current still remaining in other bunches is as low as 0.1 %.

Results

Figure 1 shows the bunch length  $\sigma_L$  and horizontal and vertical beam size  $\sigma_x$  and  $\sigma_z$  measured as a function of the beam current  $I_b$ . The beam bunch has the gaussian distribution both longitudinally and transversely. The bunch length and the sizes are independent of the beam current, and agree with the values  $\sigma_L = 81$  mm and  $\sigma_x = 0.98$  mm theoretically expected. The vertical beam size is not known apriori, since the coupling coefficient  $\kappa$  is not known.

Figure 2 shows the current dependence of the inverse beam lifetime  $1/\tau_b$  in the single bunch mode operation. The solid circles in the figure represent the observed values, which increases linealy as the beam current  $I_b$ . This indicates that the lifetime is determined by the Touschek effect.

Theoretically the Touschek lifetime  $\tau$  is expressed as

$$1/\tau = \frac{\sqrt{\pi} r_0^2 c}{e} \frac{C(\xi)}{f_0} \frac{I_b}{\gamma^3} \left\langle \frac{1}{\sigma_x v_p} \right\rangle \left( \frac{\Delta E_{max}}{E} \right)^2, \quad (1)$$

- where  $r_0$  : the classical electron radius ( =  $2.8 \times 10^{-15}$  m ),
- $c$  : the light velocity,
- $e$  : the electric charge,
- $f_0$  : revolution frequency,
- $\gamma$  : relativistic beam energy,
- $v_p$  : bunch volume ( =  $(4\pi)^{1/2} \sigma_x \sigma_z \sigma_L$  ),
- $\sigma_x$  : divergence of betatron oscillation,
- $\Delta E_{max}/E$  : RF bucket height,

$$C(\xi) = \ln(1/\xi) - 1.5 \quad \text{for } \xi < 0.01$$

$$\xi = (\Delta E_{max}/E / \gamma \sigma_x')^2$$

$\gamma E$  : Euler's constant ( = 1.78 ).

Inserting the measured value  $\sigma_x, \sigma_z, \sigma_L$  and the estimated value of  $\sigma_x' = \sqrt{E_x/\beta_x}$  into the formula (1), we get the theoretical lifetime shown with the straight line in Fig.2. In the above figure  $\Delta_p$  is added appropriately to  $1/\tau$ , which is due to the residual gas scattering.

Next we discuss about the experimental results of the beam lifetime when the beam size is expanded vertically by the excitation

of the skew quadrupole magnet. The cross section of the beam bunch changes gradually from a flat shape to a round shape with the increase of the skew field.

According to the beam size measurement the profile of the beam cross section is gaussian in both directions irrespective of the skew field strength. As shown in Fig.3, the horizontal beam size decreases slowly and the vertical size increases a little rapidly as the skew current, which is almost independent of the beam current. It is found that the area of the cross section  $\pi \sigma_x \sigma_z$  increases nearly linearly as the skew current  $I_s$ . A little increase of  $\sigma_z$  at  $I_s = 0$  (A) from 0.16 mm before installing the skew magnet to 0.19 mm after that seen in the figure is due to the

hysteresis of the remanent field of the skew magnet. The bunch length is not affected by the excitation of the skew magnet.

Now that the volume of the beam bunch increases linearly as the skew current, we expect a linear increase of the beam lifetime  $\tau_b$  as the skew current. In fact, this was observed as shown in Fig.4, where the product  $\tau_b I_b$  is given in the ordinate. We see that the experimental data run along a straight line against the skew current, independent of the beam current. This is not strange, since the increase of the bunch volume with the skew current is almost independent of the beam current as described above.

For the estimation of the Touschek lifetime it is necessary to find the value  $\sigma_x'$  as a function of the skew current, since the emittance  $\epsilon_x$  and therefore  $\sigma_x'$  changes with the coupling coefficient  $\kappa$ . In the Appendix  $\kappa$ ,  $\epsilon_x$ ,  $\sigma_x'$  as well as  $\epsilon_z$  and  $\sigma_z'$  are estimated as a function of the skew current. Then using the estimated value of  $\sigma_x'$  and the bunch volume measured, we can evaluate the Touschek lifetime  $\tau_x^t$ , where the suffix x is added because of the relation to  $\sigma_x'$ . This lifetime is represented as a product  $\tau_x^t I_b$  in Fig.4, which is about a half of the observed lifetime.

We remember that the formula (1) applies to a flat beam, where only the horizontal betatron oscillation is considered. Since in the present case the vertical beam size is expanded considerably, we should take account of the contribution of the vertical oscillation. By substituting the suffix z for x in the formula (1), we can estimate a vertical Touschek lifetime  $\tau_z^t$  from  $\sigma_z'$  which is estimated in the Appendix. In Fig.4, the lifetime is represented also as the product  $\tau_z^t I_b$ , being only a little lower than  $\tau_x^t I_b$ . Here we find an interesting fact that the total lifetime of the simple sum,

$$\tau_t^t = \tau_x^t + \tau_z^t \quad (2)$$

lies very closely to the observed data of the product  $\tau_b I_b$  as shown with a broken line in Fig.4.

If the horizontal and vertical Touschek lifetimes contribute to the beam loss independently, the total Touschek lifetime will be given as follows,

$$1/\tau_t^t = 1/\tau_x^t + 1/\tau_z^t. \quad (3)$$

But this relation of inverse sum cannot explain the experimental data shown in Fig.4. In conclusion, the horizontal and vertical betatron oscillations should be considered together for the estimation of the Touschek lifetime in a round beam.

#### Appendix

Effect of a skew quadrupole magnet

According to Guignard,<sup>5)</sup> the emittance and the coupling coefficient of the betatron oscillations under the presence of skew quadrupole field are given by

$$\epsilon_x = \epsilon_x^0 \left\{ 2 \left( \frac{\chi}{\Delta} \right)^2 + 1 \right\} / \left\{ 4 \left( \frac{\chi}{\Delta} \right)^2 + 1 \right\}, \quad (A-1)$$

$$\kappa = \frac{\epsilon_z}{\epsilon_x} = \left( \frac{c}{\Delta} \right)^2 / \left\{ \left( \frac{c}{\Delta} \right)^2 + \frac{1}{2} \right\}, \quad (A-2)$$

where  $\Delta = \nu_x - \nu_z - p$  ( $p$ : integer), and

$$C = \frac{R}{8\pi B S} \int_0^{2\pi} \sqrt{\beta_x \beta_z} \left( \frac{\partial B_x}{\partial x} - \frac{\partial B_z}{\partial z} \right) \chi \exp [i(\mu_x - \mu_z - \theta \Delta)] d\theta. \quad (A-3)$$

The strength of the skew field is represented by the field gradient of the quadrupole magnet as

$$g_{sq} = \left( \frac{\partial B_x}{\partial x} - \frac{\partial B_z}{\partial z} \right) / 2. \quad (A-4)$$

Since only one skew magnet is installed in the present case, we find

$$|C|^2 = |C_{sq}|^2 + |C_0|^2, \quad (A-5)$$

$$|C_{sq}|^2 = \beta_x \beta_z l_s g_{sq}^2 / 4\pi B S, \quad (A-6)$$

where  $|C_0|^2$  is the contribution other than the skew magnet such as the tilt of quadrupole magnets and  $\theta$  component of fringing field of bending magnets of the storage ring.

At first we estimate  $|C_0|^2$ . From the measured value  $\bar{\sigma}_z = 0.16$  mm, we find  $\epsilon_z = 0.0098$  mrad mm and  $\kappa \approx \epsilon_z / \epsilon_x = 0.033$ , where  $\epsilon_x^0 (= 0.30$  mrad mm) is the design value. Then from (A-2),  $|C_0/\Delta|^2 = 0.017$ . The betatron oscillation frequencies, measured by the RF knock out method, are  $\nu_x = 1.285$  and  $\nu_z = 1.219$ , and therefore  $\Delta^2 = 4.3 \times 10^{-3}$ . Consequently  $|C_0|^2 = 7.4 \times 10^{-5}$ .

Next  $|C_{sq}|^2$  is estimated. At 380 MeV  $B S = 1.27$  [Tm], and  $\beta_x = \beta_z = 2.0$  m at the place of the skew magnet. According to the field measurement of the skew magnet with a rotating coil,  $g_{sq} l_s = 0.056$  Is(A) [T]. Then from (A-6) we get  $|C_{sq}|^2 = 4.9 \times 10^{-5}$  Is<sup>2</sup>(A).

Inserting  $|C_0|^2$ ,  $|C_{sq}|^2$  and  $\Delta^2$  into (A-5) and (A-2) we find  $\kappa$  for a given  $I_s$ , and from (A-1) and (A-2)  $\epsilon_x$  and  $\epsilon_z$  respectively. The beam size  $\sigma_x$  and  $\sigma_z$  can be calculated using the estimated  $\epsilon_x$  and  $\epsilon_z$ , which is shown with solid lines in Fig.3. Finally the divergence  $\sigma_x'$  and  $\sigma_z'$  are obtained. Figure A-1 shows the estimated  $\epsilon_x$ ,  $\epsilon_z$ ,  $\kappa$ ,  $\sigma_x'$  and  $\sigma_z'$  as a function of the skew current.

References

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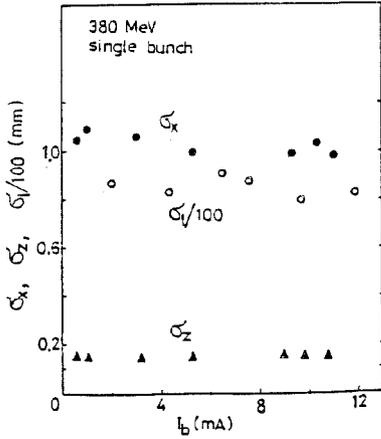


Fig. 1 Bunch length  $\sigma_l$ , horizontal and vertical beam size  $\sigma_x$  and  $\sigma_z$  measured in single bunch mode operation. Abscissa is the beam current.

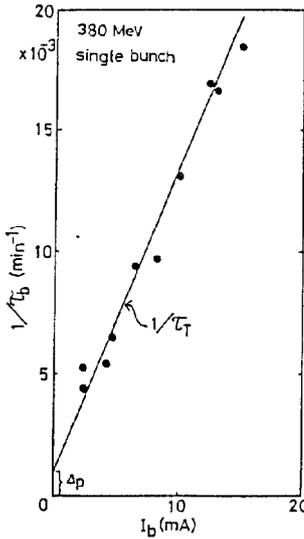


Fig. 2 Inverse beam lifetime  $1/\tau_b$  measured as a function of the beam current  $I_b$ . The solid line represents the inverse of the Touschek lifetime  $1/\tau_T$  calculated. The  $\Delta p$  in the figure is added for the residual gas scattering.

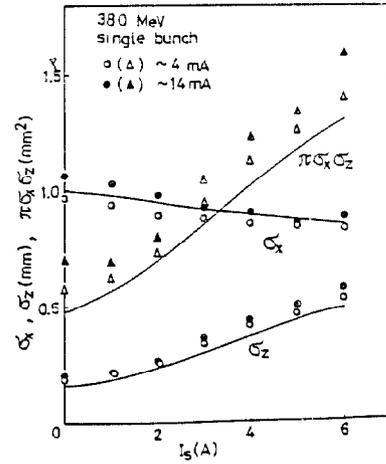


Fig. 3 Transverse beam size  $\sigma_x$  and  $\sigma_z$  measured at the beam current  $\sim 4$  mA and  $\sim 14$  mA as a function of skew current  $I_s$ . The triangles in the figure indicate the product of the measured  $\sigma_x$  and  $\sigma_z$  multiplied by  $\pi$ . The solid lines are the calculated.

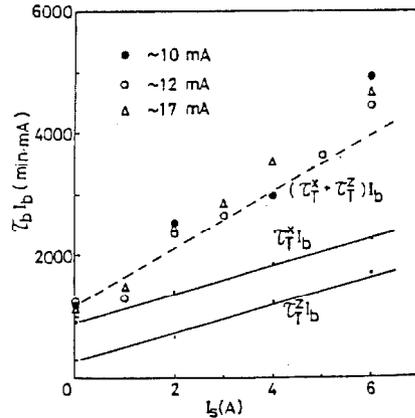


Fig. 4 Product of the beam current  $I_b$  and beam lifetime  $\tau_b$  measured at the beam current  $\sim 10$ ,  $\sim 12$  and  $\sim 17$  mA as a function of the skew excitation current. The solid lines in the figure represent the product of the beam current  $I_b$  and calculated horizontal or vertical Touschek lifetime  $\tau_T^x$  or  $\tau_T^z$ . The broken line indicates the sum of the  $\tau_T^x I_b$  and  $\tau_T^z I_b$ .

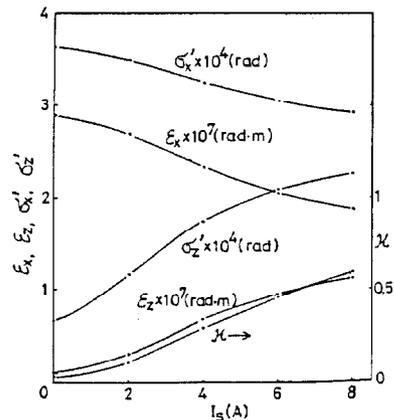


Fig. A-1 Horizontal and vertical emittances  $\epsilon_x$  and  $\epsilon_z$ , divergences  $\sigma_x'$  and  $\sigma_z'$  and coupling constant  $\kappa$  calculated. Abscissa is the excitation current of a skew quadrupole magnet.