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STUDY OF THE PLASMA FIBER ACCELERATOR

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Abstract

We study the plasma fiber accelerator concept by simulation and theory. An outgrowth of the laser beat-wave accelerator concept, the plasma fiber accelerator, sets the phase velocity of the beat plasma wave at the speed of light (or any value desired) by allowing each laser light to have a specific transverse wavenumber (or structure). Two-dimensional slab duct structures with laser beams of both transverse electric and transverse magnetic polarizations are explored. Accelerated beam dynamics in fiber beat-wave is discussed.

I. Plasma Fiber Accelerator

The idea of the plasma fiber accelerator[1,2] is to tackle simultaneously two problems; (i) the longitudinal phase mismatch between the plasma wave and the particles; (ii) the tendency of laser light to spread in the transverse direction. It is crucial to overcome these difficulties in order to scale the scheme to ultra-high energies. The plasma wave excited by the beat of two laser lights has a phase velocity equal to the group velocity of the electromagnetic waves in the plasma.[1] Since the phase velocity of the plasma wave is less than the speed of light, the plasma wave will be outrun by high energy particles during the dephasing time $\tau_d = 2\pi(\omega_0/\omega_p)^2\omega_p^{-1}$. The particles that are trapped by the plasma wave gain energy by $2(\omega_0/\omega_p)^2 mc^2$.

It was shown that the plasma fiber under appropriate conditions possesses a property to overcome this difficulty. The duct structure, in which the plasma density is low inside and the density is so high outside that the electromagnetic wave is evanescent, enables it to sustain a beat-wave phase velocity equal to any prescribed velocity including the speed of light. In addition to this benefit the plasma fiber confines the light, overcoming the natural tendency toward transverse spreading. Thus the idea of the plasma fiber plays a central role in improving the laser beat-wave accelerator in two of the most important points. It is to be noted that the optical fiber waveguide is one of the very active and important areas of optics in recent years.[3,4] It is known that the fiber structure can well sustain the shape and the amplitude of light pulses over 10^7 cm, used as an efficient communication method. The fiber may have favorable properties such as sustaining optical solitons under certain conditions.[5,6] In the usual dielectric optical fiber the index of refraction is small toward the outside so that the light beam is trapped in the duct. In the present problem, of course, the plasma dielectric constant is less than unity so that the plasma density has to rise toward the outside. This cavitated structure of fiber type is often a natural occurrence of plasmas with intense light shone.[7,8]

A way to match the phase of the accelerating field with high energy particles is to inject particles oblique to the electric field direction. In order to phase-lock, the angle θ between the particle momentum and the electric field is given by

$$\cos\theta = \left(1 - \frac{\omega_p^2}{\omega_0^2}\right)^{1/2}. \tag{1}$$

Although we match the parallel phase, we have now introduced an extraneous perpendicular acceleration, which bends the particle orbit. To correct this situation, it was proposed[9] that a static vertical magnetic field be imposed. The magnetic field is such that

$$\frac{B}{E} = \sin\theta = \frac{\omega_p}{\omega_0}, \tag{2}$$

for relativistic particles where B is out the board.

An alternative approach is to conduct the laser lights in a plasma duct where the plasma density in the middle is low and the edge density high.[1,2] We choose the ouside density so high that the electromagnetic waves are evanescent there and therefore they are trapped within the duct structure (plasma fiber). By choosing the right width of the duct we can shown that the beat-wave velocity matches the speed of light. Let us choose flat densities n inside the duct n' outside of it with duct width d. We demand frequency matching among the two lasers and the plasma wave.

$$\omega_0 - \omega_1 = \omega_p. \tag{3}$$

Further, we demand that the parallel velocity of the two lasers, which is equal to the parallel phase velocity of the plasma wave, be the speed of light (to be precise, this speed should be the particle velocity).

$$v_{gr\parallel}^{EM} \equiv \frac{\omega_0 - \omega_1}{k_{\parallel 0} - k_{\parallel 1}} = v_{ph\parallel} = c, \tag{4}$$

where

$$k_{\parallel j} = \left[\frac{\omega_j^2}{c^2} - \frac{\omega_p^2(r)}{c^2} - \left(\frac{\pi}{d}\right)^2 n_j^2\right]^{1/2} \tag{5}$$

and $j = 0$ or 1 and we choose $n_0 = n_1 + 1$, where n_j are the number of transverse nodes. In a slab model with the y-direction parallel to the laser injection direction the electromagnetic fields may take the transverse magnetic or the transverse electric forms. The transverse electric case, for example, has the E_z, B_x, and B_y fields. If the plasma density abruptly changes from the duct plasma density to a very large value with the duct width d for simplicity, the electromagnetic field inside the duct may be approximated by the wave-guide modes:

$$E_{zj} = E_{zj}^0 \sin(k_{xj}x)\cos(k_{yj}y - \omega_j t), \tag{6}$$

$$B_{yj} = -\frac{ck_{xj}}{\omega_j} E_{zj}^0 \cos(k_{xj}x)\sin(k_{yj}y - \omega_j t), \tag{7}$$

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$$B_{zj} = \frac{ck_{yj}}{\omega_j} E_{zj}^0 \sin(k_{xj}x)\cos(k_{yj}y - \omega_j t), \quad (8)$$

where $k_{xj} = \pi n_j/d$ with $j = 0$ or 1. The dispersion relation of the electromagnetic waves is

$$\omega_j = [\omega_p^2 + (k_{xj}^2 + k_{yj}^2)c^2]^{1/2}$$

where ω_p^2 is determined by the plasma density inside the duct. The condition Eq. (3) yields the following relation

$$d^{-2} = \Big\{ -[AB + 4(ED' + E'D)] \pm \sqrt{[AB + 4(ED' - E'D)^2]} \overline{-(B^2 - 4DD')(A^2 - 4EE')} \Big\} \times (A^2 - 4EE')^{-1} \quad (9)$$

where

$$A = E + E',$$
$$B = 1 - D - D',$$
$$D = \left(\frac{\omega_0}{\omega_p}\right)^2 - 1,$$
$$D' = \left(\frac{\omega_1}{\omega_p}\right)^2 - 1,$$
$$E = \left(\frac{\pi n_0 c}{\omega_p}\right)^2,$$
$$E' = \left(\frac{\pi n_1 c}{\omega_p}\right)^2.$$

By choosing $n_0 = 2$ and $n_1 = 1$, and using $(\omega_0/\omega_p)^2 \gg 1$ and $(\omega_1/\omega_p)^2 \gg 1$, we arrive at the following condition for the duct width

$$d \simeq \sqrt{3}\frac{\pi c}{\omega_p}\left(\frac{\omega_0}{\omega_p}\right)^{1/2}. \quad (10)$$

If the outside density n' satisfies the condition

$$n' \geq n + \frac{9\pi mc^2}{4e^2 d^2}, \quad (11)$$

the electromagnetic waves are evanescent outside the plasma fiber. The plasma wave phase front is straight, i.e., $\phi(z,r) \sim A(r)\cos(k_{||p}z - \omega_p t)$.

II. Simulation Study

A 2-1/2 dimensional relativistic electromagnetic particle code is employed to simulate the aforementioned scheme. The duct structure in the plasma density is preformed: the density outside of the duct is 10 times the one inside in the present example. We do this by allowing the particles which are initially outside of the duct region to be weighted with a larger charge and mass, keeping the charge to mass ratio fixed. To accomodate large weighting, Triangular Shaped Cloud interpolation[10] is utilized.

The y-direction was chosen as the direction of the incident light waves. A uniform wave front, as in a one-dimensional simulations, has been established in the duct for cases of transverse electric and also for transverse magnetic forms. Waveguide modes including x-dependence of the fields inside the duct, as in Eqs. (6), (7) and (8), were also simulated as well as other polarizations.

An example of the simulation runs with an E_x polarization with a flat wave amplitude within the duct is presented. The density profile has a duct width, $d = 22.05\Delta$ and a density ratio of outside to inside the duct of 10. The original light wavenumbers are $k_0 = 2\pi \times 22/128\Delta$ and $k_1 = 2\pi \times 15/128\Delta$, the speed of light, $c = 2.909\omega_{po}\Delta$, the quiver velocity of the light waves $v_{os_j} = 0.6c (j = 1$ and $2)$, and $v_t = 0.5\omega_{po}\Delta$, where ω_{po} is the plasma frequency in the duct and Δ is the grid spacing. The electrostatic potential of the beat plasma wave is shown in Fig. 1. The wave grows until the detuning time and saturates. the phase space plot in the y-direction shows electrons "trapped" by the beat plasma wave. The phase velocity of the plasma wave is measured in Fig. 2: v_{ph}(measured) $= 2.942\omega_{po}\Delta$. This is very close to the speed of light $c = 2.909\omega_{po}\Delta$, in agreement with the theory discussed in Sec. I. This compares with the case of no duct with parameters as stated in the previous case except $c = 3.095$ so that v_{ph}(theory) $= 2.910$. The phase velocity measured in this simulation, v_{ph}(measured) $= 2.916$ so that $|c - v_{ph}|/c \simeq 0.06$, while with the duct $|c - v_{ph}|/c \simeq 0.01$ (within a measurement error).

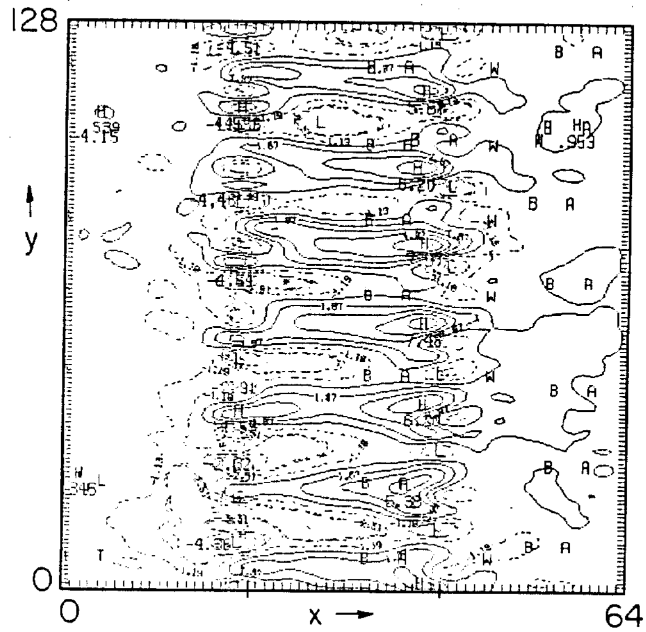

Figure 1 — Countour plot of the electrostatic potential at *time* $= 8\omega_{po}^{-1}$ due to the beat wave created in the fiber structure.

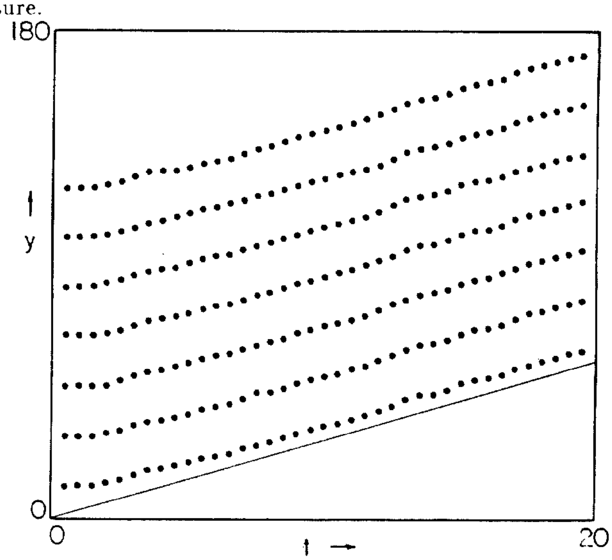

Figure 2 — Measured phase velocity of the beat wave in the fiber. Dots are the y-positions measured at discrete times of the peaks of the plasma wave whose Fourier component is $k_y = 2\pi \times 7/128\Delta$. In this diagnostic the spatial average from $x = 21\Delta$ to 43Δ is taken. The solid line indicates the speed of light.

III. Beam Stability

Let us consider the phase stability of a particle beam in the plasma fiber accelerating fields.

One goal of a plasma fiber accelerator is to provide phase velocity matching between the plasma wave and the particle. For ultrarelativistic particles, this is obtained if the phase velocity is c. In that case the phase-energy equations are:

$$\frac{d\gamma}{dt} \cong \frac{qE_o}{mc}\cos\phi \tag{12}$$

$$\frac{d\phi}{dt} = k_p c(\beta - 1) \cong \frac{-k_p c}{2\gamma^2} \tag{13}$$

where $\phi = k_p x - \omega t = k_p(x - ct)$, E_o is the electric field and k_p is the plasma wave number. The equations can be integrated to obtain

$$(\sin\phi_i - \sin\phi_f) = \frac{k_p(mc^2)}{2qE_o}\left[\frac{1}{\gamma_i} - \frac{1}{\gamma_f}\right] \tag{14}$$

as in a conventional rf linac, where ϕ_i, ϕ_f are initial and final phases; γ_i, γ_f are the initial and final energies.

Acceleration can continue indefinitely if the motion is ultra relativistic, that is

$$\frac{k_p(mc^2)}{2qE_o\gamma_i} \lesssim 1 \quad . \tag{15}$$

At typical plasma accelerator parameters, $(k_p = (2\pi/2.5 \times 10^{-5}), qE_o = 10GeV/m)$, this implies a minimum initial energy $E_i = \gamma_i mc$ of only $\sim 5MeV$ for electrons, which is easily satisfied.

However, for muons, this condition requires moderately high energies ($E_i \gtrsim 100GeV$) and, for protons, very high energies ($E_i \gtrsim 10TeV$) are required. Then continuous acceleration may be obtained by matching the phase velocity of the plasma wave to the particle velocity. In the plasma fiber accelerator this is done by the matching condition:

$$\beta_c = \frac{\omega_p}{k_0 - k_1}.$$

As $\beta \to 1$, the same condition is obtained as for the $\beta = 1$ ultrarelativistic fiber accelerators, and as

$$\beta \to \sqrt{1 - \left(\frac{\omega_p}{\omega_1}\right)^2} \quad \left(\gamma_w = \frac{\omega_1}{\omega_p}\right) \tag{16}$$

$d \to \infty$. and the unchanneled plasma beat wave accelerator is obtained. Lower values of $\beta(\gamma_w < \omega_1/\omega_p)$ cannot be matched. Continuous acceleration can be obtained by matching the plasma width to a synchronous particle velocity as the beam is accelerated. This implies shrinking the duct width toward the ultrarelativistic value during acceleration.

An alternate approach to stable acceleration is the surfatron[9], in which a transverse magnetic field is imposed. A transverse magnetic field may also be imposed upon the fiber accelerator. In a fiber surfatron the relevant fields are a longitudinal electric field

$$\vec{E} = E_o \sin(k_w x - \omega_p t)\hat{x}$$

and the magnetic field

$$\vec{B} = B \quad \hat{z}$$

where

$$k_w = k_0 - k_1 = \frac{2\pi}{\beta_o\lambda_p}$$

which is the same as for the surfatron except β_0 is the fiber β, rather than the plasma phase velocity.

Particle dynamics are, in fact, identical to the surfatron with stable accelerations and phase-energy oscillations about the stable phase.[11] The difference is that $\gamma_p \cong \omega_0/\omega_p$ is replaced by $\gamma_0 = \sqrt{1/1 - \beta_o^2}$, and, as noted above, $\gamma_0 > \gamma_p$.

Therefore, the stable acceleration rate found from

$$m_0 c^2 \frac{d\gamma_s}{dt} = qB\gamma_o\beta_o c$$

is larger by a factor of γ_o/γ_p. The stability constraint

$$\gamma_o B \lesssim E_o$$

requires correspondingly greater plasma fields.

As in the surfatron, the accelerated beam travels at an angle $\theta \cong 1/\gamma_0$ with respect to the normal to the wavefront, in the plane perpendicular to B. This angle is most trivially accomadated by a 1-D fiber (plane) with the plane aligned along the transverse motion direction. A 2-D fiber must follow the stable particle direction for extended accelerations and it is unclear whether this is possible while maintaining the structure.

Acknowledgements

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