A NEW FORMULATION FOR LINEAR ACCELERATOR DESIGN*

Z. D. FARKAS

Stanford Linear Accelerator Center
Stanford University, Stanford, California, 94305

ABSTRACT

We define three basic, calculable and measurable parameters. With these parameters we derive expressions for the radio frequency induced, beam induced, and loaded section voltages and average section gradients. Unlike the present well known expressions these alternate expressions are valid for continuous wave, pulsed, and single bunch beams, for both lossy and lossless sections and for both standing wave and travelling wave sections. We use these alternate expressions to maximize the efficiency and the gradient for a given peak power.

INTRODUCTION

Three local parameters characterize a traveling wave accelerator section: the elastance per unit length s, the group velocity v_g, and the unloaded (internal) time constant T_o. They are defined as follows:

$$s = \frac{E^2}{w} \tag{1}$$

$$v_g = \frac{P}{w} \tag{2}$$

$$T_o = \frac{2w}{p_d} \tag{3}$$

where E is the accelerating gradient, w is the energy stored per unit length, P is the power transmitted and p_d is the power dissipated per unit length. As the linear dimensions vary as the frequency f, we infer from the above definitions that for the same group velocity and same mode

$$s \propto f^2 \qquad \text{and} \qquad T_o \propto f^{-3/2} \tag{4}$$

We can express the rf induced gradient, the beam induced gradient and the difference between the rf and the beam induced section gradients in terms of the three parameters. With these parameters, and given the rf peak power and pulse energy into the section we obtain the beam voltage and total energy transferred to the beam. The ratio of energy in each beam pulse to the energy in each rf pulse is the conversion efficiency, or simply the efficiency.

RF ENERGY TO BEAM ENERGY CONVERSION EFFICIENCY

We will derive expressions for the efficiency for two special cases: long beam pulse and single bunch. The rf induced accelerating gradient which is the same for both cases, is derived in Ref. 1 for the general case of variable group velocity along the section. For the special case of zero group velocity gradient it is derived in the Appendix A and is

$$\bar{E}_a = \frac{V_o}{L} = \sqrt{\frac{\eta_s s P_o T_f}{L}} \quad , \quad \eta_s = \frac{(1-e^{-\tau})^2}{\tau^2} \tag{5}$$

$$\tau = \frac{T_f}{T_o} \quad , \quad T_f = \frac{L}{v_g} \tag{6}$$

where η_s is the section efficiency, s is the elastance per unit length (MΩ/μs-m), P_o is the section power input (MW), T_f is the section fill time (μs), τ is the section attenuation in nepers, and L is the section length (m). The section efficiency η_s is the energy required for a given voltage divided by the energy required for the same voltage by an identical but looseless section.

<u>Long Beam Pulse</u>

From the Appendix B the expressions for the beam induced and loaded gradients are

$$\bar{E}_b = \frac{\eta_i s I_o T_f}{4} \quad , \quad \eta_i = 2\left[\frac{1}{\tau} - \frac{1-e^{-\tau}}{\tau^2}\right] \tag{7}$$

$$\bar{E} = \bar{E}_a - \bar{E}_b = \bar{E}_a - \frac{\eta_i I_o T_f}{4} \tag{8}$$

Assuming a beam pulse of length T_b is turned on as soon as the section is full the rf energy to beam energy conversion efficiency is

$$\eta = \frac{\bar{E} I_o T_b L}{P_o(T_f+T_b)} = \frac{\eta_s s I_o T_b T_f}{\bar{E}_a(T_f+T_b)}\left[1 - \frac{\eta_i s I_o T_f}{4\bar{E}_a}\right] \tag{9}$$

<u>Single Bunch</u>

If we inject a single bunch of charge q at T_f, the self-induced effective field acting on the bunch is:[1] $\bar{E}_b = sq/4$. In Ref. 1, $s/4$ is called the loss parameter k_1. The efficiency of transforming rf energy to bunch energy is

$$\eta = \frac{(\bar{E}_a - \bar{E}_b)Lq}{P_o T_f} = \frac{\eta_s s(\bar{E}_a - \bar{E}_b)q}{\bar{E}_a^2} = \frac{\eta_s sq}{\bar{E}_a}\left[1 - \frac{sq}{4\bar{E}_a}\right] \tag{10}$$

In the single bunch mode, when the group velocity is close to the particle velocity, assumed to be c, then we do not have to fill the section before we inject the beam pulse and the effective fill time is diminished by the factor $f_v = 1 - (v_g/c)$. The required klystron pulse width is reduced by this factor and η in Eq. (9) is improved by the inverse of this factor. In both cases for light beam loading the rf energy to beam energy conversion efficiency varies as the elastance.

SECTION DESIGN: CHOICE OF GROUP VELOCITY AND FREQUENCY

Define the section elastance s_s, and the section reactance x_s:

$$s_s = \frac{\bar{E}_a^2}{w_o} \quad , \quad x_s = \frac{\bar{E}_a^2}{p_o} \tag{11}$$

Here p_o and w_o are respectively the section input peak power per meter and pulse energy per meter. Using (5) and (11) we obtain

$$s_s = \eta_s s f_v \quad , \quad x_s = \eta_s s T_f = \eta_s s \frac{L}{v_g} \tag{12}$$

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.

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The section elastance determines the energy per unit length and the section reactance determines the peak power per unit length needed to attain a given gradient. Figure 1 shows a plot of x_s and s_s as a function of fill time for a 2 cm aperture, 1.44 μs internal time constant, disk-loaded waveguide (DLWG) operating at 2856 MHz. Its length is 3 m and its fill time is 0.82 μs and hence $\tau = 0.57$ and $\eta_s = 0.581$. The elastance is 76.4 MΩ/mμs and the group velocity is 3.66 m/μs. The fill time is a compromise: we accept a lower efficiency in order to reduce the peak power requirement. The local and section parameters are listed in the first line of Table 1.

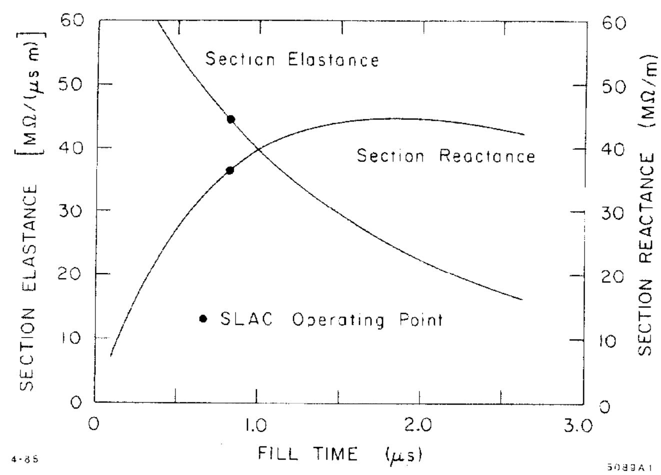

Fig. 1. TW section elastance and reactance vs fill time.

Table 1. Local and section parameters for a 2 cm aperture DLWG

freq MHz	diam cm	s MΩ/mμs	v_g m/μs	T_o μs	T_f μs	L m	s_s MΩ/mμs	x_s MΩ/m
2856	10.5	76.4	3.66	1.44	0.82	3	44.4	36.4
11400	2.8	267	96	.585	0.33	32	226	51.2

For a given aperture there is a frequency that will maximize the elastance. As we shall see in the following example, the elastance can be increased considerably over the value at 2856 MHz.

We now increase the aperture to 8 cm and increase the frequency so that the aperture returns to 2 cm as illustrated in Fig. 2. With our assumed light loading the design is the same for both a single bunch and long beam pulse. But when operating in a single bunch mode the velocity factor increases the section elastance by a factor of 1.46. The resulting parameters with maximized elastance are listed in the second line of Table 1. We do pay for the 5 fold increase in elastance with a high peak power requirement, but this can be remedied with efficient peak power multiplication.

High group velocity and high frequency combination afford a mechanical advantage: the accelerator becomes a long thin tube which can be shaped into a zig-zag structure.

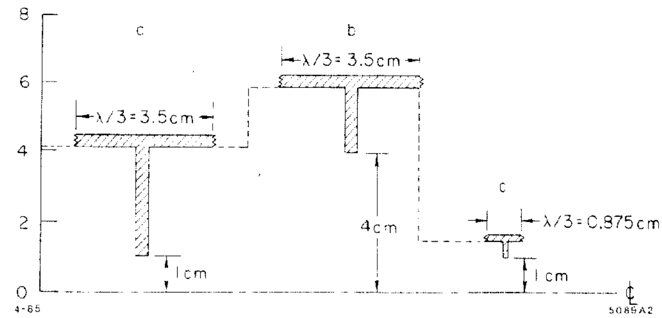

Fig. 2. Evolution of fixed aperture design from 2856 MHz to 11400 MHz.

STANDING WAVE SECTION DESIGN

Define the internal and the external standing wave (SW) section time constants as

$$T_o = 2W/P_d \quad , \quad T_e = 2W/P_e \tag{13}$$

W is the energy stored in the cavity, P_d is the power dissipated in the cavity, P_e is the power emitted by the cavity. The stored energy at the end of the input pulse T_f is[2]

$$W = \frac{0.5\alpha^2(1-e^{-\tau})^2}{T_f/T_e} P_o T_f \tag{14}$$

$$\alpha = \frac{2}{1+T_e/T_o} \quad , \quad T_L = \frac{T_e \alpha}{2} \quad , \quad \tau = \frac{T_f}{T_L} \tag{15}$$

Define
$$s = \frac{V^2}{WL} = \frac{\bar{E}_a^2}{W/L} \tag{16}$$

where V is the accelerating voltage. From (14) and (16) the rf induced accelerating gradient is

$$\bar{E}_a = \sqrt{\frac{\eta_s s P_o T_f}{L}} \quad , \quad \eta_s = \frac{0.5\alpha^2(1-e^{-\tau})^2}{T_f/T_e} \tag{17}$$

Figure 3 shows a plot of T_e/T_o that maximizes η_s for a given T_f/T_o. The T_e obtainable from this plot matches a source with a given pulse length to a cavity with a given T_o. If $T_f \gg T_o$ then we have the steady state match condition of $T_e = T_o$. If $T_o \gg T_f$ then $\alpha = 2$, $\tau = T_f/T_e$ and the section efficiency has a maximum with respect to T_e of 0.815 at $T_f = 1.257 T_e$.

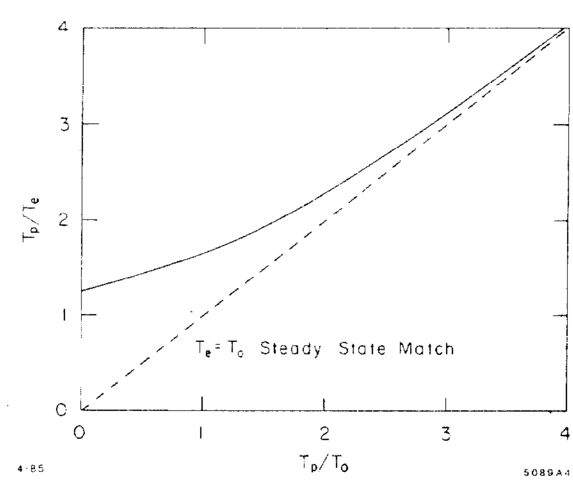

Fig. 3. Pulse matching.

We see that the constants that characterize a SW section are the same as the ones that characterize a traveling wave section with the following exceptions. The lossless efficiency is 0.815 rather then unity as for a TW section, the group velocity is replaced by the external time constant, the elastance per unit length and the internal time constant is that of the whole section rather then obtained from per unit length quantities. The elastance and internal time constant can be obtained from computer codes such as SUPERFISH.

Figure 4 shows a plot of x_s and s_s as a function of the fill time for a SW section with the same elastance, internal time constant and length as a SLAC section. Just as with a TW section the section elastance approaches an asymptotic maximum as the fill time gets much smaller than the internal time constant. Unlike in a TW section in a SW section there is a slight increase in stored energy if the source pulse width increases beyond the fill time.

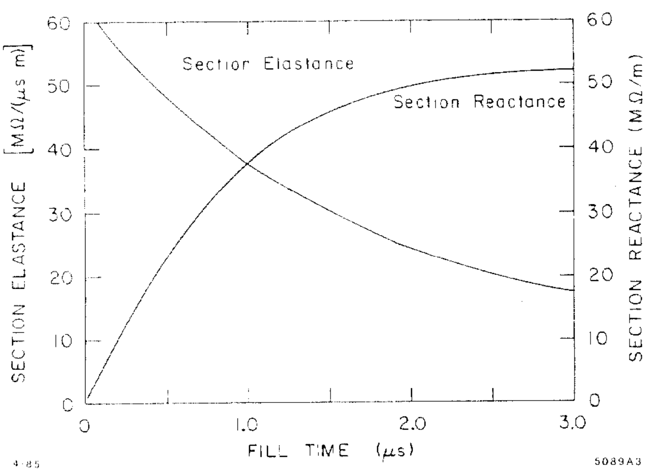

Fig. 4. SW section elastance and reactance vs fill time.

A PROPOSAL

We propose the use of the three parameters s, v_g, and T_o for accelerator design. As we have shown, they are convenient and sufficient to design a TW or a SW accelerator section, as well as a superconducting section.[3] These parameters are clearly definable, have names, are directly measurable and are useful: that is they give information. We believe they simplify the present symbol soup. From the three parameters we can obtain the familiar parameters: the quality factor Q, the shunt resistance per unit length r, r/Q, $\omega r/Q$, and the loss parameter k_1:

$$Q = \frac{\omega T_o}{2}\ , \quad r = \frac{sT_o}{2}\ , \quad r/Q = \frac{s}{\omega}\ , \quad 4k_1 = \frac{\omega r}{Q} = s \qquad (18)$$

With our parameters the familiar expressions for the no-load and beam induced voltages can be converted to expressions that do not become indeterminate as the resistivity of the section material approaches zero.

APPENDIX A: UNLOADED VOLTAGE

From (2) and (3) we have

$$p_d = \frac{-2P}{v_g T_o} \qquad (A1)$$

Approximating p_d by dP/dz Eq. (A1) can be solved to obtain

$$P(z) = P_o e^{-2z/v_g T_o} \qquad (A2)$$

Here P_o is the section input power. From (1), (2) and (A2)

$$E = \sqrt{\frac{sP}{v_g}} = E_o e^{-z/v_g T_o}\ , \quad E_o = \sqrt{\frac{sP_o}{v_g}} \qquad (A3)$$

Here E and E_o are the accelerating gradient as a function of z and at the section input respectively. We integrate Eq. (A3) and obtain

$$V_o = \int_0^L E e^{-z/v_g T_o} = \left(\frac{1-e^{-T_f/T_o}}{T_f/T_o}\right)\sqrt{sP_oT_fL} = \sqrt{\eta_s s P_o T_f L} \qquad (A4)$$

APPENDIX B: BEAM INDUCED VOLTAGE

From consideration of conservation of energy and using $P = (v_g/s)E^2$

$$\frac{dP}{dz} = I_o\sqrt{\frac{s}{v_g}P} - \frac{2P}{v_g T_o}\ . \qquad (B1)$$

$$P = \frac{I_o^2 T_o^2 s v_g}{4}(1 - e^{-z/v_g T_o})^2 \qquad (B2)$$

$$E = \frac{I_o T_o s}{2}(1 - e^{-z/v_g T_o}) \qquad (B3)$$

$$V_b = \frac{I_o T_o s}{2}[z - v_g T_o(1 - e^{-z/v_g T_o})] \qquad (B4)$$

$$V_b = 2\left[\frac{1}{\tau} - \frac{1-e^{-\tau}}{\tau^2}\right]\frac{I_o T_f s L}{4} \qquad (B5)$$

ACKNOWLEDGEMENT

I am grateful to P. Wilson for his help and to S. StLorant for editing this note.

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