AZIMUTHAL SHAPING OF CYLINDRICAL ACCELERATING CAVITIES FOR IMPROVED HIGHER ORDER MODE EXTRACTION*

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Summary

In cylindrically symmetric accelerator cavities equipped with higher order mode (HOM) couplers, the two polarizations of each deflecting mode are often fixed by the couplers themselves, leaving one polarization inadequately damped. This problem can be overcome by deliberate breaking of the cell's cylindrical symmetry. We describe a quasi-cylindrical cavity profile in which the polarization of the dipole, quadrupole, and sextupole HOM's are fixed at $\pm\pi/4$, $\pm\pi/8$, $\pm\pi/12$ with respect to the intended coupler orientation, providing a maximally favorable situation for damping by a single coupler. A single cell S-band cavity was built to test the practicality of our design. The experimental data, in the range 3-7 GHz, conclusively shows that the mode polarizations can be controlled with only minor perturbations to the cell's cylindrical symmetry.

The design of superconducting microwave cavities for particle accelerators is a multifaceted task. A cavity must support high electric fields (presently about 10 MV/m) along the particle trajectory without thermal quench, electric field emission, or multipacting. In addition, adequate external coupling must be provided for all higher order modes (HOM's) which are likely to cause particle beam instabilities. There are a number of constraints which make the latter requirement difficult to meet. HOM couplers must not distort the electromagnetic fields of the accelerating mode in any way which limits the maximum achievable accelerating gradient; couplers must be attached to the cavity without promoting thermal breakdown through local surface field enhancement; they must not stimulate electron loading by providing surface geometries favorable for multipacting. Couplers must also be compatible with chemical processing procedures used to clean and prepare niobium cavity surfaces. Structures which are not open for drainage or rinsing are unsuitable for the ultra-clean, ultra-high vacuum environment required by superconducting accelerator cavities. Finally, each coupler which is connected to a room temperature microwave load contributes a substantial heat leak (about 1 W) as well as a sizable amount of hardware complexity [1]; the fewer couplers required, the better.

Recent developments in cavity design have favored the use of cylindrically symmetric cavities. Such structures are easily fabricated, free of field curvature in the accelerating mode, and amenable to numerical analysis via computer programs such as LALA [2], SUPERFISH [3], and URMEL [4]. In particular, knowledge of the cavity field distributions has allowed detailed calculations of one-point and two-point multipacting orbits. This has inspired the elliptically-profiled cavity shown in Figure 1, which is relatively free from multipacting at the microwave frequencies and field strengths in present use [5]. The introduction of coupling holes, loops, or probes into the cavity cells would destroy the favorable elliptical geometry, promoting multipacting and also interfering with the demanding task of chemical cleaning. For these reasons, we believe that it is wise to leave the cavity cells undisturbed, and append the rf couplers to the ends of the cavity structure (Fig. 1).

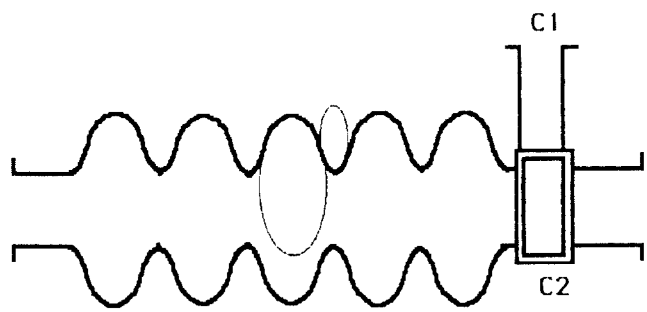

Figure 1: Longitudinal profile of five-cell elliptical cavity with HOM couplers attached to right end cell. The cell profile is generated by two ellipses which merge smoothly at an angle of 75° with respect to the cavity axis.

There are two disadvantages to HOM end-couplers. First, they provide weaker external damping than couplers which directly penetrate the cavity cell walls. This necessitates more exacting design in order to meet beam stability criteria. The second consideration arises from the cylindrical symmetry of the cells. Each deflecting mode of the ideal cavity is doubly degenerate, having fields which vary azimuthally as $\cos(m\theta-\theta_o)$ or $\sin(m\theta-\theta_o)$ about the axis of symmetry. (The m = 0, 1, 2 and 3 modes are known as monopole or accelerating, dipole, quadrupole, and sextupole modes, respectively.) The polarization axes (e.g., the value of θ_o) selected by each mode pair are determined either by accidental deviations from perfect cylindrical symmetry introduced during fabrication, or by the perturbing influence of the couplers themselves. It has been our experience, using two perpendicular waveguide end-couplers, that several destabilizing HOM's will be polarized so that they do not "see" the couplers, resulting in unacceptably high external Q's. In the following sections, we will describe this situation more closely, and show how it can be avoided by proper cavity cell shaping.

It is easy to understand why simple coupling schemes employing one or two end-couplers are only marginally acceptable. Consider the structure shown in Figure 2, in which a cylindrically symmetric cavity is equipped with two identical waveguide end-couplers. The cavity symmetry is reduced by the couplers to the symmetry group C_2, with a reflection plane σ bisecting the angle 2ϕ between the couplers. The allowed resonant modes have either odd or even symmetry under reflection through σ, and can be approximated by the unperturbed (cavity cells without couplers) degenerate eigenfunctions $\psi_{i+} = \mu_i(r,z) \cos m\theta$ and $\psi_{i-} = \mu_i(r,z) \sin m\theta$. Here, θ is measured with respect to the symmetry plane, and ψ_{i+} (ψ_{i-}) is a representative component of the vector electromagnetic field with even (odd) reflection symmetry. Because of the couplers, the two eigenfunctions are no longer degenerate, and the difference in their resonant frequencies is a measure of the polarization "pinning force" aligning ψ_{i+} along σ and ψ_{i-} at an angle $\pi/2m$ with respect to σ. The external coupling to the two modes will be proportional to $\cos^2 m\phi$ and $\sin^2 m\phi$, respectively. The essential point is that no value of ϕ will be favorable for both polarizations of the dipole, quadrupole, and sextupole modes which are known to influence beam stability. Needless to say, the situation is worse for a single end-coupler.

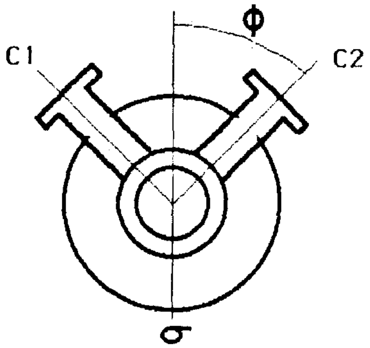

Figure 2: Cylindrically symmetric cavity with two end-couplers, viewed along cavity axis, showing mirror symmetry plane and coupler separation angle.

The origin of the problem is that the mode polarizations are all referenced to the same symmetry plane, which in turn is fixed by the couplers themselves. We can avoid this difficulty by deliberately perturbing the cavity cells, thereby pinning the mode polarizations, and then orienting the couplers to take maximum advantage of this pinning. For example, consider a "pillbox" cavity with a sinusoidally modulated wall radius, $r = r_o + \alpha\cos(2n\theta-\theta_o)$. The Slater perturbation theorem [6] predicts that, due to the varying radius, a cavity resonance will experience a frequency shift Δf given by

$$\frac{\Delta f}{f_o} = \frac{\int_{\Delta V} (\mu H^2 - \varepsilon E^2)\, dV}{4U}$$

where f_o is the unperturbed resonant frequency ($\alpha = 0$), ΔV is the volume removed or added by perturbing the radius, and U is the stored energy in the cavity. An m-fold symmetric mode will experience a frequency shift, and thus a polarization pinning force, proportional to

$$\alpha\int_0^{2\pi}(1 + \cos 2m\theta)\cos(2n\theta-\theta_o)d\theta$$

which vanishes for m, n > 0 unless m = n. In the general case, $r - r_o = \sum_n \alpha_n \cos(2n\theta-\theta_n)$, and members of each m-fold symmetric mode pair will be aligned at angles of 0 and $\pi/2m$ with respect to the axis of symmetry (at θ_m) of the mth Fourier component of the wall radius modulation. By proper choice of phases θ_m, it is possible to provide reliable external coupling using a single HOM end-coupler. Figure 3 illustrates the idea: the first three Fourier components of $r(\theta)$ are rotated by angles $\theta_1 = \pi/4$, $\theta_2 = \pi/8$, and $\theta_3 = \pi/12$ from the wave guide coupler axis. For both polarizations of all three cases, the external coupling will be proportional to $\cos^2(\pi/4) = 1/2$. No modes are left uncoupled.

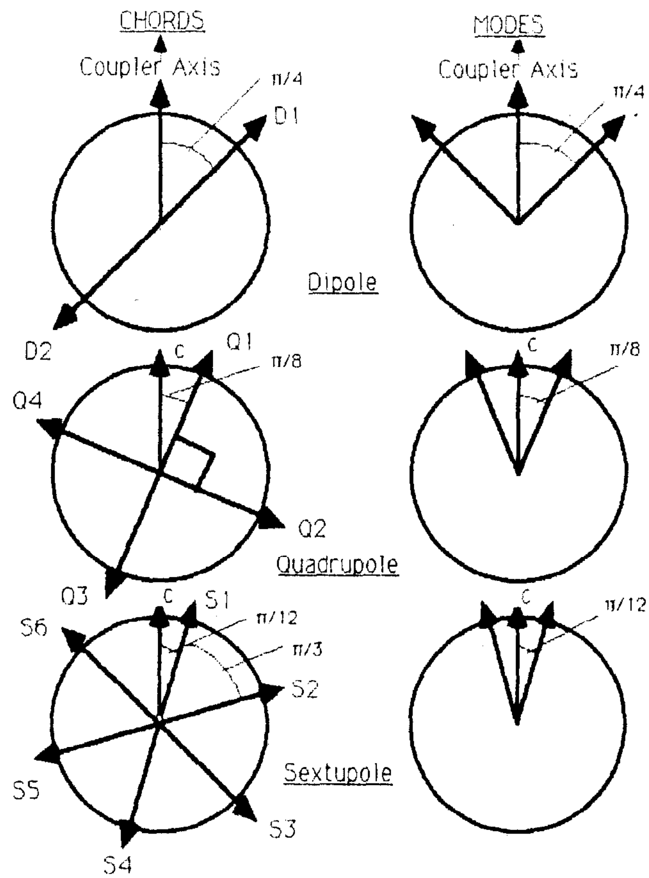

Figure 3: Orientation of dipole (D), quadrupole (Q) and sextupole (S) modes with respect to coupler axis C. $\theta_1 = \pi/4$, $\theta_2 = \pi/8$, $\theta_3 = \pi/12$.

The practicality of this approach was tested in the following way. A single cell elliptically-profiled cavity was machined in two matching halves using a numerically controlled milling machine. The azimuthal boundary at the cavity midplane consists of twelve chords joined by circular arcs which meet the chords tangentially (Figure 4). The average radius at the cell midplane is $r = 4.66$ cm, chosen to fix the fundamental accelerating mode (TM_{010}) resonant frequency close to 3.0 GHz. The width of each chord scales linearly with wall radius, vanishing at $r_{min} = 2.06$ cm, where the two ellipses which generate the cell profile meet. For $r < r_{min}$, the cavity is cylindrically symmetric.

Two chords fix the orientation of the dipole modes; four and six are needed for the quadrupole and sextupole modes, respectively. The cavity shape was determined by first building an elliptical cavity with two equal and diametrically opposed chords. The frequency splittings for dipole modes were found to be about 50 MHz, far greater than those encountered in practice due to fabrication tolerances or coupler perturbations; hence, the two chords were sufficient to pin the dipole mode polarizations. The lengths of the quadrupole and sextupole chords were determined by Fourier analysis: a trial solution (chord lengths plus connecting arcs) for $r(\theta)$ at the cavity midplane was Fourier analyzed, and then iteratively corrected until α_2, $\alpha_3 \simeq \alpha_1/2$ (smaller Fourier coefficients are sufficient to pin the quadrupole and sextupole modes). The final dimensions of our S-band cavity are given in Figure 4.

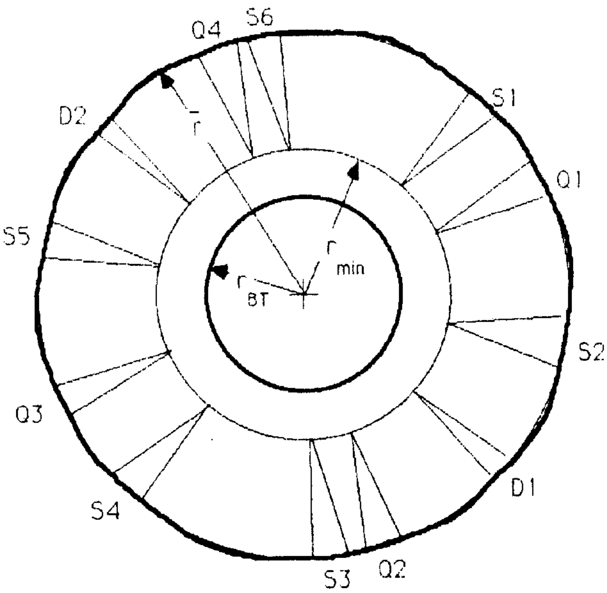

Figure 4: Mid-section view of S-band cavity, showing 12 chords used to pin the HOM's. The average mid-plane radius $\bar{r}$ = 4.66 cm. The chords vanish at r_{min} = 2.06 cm, and the beam tube radius is r_{BT} = 1.75 cm. The chord widths at the midplane are D_i = .326 cm; Q_i = .549 cm; S_i = .646 cm.

Table 1 summarizes the cavity's performance in the range 3-7 GHz. The performance of a cylindrically symmetric elliptical cavity having a midplane radius $r = \bar{r}$ is included for comparison. All modes of the twelve-chord cavity are properly oriented for energy extraction through a port at 0°. The frequency splittings induced by cell shaping are much larger than those encountered in nominally cylindrical cavities, and are obtained with rather small departures from cylindrical symmetry. With the exception of one mode (5471 MHz), the induced splittings dominate those which arise due to end-couplers, and should therefore fix the mode orientations in practical accelerator cavities. We do not anticipate that the geometric perturbations introduced will affect the multipacting behavior. If the cavity cell is shaped by deep-drawing, and if a numerically-controlled milling machine is available to cut the dies, then the twelve-chord cavity is no more difficult to fabricate than an ordinary cylindrically symmetric cavity.

Table 1: Frequency splittings of HOM's for nominally cylindrical and 12-chord cavities. All modes of the latter are properly oriented for energy extraction through a single coupler at 0°.

	CIRCULAR CAVITY		TWELVE-CHORD CAVITY		
m	f(MHz)	Δf	f(MHz)	Δf	Orientation
0	2952.20		2964.30		
1	3757.18	0.14	3759.61	11.25	±45°
	3757.32		3770.86		
1	4122.00	0.02	4110.45	51.05	±45°
	4122.02		4161.50		
2	5202.66	0.46	5202.48	28.02	±22.5°
	5202.72		5230.50		
1	5275.44	0.12	5274.80	11.27	±45°
	5275.56		5286.07		
1	5468.95	0.21	5471.56	1.76	±45°
	5468.74		5473.32		
2	5519.98	0.16	5515.36	39.99	±22.5°
	5519.82		5556.35		
0	5589.22		5607.17		
0	5659.73		5678.96		
0	5972.97		5997.25		
1	6140.02	0.54	6141.72	6.11	±45°
	6139.48		6147.83		
1	6271.58	0.50	6277.85	7.60	±45°
	6271.08		6285.45		
3	6355.80	0.17	6354.72	42.59	±15°
	6355.63		6397.31		
0			6667.63		
3			6811.59	29.42	±15°
			6841.01		
1			6955.23	6.81	±45°
			6962.04		
1			7216.53	21.65	±45°
			7238.18		

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