PUMP DEPLETION AND LASER STAGING FOR BEAT-WAVE ACCELERATOR

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Abstract

The beat-wave accelerator (BWA) scheme is promising for a large acceleration gradient for high energies. Some of the concerns with this accelerator concept are (i) the longitudinal dephasing between the speed of high energy particles and that of plasmons and (ii) the laser pump depletion. The latter problem is analyzed in detail here. To cope with these problems and also to ease some of the requirements on laser and plasma parameters, we propose an accelerator architecture for BWA with the following features: (i) distributed laser staging optically connected by the master oscillator; (ii) optical delay lines synchronizing the timing and phase of each laser stage; (iii) appropriate mirror systems; and (iv) optical guides (plasma fiber) to trap laser beams by acoustic transducers or other mechanism. If necessary, additional collector devices may be added to collect any unused portion of laser power and to convert it for reuse.

I. Growth of the Plasma Wave and Pump Depletion

We consider the self-consistent growth of the plasma wave ω_p driven up from the beating of two high frequency, ω_1, ω_2, linearly polarized laser beams. The laser fields are $E_j \sin(k_j x - \omega_j t + \phi_j)\hat{e}_y$ and $B_j \sin(k_j x - \omega_j t + \phi_j)\hat{e}_z$ with $B_j = (ck_j/\omega_j)E_j$. The plasma wave is $E_p \cos(k_p x - \omega_p t + \phi_p)\hat{e}_x$ with ideal, resonant $\omega_1 - \omega_2 = \omega_p$ growth rate given by $\mathcal{E} \equiv eE_p/m\omega_p c = \lambda t \omega_p$ where $\lambda \cong a_1 a_2/4$ is the ideal growth rate given in terms of the dimensionless quiver velocities a_1 and a_2. We now investigate how this ideal growth can be limited due to variations in the laser-plasma wave phase relation $\psi = \phi_1 - \phi_2 - \phi_p$ due to pump depletion.

From the transverse electromagnetic wave dispersion relation $\omega_j = (c^2 k_j^2 + \omega_p^2)^{1/2}$ it follows that the phase velocity of the beat wave $v_p = (\omega_1 - \omega_2)/(k_1 - k_2) \simeq d\omega_1/dk_1 = c(1 - \omega_p^2/\omega_1^2)^{1/2}$ equals the group velocity of the laser pulse for $\omega \gg \omega_p$. The Lorentz transformation to the beat wave frame is given by $\beta_p = \Delta\omega/\Delta kc = \omega_p/k_p c$ and $\gamma_p = (1-\beta_p^2)^{-1/2} = \omega_1/\omega_p$. The energy gain in the simple beat wave accelerator[1] occurs from reflection off the electrostatic potential $\varphi_{wave} = \gamma_p \varphi_p$ in the wave frame and is given by $\Delta E = 2mc^2\gamma_p^2 \mathcal{E}_{max}$ where $\mathcal{E}_{max} = max(eE_p/m\omega_p c)$.

The energy density in the transverse laser waves is $W_j = \langle (E_j^2 + B_j^2)/8\pi - \frac{1}{2}mnv_{y_j}^2 \rangle = (E_j^2/16\pi)\left[1 + (c^2k_j^2 + \omega_p^2)/\omega_j^2\right] = E_j^2/8\pi$ and in the longitudinal plasma wave is $W_p = \langle E_p^2/8\pi + \frac{1}{2}mnv_x^2 \rangle = (E_p^2/16\pi)(1 + \omega_p^2/\Delta\omega^2) = E_p^2/8\pi$. We define the dimensionless quiver or oscillation velocities by

$$a_j = \frac{eE_j}{m\omega_j c} \qquad \text{and} \qquad \mathcal{E} = \frac{eE_p}{m\omega_p c} = \frac{\omega_p \xi}{c} \tag{1}$$

where in the last equation we also introduce the amplitude of the Lagrangian displacement ξ of the thermal electrons in the plasma wave. The Poisson equation gives the exact linear relation between E_p and ξ through $E_p = 4\pi en\xi$ where $-en\xi$ is the surface charge density introduced by the displacement of ξ of the electrons at x_o from the ions by $x = x_o + \xi(x_o, t)$, unless the wave breaking takes place. The Lagrangian equation of motion for the plasma wave is linear $\partial_t^2 \xi + \omega_p^2 \xi = 0$ but describes steepening of the plasma wave through the inversion of the Lagrangian equation for $x_o = x_o(x,t)$. For a sinusoidal oscillation $\xi \sin(k_p x)$ the inversion becomes multivalued when $dx/dx_o = 1 + k_p \xi \cos(k_p x) = 0$ which describes wave breaking at the limit $k_p \xi = 1$. For plasma waves with $v_p = \omega_p/k_p \simeq c$ the breaking limit gives $\mathcal{E} = E_p/m\omega_p c = \omega_p \xi/c = 1$.

The relativistic equation of motion[2] for $\xi(x_o, t)$ follows from calculating the rate of increase of $p_x = m\gamma(\dot{\xi})\dot{\xi}$ due to the resonant frequency part of the Lorentz force $-e\left[E_x + (v_y^{(1)}B_z^{(2)} + v_y^{(2)}B_z^{(1)})/c\right]$ to obtain

$$\gamma_\xi^3 \frac{\partial^2 \xi}{\partial t^2} + \omega_p^2 \xi = 2\Delta\omega c\lambda \sin\left[\Delta k(x_o + \xi) - \Delta\omega t + \Delta\varphi\right] \tag{2}$$

where

$$\lambda = \frac{a_1 a_2}{4\beta_p} \qquad \text{and} \qquad \Delta\varphi = \varphi_1 - \varphi_2 .$$

For small λ the effective plasma fequency becomes $\omega_p\left[1 - \frac{3}{8}(\omega_p^2\xi^2/c^2)\right]^{1/2}$ and the linear resonant growth $\mathcal{E} = \omega_p\xi/c = \lambda t \omega_p$ stops when $\mathcal{E}_{RL} = 4(\lambda/3)^{1/3}$ at $\omega_p t_s = 3.89\lambda^{-2/3}$.

To close the system we calculate the effect of the plasma wave on the transverse wave amplitude and phase $\hat{E}_j(x,t) = E_j \exp(i\phi_j)/2i$ where $E_j(x,t) = \mathrm{Re}\hat{E}_j \exp(ik_j x - i\omega_j t)$. Reducing the $(\partial_t^2 - c^2\partial_x^2)E_y = -4\pi\partial_t j_y$, we obtain

$$(\partial_t + v_j\partial_x)\hat{E}_j = -2\pi\langle j_y^{n\ell} e^{-ik_j x + i\omega_j t}\rangle \tag{3}$$

where $j_y^{n\ell} = -e\delta n_p v_y = (4\pi)^{-1} v_y \partial E_p/\partial x$. Introducing the envelope approximation in Eq. (2) with $E_p(x,t) = \mathrm{Re}\hat{E}_p \exp(ik_p x - i\omega_p t)$ we obtain for the closed system of waves

$$(\partial_t + v_1\partial_x)\hat{E}_1(x,t) = -\frac{ek_p}{2m\omega_2}\hat{E}_p^* \hat{E}_2 \tag{4}$$

$$(\partial_t + v_2\partial_x)\hat{E}_2(x,t) = \frac{ek_p}{2m\omega_1}\hat{E}_1 \hat{E}_p^* \tag{5}$$

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$$(\partial_t - i\frac{3}{16}\omega_p|\epsilon|^2)\hat{E}_p(x,t) = \frac{e\omega_p\Delta k}{2m\omega_1\omega_2}\hat{E}_1\hat{E}_2^* \tag{6}$$

where only two original electromagnetic waves are retained. The rate of transfer of energy from the lasers to the plasma is determined by

$$\begin{aligned} T &= \hat{E}_1^*\hat{E}_2\hat{E}_p + \hat{E}_1\hat{E}_2^*\hat{E}_p^* \\ &= \frac{1}{4}E_1E_2E_p\sin(\psi) \end{aligned} \tag{7}$$

where $\psi = \phi_2 + \phi_p - \phi_1$ with rates given by

$$(\partial_t + v_1\partial_x)W_1 = -\Big(\frac{ek_p}{2m\omega_2}\Big)T \tag{8}$$

$$(\partial_t + v_2\partial_x)W_2 = \Big(\frac{ek_p}{2m\omega_1}\Big)T \tag{9}$$

$$\partial_t W_p = \Big(\frac{e\omega_p\Delta k}{2m\omega_1\omega_2}\Big)T \tag{10}$$

From Eqs. (8)-(10) it follows that $\Delta(W_1/\omega_1) = -\Delta(W_2/\omega_2) = -\Delta(W_p/\omega_p)$ when $k_p = \Delta k$, a manifestation of the Manley-Rowe relation. For $v_1 \cong v_2 = v_p$ the transformation of laser energy to plasma wave energy is given by

$$(\partial_t + v_p\partial_x)(W_1 + W_2) = -\frac{ek_p\Delta\omega}{2m\omega_1\omega_2}T \tag{11}$$

$$\partial_t W_p = \frac{e\Delta k\omega_p}{2m\omega_1\omega_2}T. \tag{12}$$

We use the conservation of the total energy to calculate the depletion of the laser energy $W_\ell \equiv W_1 + W_2$ by the growth of the plasma wave

$$(\partial_t + v_p\partial_x)W_\ell(x,t) = -\frac{\partial W_p(x,t)}{\partial t}. \tag{13}$$

Since $W_p/W_\ell = (\omega_p^2/\omega^2)(\mathcal{E}^2)/(a_1^2 + a_2^2) \simeq (\omega_p^2/\omega^2)(\mathcal{E}^2/8\lambda) \ll 1$, in the first approximation the solution of Eq. (13) is $W_\ell = W_\ell(x - v_pt)f(t/\tau_{pd})$ with depletion time τ_{pd} given by

$$\tau_{pd} = W_\ell/(\partial W_p/\partial t) = \tau_\ell(\omega^2/\omega_p^2)(4\lambda/\mathcal{E}_m^2) \tag{14}$$

where $c\tau_\ell$ is the laser pulse length. For $\mathcal{E}_m = \lambda\tau_\ell\omega_p$ the depletion length $L_{pd} = c\tau_{pd} = 4(c/\omega_p)(\omega^2/\omega_p^2)/\mathcal{E}_m$ and for $\tau_\ell\omega_p = 4\lambda^{-2/3}$ (if we set τ_ℓ for the relativistic detuning time) and $\mathcal{E}_m = 4(\lambda/3)^{1/3}$ the depletion length is

$$L_{pd} = (c/\omega_p)(\omega^2/\omega_p^2)(3/\lambda)^{1/3}. \tag{15}$$

The pump depletion length is only slightly larger than the dephasing length. Thus the pump depletion effect may limit the main attractiveness of the surfatron.[3]

Since the pump depletion time τ_{pd} is long compared with growth period $1/\omega_p\lambda$ of the plasma wave, it is convenient to calculate the decay of the laser pulse in the wave frame where

$$\frac{\partial W_\ell}{\partial t'}(x',t') \cong v_p\frac{\partial W_p}{\partial x'}(x',\lambda(x',t')), \tag{16}$$

and the loss of total laser energy $E'_\ell = \int W_\ell d^3x'$ occurs from the flux of plasma waves $v_pW_p(x' = -L_\ell)$ out the rear of the laser pulse

$$\frac{d}{dt'}E'_\ell = \int\int_{-L_\ell}^{0}\frac{d}{dt'}W_\ell da dx' = -v_pA_pW_p(x' = -L_p). \tag{17}$$

Equation (17) shows that the laser pulse decays most rapidly when $\partial W_p/\partial x'$ takes maximum.

II. Laser Staging for the Beat − Wave Accelerator

The pump depletion time is not much different from the longitudinal dephasing time[4] and the acceleration time.[1] It is thus important to devise a method of supplying fresh laser power over the pump depletion length L_{pd} or that of amplifying the laser beam while it is used for acceleration (*in situ* amplification). We propose a concept of laser staging for the beat-wave accelerator as a countermeasure of this probem.

In order to cope with the pump depletion and/or the longitudinal dephasing, we inject fresh laser beams at a certain interval of length over L_{pd} given by Eq. (15). In order to ease the difficulty to compensate the consumed laser power used for acceleration, distributed amplifiers amplify the master laser beam at the site of the acceleration. In order to cope with the matching of the phase in different acceleration modules, all modules share one master oscillator. In order to adjust the optical path difference, we introduce path adjusters. Figure 1 shows the schematic description of our laser staging concept.

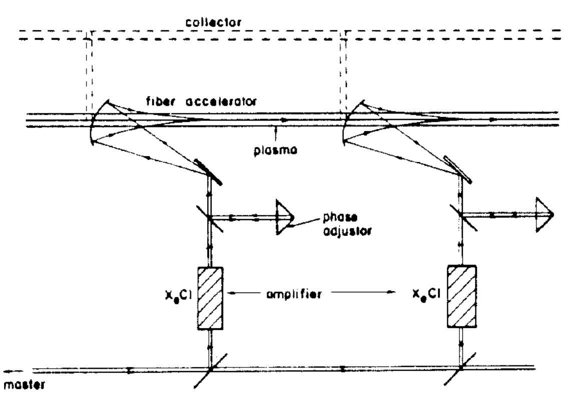

Fig. 1 - Schematic concept of laser staging for the beat-wave accelerator.

A similar concept had been reported by Pellegrini[5] for far field accelerator concepts. This architecture of the beat-wave accelerator consists of the following features: (i) distributed laser staging optically connected by the master oscillator; (ii) optical delay lines synchronizing the timing and phase of each laser stage; (iii) appropriate mirror and filter systems; (iv) optical guides (plasma fiber)[4] to trap laser beams created by acoustic transducers or other mechanisms. If necessary, additional collector devices may be added to collect any unused portion of laser power and to convert it for reuse.

The individual accelerator module should stack up in tandem until the desired energy of particles is reached. Each module is separated by a distance of the laser pump depletion length plus the mirror focal length. As we have noted in the above, under the circumstance of most typical operations of the beat-wave accelerator the longitudinal dephasing length is close to the pump depletion length. Therefore, this concept of laser staging conveniently satisfies both requirements for the module repetition length arising from the pump depletion and from the dephasing simultaneously.

Although more research is due in the future, excimer lasers seem favorable lasers for staging *in situ* for several reasons, including possible high efficiency, possible high repetition rate, reasonable gain, fairly short wavelength, etc. One example is a XeCl laser. The XeCl laser seems to have a good potential for a fairly large number of repetitions with good efficiency and robustness. Its wavelength is 0.3μ. Also XeF, KrF possible.

The laser staging makes the length of accelerator longer because of the focal length of the many added mirrors. On the other hand it makes the beat-wave accelerator concept much more realistic. Compare this with the earlier design by Ruth and Chao.[6]

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