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# A PERIODIC PLASMA WAVEGUIDE ACCELERATOR

F. T. Cole *
Argonne National Laboratory **

## INTRODUCTION

The increasing cost of synchrotrons and storage rings has given new interest in the search for new methods of acceleration. The primary goal of this search is very large accelerating fields, because the cost of an accelerator to reach TeV energies is dominated by costs that scale with length. Very large electric fields are possible in plasmas and in lasers and many geometries are being studied that make use of plasmas, lasers, or combinations of them.

In a plasma accelerator, the plasma can have several different functions. It may act as a medium for the propagation of accelerating electric-field waves. In addition, these waves may also act as a source of the energy needed to accelerate particles. Accelerators using various waves in plasmas have been built and studied in many laboratories.[1,2,3]

The device proposed here is an attempt to separate the two functions of providing a medium and providing an energy source. A relatively low-energy electron beam is used as a non-neutral plasma only to make a slow-wave medium for the propagation of an externally generated wave. The wave is a TM electromagnetic wave and the device may be thought of as a conventional electron linear accelerator with the evacuated volume and metallic envelope replaced by the electron beam. A separate second beam, which may be electrons or heavier particles, is accelerated. The application in mind here is a single-pass collider.

A slow electromagnetic wave can propagate in this medium because the electron beam moves in a periodically varying external longitudinal magnetic field that produces periodic density variations. The plasma frequency therefore varies periodically and many different space harmonics can propagate. The TM wave in synchronism with the second beam will accelerate that beam. It is also possible to decelerate the second beam and thus to use the structure as a travelling-wave tube. Figure 1 is a sketch of such a plasma waveguide. Here the periodic external field is produced by alternating washers of ferromagnetic and nonferromagnetic material. Many other ways of producing such a field have been used[4] and could be applied here.

Each plasma-waveguide section is a few meters in length and has its own electron gun to provide the low-energy beam. Modular sections of guide form a linear sequence through which the accelerated particles pass, as in a conventional linear accelerator.

This periodic plasma waveguide differs from the earlier work of Fainberg[1] in its use of a periodic field. It differs from other devices[2,3,4] that use an electron beam in a periodic external field in that these other devices utilize coherent transverse motion of the beam as a source of energy, while this devices uses longitudinal motion to create a medium. Tajima[5] has proposed a somewhat similar device, a hollow-beam periodic plasma waveguide at laser frequency.

The accelerating field attainable in the plasma waveguide is limited by trapping of the low-energy beam electrons in the field, with consequent breakup of the medium. Accelerating fields of the order of 1 GV/m appear to be possible in principle at millimeter wavelengths.

This paper gives discussions of the trapping limit, the motion of the low-energy beam, and the dispersion relation for electromagnetic waves. Some of this discussion has been presented in Ref. 6 and is included here for completeness. Discussion of the motion of the second, accelerated beam is not given here, because that motion will be relatively conventional.

## TRAPPING AND FIELD LIMITS

We calculate the trapping of a low-energy electron in a bucket of the accelerating field. The calculation here is in the spirit of Ruth and Chao.[7] The single-particle Hamiltonian of a particle in a travelling wave of frequency $\omega_0$ and wave number k is

$$H = c\sqrt{p^2 + m^2c^2} - \frac{eE_0}{k}\cos(\omega_0 t - kz) , \qquad (2.1)$$

where $E_0$ is the amplitude of the field and $v_0=\omega_0/k$ is its phase velocity. We make a canonical transformation to the wave frame with the generating function

$$F(P,z,t) = P(z - v_0 t) . \qquad (2.2)$$

The new Hamiltonian is

$$\bar{H} = c\sqrt{P^2 + m^2c^2} - \frac{eE_0}{k}\cos kZ - v_0 P , \qquad (2.3)$$

where $Z = z - v_0 t$. The bucket is centered on the momentum $P = P_0 = m\gamma_0 v_0$ of the high-energy second beam. There is a stable fixed point at $Z = 0$, $P = P_0$. There are unstable fixed points at the bucket limits $Z = \pm\pi/k$, $P = P_0$. On the separatrix through these unstable fixed points, the Hamiltonian has the value

$$\bar{H} = mc^2\left(\frac{1}{\gamma_0} + \delta\right) , \qquad (2.4)$$

where

$$\delta = \frac{eE_0}{kmc^2} . \qquad (2.5)$$

The equation of the upper and lower branches of the separatrix is

$$P = mc\left[\beta_0\gamma_0 A \pm \gamma_0\sqrt{A^2\gamma_0^2 - 1}\right] , \qquad (2.6)$$

with

$$A = \frac{1}{\gamma_0} + \delta(1 + \cos kZ) . \qquad (2.7)$$

The low-energy beam will be outside the bucket and therefore untrapped if the bucket half-height in momentum is smaller than the height of the bucket center, that is, if $\gamma_0\sqrt{A^2\gamma_0^2 - 1} < \beta_0\gamma_0^2 A$ , from which $A^2 < 1$. At the bucket center $Z = 0$, this gives

$$\frac{2eE_0}{kmc^2} < 1 - \frac{1}{\gamma_0} . \qquad (2.8)$$

At low energy ($\gamma_0 \cong 1$), the maximum accelerating field is small, but for high energy and small wavelength, the attainable field can be large. For example, at $\lambda = 1$ mm,

$$E_0 < \frac{\pi}{2}\cdot 10^9 \text{ V/m} . \qquad (2.9)$$

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## MOTION OF THE LOW-ENERGY ELECTRON BEAM

We neglect nonlinear terms in the motion of the electron beam and therefore omit transverse components of the external field. Thus we take the field to be of the form

$$B_z = B_0 + \sum_{n\neq 0} B_n e_n \,, \qquad (3.1)$$

where $e_n = \exp(ink_0 z)$ and $k_0$ is the wave number of the external field. The cyclotron frequency is

$$\omega_c = -\frac{B}{m\gamma_c} = \omega_{c0} + \sum_{n\neq 0} \omega_{cn} e_n \,. \qquad (3.2)$$

The radial electric-field and azimuthal magnetic-field components of the self field combine to give a force on a particle at the edge of the beam

$$\frac{1}{2}\,\frac{m\omega_p^2}{\gamma}\,\frac{r_b^2}{r} \,, \qquad (3.3)$$

where $\omega_p^2 = 4\pi e^2 n/m\gamma$ is the plasma frequency and $r_b$ is the beam radius. The plasma frequency varies periodically with z as

$$\omega_p^2 = \omega_{p0}^2 + \sum_{n\neq 0} \omega_{pn}^2 e_n \,, \qquad (3.4)$$

and our first objective is to find $\omega_{pn}^2$.

If there were no longitudinal variation, the beam would have a uniform radius $r_0$ related to the constant canonical angular momentum $p_\theta$ by

$$\frac{p_\theta}{m\gamma r_0^2} = \omega_{c0}^2 - \omega_{p0}^2 \,. \qquad (3.5)$$

We retain only linear terms in $x = r - r_0$ and find the equation of motion

$$\frac{d^2x}{dz^2} + \frac{1}{v_0^2}\left(\omega_c^2 - \omega_p^2\right)x = 0 \,, \qquad (3.6)$$

where $z = v_0 t$. We seek a phase-amplitude solution

$$x = A(z)\cos\phi(z) \,, \qquad (3.7)$$

with

$$A = A_0 + \sum_{n\neq 0} A_n e_n \,. \qquad (3.8)$$

We find

$$A_n = \frac{2\omega_{cn}\omega_{c0}}{\omega_{c0}^2 - \omega_{p0}^2 - k_0^2 v_0^2} A_0 \qquad (3.9)$$

and

$$\omega_{pn}^2 = -2\omega_{p0}^2 \frac{A_n}{A_0} \,. \qquad (3.10)$$

This scalloped envelope and the effect of the resonance denominator have been predicted and observed.[8]

## ELECTROMAGNETIC FIELDS AND DISPERSION RELATIONS

Our purpose is to consider propagation of electromagnetic waves in the periodic medium. The external magnetic field has only a small direct effect on this propagation and we omit it here. Thus we lose the coupling between TE and TM waves and have a simple wave equation for the TM wave

$$\frac{\partial^2 E_z}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E_z}{\partial r}\right) + \frac{1}{c^2}\left(\omega^2 - \omega_p^2\right)E_z = 0 \,, \qquad (4.1)$$

where the time dependence is taken in the form $\exp(i\omega t)$. Here $\omega_p^2$ is a periodic function of z given by Eq. (3.10). Note the difference from the conventional disc-loaded waveguide and from the hollow-beam case. There the wave equation does not have periodic coefficients, but instead periodic boundary conditions. In the case treated here, we can separate variable with only one transverse wave number $k_r$:

$$E_z = R(r)E(z) \qquad (4.2)$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right) + k_r^2 R = 0$$
$$\frac{d^2E}{dz^2} + \frac{1}{c^2}\left(\omega^2 - \omega_p^2 - c^2k_r^2\right)E = 0 \,. \qquad (4.3)$$

The radial equation is satisfied by Bessel functions of order zero. Boundary conditions on a conducting surface at larger radius can be satisfied by this solution. The longitudinal equation satisfies Floquet's theorem. Its solution is therefore of the form

$$E = \sum_n E_n e^{ik_n z}$$
$$k_n = k_z + nk_0 \,. \qquad (4.4)$$

Substitution of this form into Eq. (4.3) gives a set of recursion relations in the space harmonics $E_n$

$$D_n E_n = \sum_{m\neq 0} \omega_{pm}^2 E_{n-m}$$
$$D_n = \omega^2 - \omega_p^2 - c^2\left(k_r^2 + k_n^2\right) \,. \qquad (4.5)$$

When the $\omega_{pm}$ are zero for $m \neq 0$, the right hand side vanishes and Eq.(4.5) reduces to the usual dispersion relation for electromagnetic waves in a plasma

$$D_0 = \omega^2 - \omega_p^2 - c^2\left(k_r^2 + k_z^2\right) = 0 \,. \qquad (4.6)$$

In the more general case with $\omega_{pm}^2 \neq 0$, Eq. (4.5) can be solved numerically. As an example, let us consider the case with three harmonics different from zero, $\omega_{c0}$, $\omega_{c1} = \omega_{c,-1}$. The plasma-frequency harmonics $\omega_{p'} = \omega_{p,-1}$ are then given by Eq. (3.10). There are now just two parameters in the dispersion relation

$$\zeta = \omega_{p0}^2 + c^2k_r^2$$
$$\eta = \omega_{p1}^2 \,. \qquad (4.7)$$

In Fig. 2, we plot computed dispersion curves for several different values of ζ and η, which are expressed in units of $ck_0$ for plotting. It can be seen that there are three branches of the dispersion curve, including one that can have phase velocity $v_\phi <$ c. A locus of points such that $v_\phi = c$ as a function of ζ and η can be found and such a curve is plotted in Fig. 3., where we have taken $k_z = 1/4\ k_o$, a π/2 structure.

The existence of this curve shows that there are physically realizable parameters for which a slow accelerating wave can propagate. These parameters may not be trivial to achieve. The beam must be finite in radial extent and therefore $k_r$ must be different from zero. Even if $\omega_{p0}^2 << c^2k_r^2$, physically realizable operating points must be a finite distance up the curve of Fig. 3. This means that $\omega_{p1}^2$ must have an appreciable value, which in turn implies that the electron-beam density and current must be large.

## CONCLUSION

An electron beam can exhibit periodically varying density in a periodic external field (in plasma-physics language, a standing cyclotron wave) and slow TM electromagnetic waves will propagate in the medium. It appears possible in principle to accelerate a second beam of particles in this slow wave. Initial consideration of parameters indicates that an electron-beam current in the kiloamp range will be needed to produce the plasma frequency needed.

Questions of stability of the electron-beam motion, stability of the accelerated beam, coupling of TM and TE waves and possible use of hollow beams remain to be discussed.

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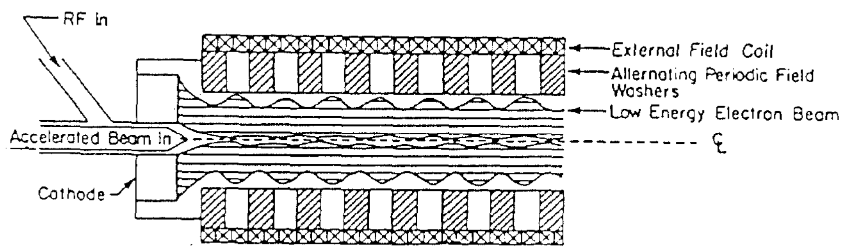


Fig. 1 A Sketch of a Periodic Plasma Waveguide

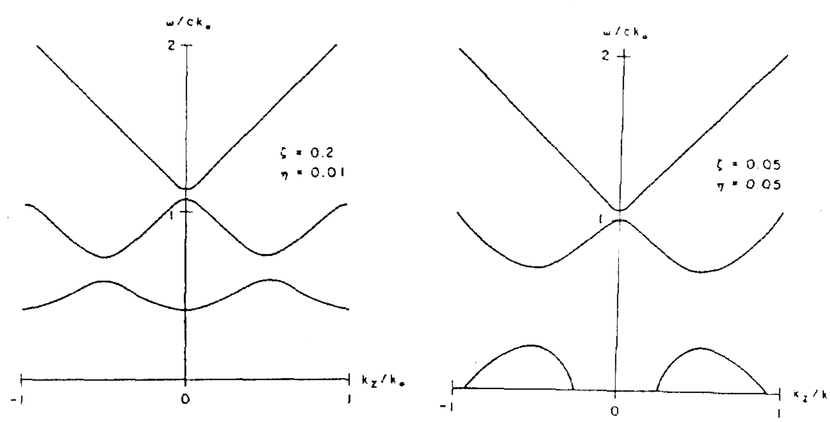


Fig. 2 Dispersion Curves

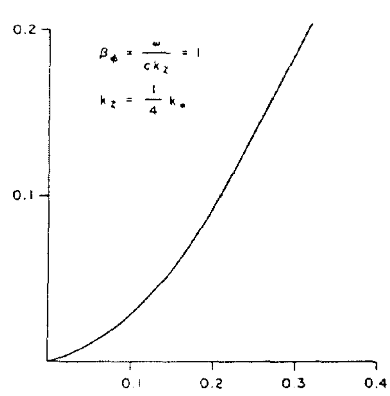


Fig. 3 Locus of Points for which $v_\phi = c$ in a $\pi/2$ Structure