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COLLECTIVE ION ACCELERATION BY MEANS OF VIRTUAL CATHODES

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1. Introduction

Experiments on collective ion acceleration by means of the formation of a virtual cathode have been carried out for a number of years in the Soviet Union and in the United States. Recently, there has been renewed interest in the subject as a possible means of accelerating ions to very high energies. By understanding the physics underlying the acceleration process it may be possible to determine the feasibility of virtual cathode staging for very high energy ion production.

For this reason, a theoretical and computational effort is underway at Los Alamos in order to clarify the basic issues of collective ion acceleration by means of virtual cathodes. To support the theoretical effort, simulations were done with the fully electromagnetic and relativistic particle-in-cell code ISIS (in a one-dimensional mode) and the electrostatic one-dimensional code BIGONE. In the simulations, an electron beam of density 6 x 10^{11} cm^{-3} is injected into a one-dimensional box of length L . To supply the necessary ions for collective acceleration, a plasma source containing both ions and electrons was initialized near the emitting boundary (Fig. 1). This is similar to the experimental configuration of Destler's group at Maryland[1].

Of prime interest in this study was to understand the dynamics of virtual cathode formation and the dynamics of the acceleration process for the ions. In particular, the question of whether the ions are accelerated by a moving potential well[2] or hydrodynamic pressure due to ambipolar expansion[3] is of primary interest. For this reason, the simulations were done in one-dimension in order to isolate the basic physical processes underlying collective ion acceleration from simple geometric effects. These simulations were then compared to 2-1/2 dimensional ISIS runs.

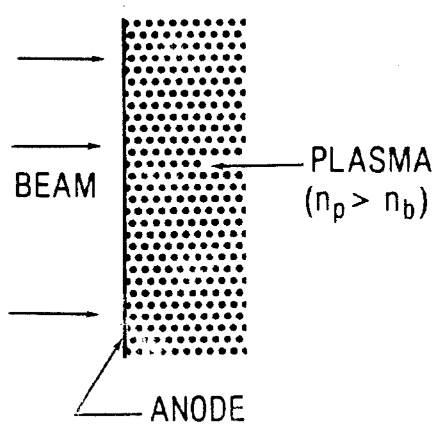

Fig. 1. Schematic drawing of a relativistic electron beam injected into a drift region containing a localized source of plasma. The collector plate, which is at a distance of L cm away from the emitter, is not shown.

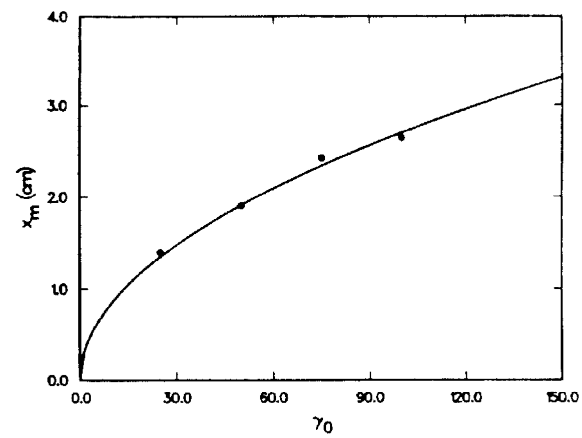

Fig. 2. Virtual cathode position x_m as a function of initial beam energy γ_0. The solid curve shows $\gamma_0^{1/2}$ scaling.

2. Virtual Cathode Formation

Consider a relativistic electron beam injected into a box of length L such that $\varphi(x=0)=0$ and $\varphi(x=L)=0$. A time-dependent analysis of the problem up to the first electron turning point was presented by Poukey and Rostoker[2] for the one-dimensional problem, but with the boundary conditions $\varphi(x=0)=0$ and $E(x=\infty)=0$. Even so, their results should describe the problem at hand if $x_m/L \to 0$, where x_m is the position of the virtual cathode.

To study the formation and dynamics of the formation of a virtual cathode, simulations were done in order to verify a simple one-dimensional steady-state model, and to provide a benchmark for understanding the general time-dependent problem. In the simulations, electrons are injected into a drift-tube consisting of two grounded end plates. Plots of the electric potential, electric field, and virtual cathode position were obtained for different values of the injection velocity.

To calculate the beam current needed for virtual cathode formation, the space charge limiting current for planar equipotential boundaries must be calculated. This was done using the Jones-Lemons[4] generalization to the Birdsall and Bridges[5] equation:

$$J_{SL} = 2.71\ (\gamma_0^{1/2}-0.84711)^2(\xi_m/\xi_0)^2/L^2 \qquad (1)$$

where J_{SL} is the space charge limiting current density in kA, and (ξ_m/ξ_0) is the Jones-Lemons relativistic correction factor[4].

According to the time-dependent theory, a virtual cathode should form at a distance on the order of an electron beam skin depth c/ω_b from the emitter. The earliest electrons should propagate across the box at the injection

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velocity v_0. This effect could be easily seen in the simulations. After the beam front leaves the box, the gross features of the phase space are constant, though the history plots show definite virtual cathode oscillations. The oscillations for low beam $\gamma_0 = \sqrt{(1 - v_0^2/c^2)}$ show both higher frequencies and amplitudes than the corresponding simulations for large γ_0.

The position of the virtual cathode was also measured as a function of the beam γ_0 for different values of the beam injection velocity. To minimize the effects of the oscillations in the virtual cathode position, these runs were done for large values of γ_0. As Fig. 2 shows, the agreement between the simulations and the Poukey-Rostoker theoretical scaling (x_m proportional to $\gamma_0^{1/2}$) is quite good.

For late times in the simulations, a "steady state" was reached. By this we mean all physical quantities averaged over the virtual cathode oscillations were essentially unchanged for all time. A simple, steady-state model was then formulated which assumes *a priori* that a virtual cathode forms at the point $x = x_m$. Assuming the beam makes up three species (the injected particles, the reflected particles, and the transmitted particles), Poisson's equation, the equation of continuity, and the conservation of energy equation $(\gamma - \gamma_0)\, mc^2 = e\varphi$ for each species can be used. The solutions cannot be integrated exactly in the general case, but in the non-relativistic regime they can be obtained exactly. The result for the potential $\bar{\varphi}$ $(\equiv e\varphi/(\gamma-1)mc^2)$ can be written:

$$\bar{\varphi}(\bar{x}) = \left[1 - 3\,\bar{x}\,\sqrt{(1 - T/2)}\right]^{4/3} - 1 \tag{2a}$$

for $x < x_m$, and

$$\bar{\varphi}(\bar{x}) = \left[1 - 3\,(\bar{L} - \bar{x})\,\sqrt{(T/2)}\right]^{4/3} - 1 \tag{2b}$$

for $x > x_m$. In Eqs. (2) the dimensionless lengths are overstruck by bars and are normalized to the beam skin depth $\delta = v_0/\omega_b$. The quantity T (which is a function of $\bar{x}_m$ only) is the transmission coefficient of the beam; that is, the ratio of the transmitted current to the injected current. Comparisons between the theoretical curve of Eqs. (2) and a typical potential plot from the one-dimensional simulations is surprisingly good.

The steady-state solutions above impose a requirement on the virtual cathode position as a function of the initial beam density n_0, the initial beam velocity v_0, and the length L of the gap spacing. In terms of dimensionless quantities, this equation can be written:

$$\bar{L} = \bar{x}_m + (9 - \bar{x}_m^{-2})^{-1/2} \tag{3}$$

In Fig. 3 we plot the theoretical curve for Eq.(3) with the corresponding simulation data points. The simulations were made with values of L ranging from 0.1 to 2.0 cm. Identical values for the initial beam density n_0 and velocity v_0 were used in these runs. Because of the variation in virtual cathode position due to the virtual cathode oscillations in this nonrelativistic case ($\gamma_0 = 1.01$), the data points correspond to a simple average of the maximum and minimum amplitudes.

An interesting corollary to Eq. (3) can easily be derived. If

$$9\bar{L}^2 - 8 > 0 \tag{4}$$

or $L < (8/9)^{1/2}\delta$, there will be no real solution for $\bar{x}_m$ in terms of a given $\bar{L}$. Hence, in the nonrelativistic regime, a necessary condition for the formation of a virtual cathode to form is that the drift tube length L must satisfy $L > 0.95\,(v_0/\omega_b)$, where v_0 and ω_b are the initial beam velocity and plasma frequency, respectively. This limit can be seen graphically in Fig. 3.

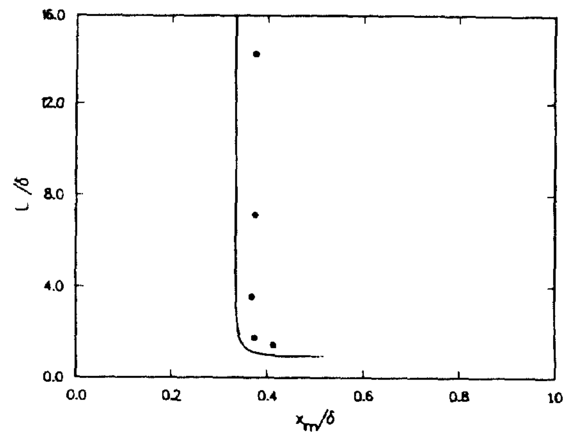

Fig. 3. Variation of virtual cathode postition x_m/δ as an implicit function of the gap spacing L/δ from Eq. (3). The data points are from one-dimensional simulations.

3. Collective Acceleration

When a plasma is introduced into the box, the beam space charge can be neutralized by this plasma. The beam then propagates a distance equal to c/ω_b beyond the plasma boundary, until a virtual cathode forms. In the "moving virtual cathode theory", the resulting electron space charge in the thin sheath beyond the plasma pulls the ions with it in a bootstrap process. The motion of the potential well with respect to time can be derived in a simple fashion from Eq.(2), and will be discussed in a later study.

In the one-dimensional runs with an ion-to-electron mass ratio $m_i/m_e = 1836$, no virtual cathode motion was seen. Typically, these runs had a rich source of ions in the form of a localized plasma of density 50 times the injected electron beam density. When the electron-to-ion mass ratio was reduced to 20, as in the work of Mako and Tajima[6], the virtual cathode was observed to move downstream from the emitter plate. Whether or not this virtual cathode movement is simply a result of the artificially low mass ratio of 20 or a real physical effect has not yet been determined.

In an accompanying paper[3], an alternative theory to the moving virtual cathode model is proposed. Some calculations in relation to this model seem to suggest that the ion currents necessary for the existence of a moving virtual cathode are prohibitively large. Instead, the simulations seem to suggest that the ions are influenced by electron pressure in the drift region, and that they move in a distinctly self-similar fashion. An interesting prediction of this model is the absence of any restriction on the magnitude of the accelerated ion current.

The one-dimensional simulations with equipotential boundary conditions ($\varphi=0$) show that the measured maximum ion energy is a little over three times the initial beam energy (Fig. 4). This agrees with the results of Ref. 6. In the simulations using the Poukey and Rostoker boundary conditions, the measured maximum ion energy increases up to twenty times the initial beam energy. In the two-dimensional simulations the maximum ion energy scaled as $(\nu/\gamma_0)^{1/2}$, where ν is Budker's parameter.

In conclusion, we have presented a simple calculation for the electric potential and electric field in a one-dimensional virtual cathode formed by the injection of a relativistic electron beam into a drift tube region. The theoretical predictions seem to agree in a time-averaged sense with a simple steady-state model. A nonrelativistic condition necessary for the formation of a virtual cathode has been derived for the drift tube length L in terms of the initial beam velocity and density. The peak ion energy in the one-dimensional collective acceleration simulations was found to be three to four times the initial beam energy, in agreement with the results of previous authors. Further work is needed to extend the simple analytic model to relativistic, and perhaps time-dependent, regimes. Finally, it is planned to use the results of this study to design future accelerators with significantly higher values of ion energy.

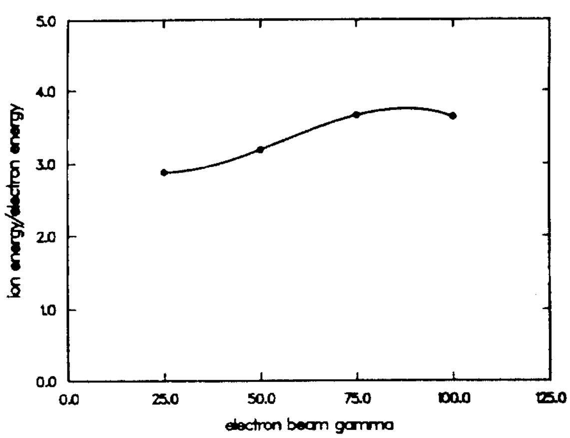

Fig. 4. Ratio of maximum ion energy to initial beam energy as a function of γ_0.

Acknowledgments

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References.

1. J.T. Cremer, P.G. O'Shea, and W.W. Destler, Bull. Am. Phys. Soc. **29**, 1274 (1983).

2. J.W. Poukey and N. Rostoker, Plasma Physics **13**, 897 (1971).

3. R.J. Faehl, W.K. Peter, this conference.

4. Michael E. Jones and Don S. Lemons, Proc. 1984 IEEE Conf. Plasma Sci., St. Louis, Mo, 1984. (IEEE Publication 84CH1958-8, p. 2).

5. C.K. Birdsall and W.B. Bridges, *Electron Dynamics of Diode Regions*, (Academic Press, New York, 1966), Ch. 3.

6. F. Mako and T. Tajima, Phys. Fluids **27**, 1815 (1984).