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# On the Choice of Positron-Producing Energy in Linac-Injected $e^{\pm}$ Colliders

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## Summary

The optimum positron-producing energy in linac injected $e^{\pm}$ colliders is investigated from the averge luminosity point of view. It has been found that with a fixed injector energy, $E_t$, there exists a minimum injection time at $E_+/E_t=1/4$, where $E_+$ is the positron-producing energy. This corresponds to the maximum of average luminosity when no single beam instability occurs. On the other hand, when the storable currents are limited by fast head-tail instability, the minimum injection time takes place at $E_+/E_t=1/3$ while the maximum of average luminosity shifts to lower value. These results can be understood because, besides the two competing physical processes, i.e. the number of positrons produced favors higher $E_+$ and the damping time favors lower $E_+$, now lower $E_+$ gives higher instability threshold. For numerical calculations, the parameters of Beijing Electron Positron Collider (BEPC) are used.


## Introduction

Among $e^{\pm}$ colliders with linac as injectors, the choices of positron-producing energy are widely different. So, there arises the question whether there is an optimum value from the average luminosity point of view for a fixed investment on the linac with less than full energy injection.

## General Remarks[1][2][3]

From basic definition, the luminosity is given by, under usual simplified conditions:

$$\hat{L} = \frac{I^2}{4\pi f e^2 \varepsilon_x \beta_y^*} \tag{1}$$

where one bunch per beam and equal positron and electron currents, I, are assumed. The current is limited by the maximum allowable beam-beam tune-shift:

$$\Delta Q_{x,y} = \frac{r_e}{2\pi f e}\frac{I}{\gamma \varepsilon_x} \leqq \text{Const.} \tag{2}$$

where f is the revolution frequency of the beams, $\varepsilon_x$, horizontal beam emittance, $\beta_y^*$, vertical $\beta$ function at interaction point. $r_e$, classical electron radius and $\gamma=E/m_\circ c^2$. So

$$\hat{L} = \frac{\pi f}{r_e^2}(\Delta Q)^2 \varepsilon_x \gamma^2 \frac{1}{\beta_y^*} \tag{3}$$

Equs (2) and (3) mean that if the emittance is held constant with energy the currents stored will vary as $\gamma$ and the luminosity as $\gamma^2$. Because of different loss mechanisms, beam currents, I, decay approximately according to

$$I = I_\circ e^{-t/T} \tag{4}$$

where T is the overall beam lifetime. Substitute (4) into (1):

$$\hat{L}(t) = \frac{I_\circ^2}{4\pi f e^2 \varepsilon_x \beta_y^*} e^{-2t/T_\circ} = \hat{L} e^{-2t/T_\circ} \tag{5}$$

the average luminosity, by integration is

$$\bar{L} = \hat{L}\left(\frac{\tau_c}{\tau_i+\tau_c}\right)\left(\frac{1-e^{-2\tau_c}}{2\tau_c}\right) \tag{6}$$

where $\tau_c=T_c/T_\circ, \tau_i=T_i/T_\circ; T_c, T_i$ are collision time and injection time respectively and $\hat{L}$ the peak luminosity at the begining of collision.

From equ.(6), one obtains that when

$$e^{-2\tau_c} = \frac{1}{1+2(\tau_i+\tau_c)} \tag{7}$$

The average luminosity has a maximum:

$$\bar{L}_{max} = \hat{L} e^{-2\tau_c} \tag{8}$$

We shall use suffix f.e. to stand for fixed emittance meaning the emittance is not changed with energy through the use of wiggler magnets. Other mode of operation, such as maintaining the beam-beam tune shift constant through adjusting $\varepsilon_x$ for different currents also exists. However, in order to save space, only f.e. case will be given as illustration.

## Optimization of Average Luminosity at the Designe Current —— no Single Beam instability

Now assume there is no single beam instability limiting the current that can be stored in the ring. Let us consider the question for an injector with total energy(or investment) fixed, what is the optimum positron-producing energy corresponding to maximum $\bar{L}_{max}$ ?

The injection time $T_i$ composes of three parts which can be expressed as the following when the operating energy $E_{opt} >$ the injection energy $E_i$

$$T_i = \frac{N_b}{Y(E_+)n_-\eta_+} T_x + \frac{N_b}{n_-\eta_-} T_x + T_r \tag{9}$$

where $T_r$ is the combination of switching time, magnet standardization time and ramping time. Y is the number of positrons captured for acceleration per incident electron per GeV energy, $N_b$ is the required number of particles per beam, $n_-$ is the number of electrons per linac pulse, $\eta_\pm$ is the storage ring capture efficiency for positron and electron respectively, taking as practically constant over the energy range consered, and $T_x$ is the transverse damping time related to lattice parameters by, using well known notations,

$$T_x = \frac{4\pi}{C_\gamma c}\frac{R\rho}{E_i^3} = \frac{K}{E_i^3(\text{Gev})} \tag{10}$$

Y for a concrete installation, depends on targeting, focusing, bunching and geomatrical arrangments. However, there are theoretical and experimental evidences that it is practically a constant. Chehab etc.[4] give the experimentally measured result of $3.3\times10^{-2} e^+/e^-$. Gev for one Gev incident beam, while the theoretical result of James etc.[5] is $2.7\times10^{-2} e^+/e^-$ Gev for 1 Gev beam and $3.9\times10^{-2} e^+/e^-$ Gev for 80 Mev beam. BEPC preliminary design [6][7] gives a calculated figure of $3.1\times10^{-2} e^+/e^-$ Gev for 370 Mev beam. For the following analysis, we will take it as a constant.

Equ. (9) can be written as:

$$T_i \simeq (N_b T_x / Y E_+ n_- \eta_+) + T_r \tag{11}$$

Substitute (10) into (11) and remembering $E_t=E_+ + E_i$, one has

$$\tau_i = (N_b K / T_\circ Y n_- \eta_+ E_t^4 (E_+/E_t)(1-E_+/E_t)^3) + \tau_r \tag{12}$$

when

$$E_+ = E_t/4 \tag{13}$$

$\tau_i$ assumes a minimum given by

$$\frac{(\tau_i)_{min} - \tau_r}{N_b K/T_\circ n_- \eta_+ Y E_t^4} = \frac{4^4}{3^3} \tag{14}$$

Fig. 1 (a) shows the variation of $\tau_i$ vs $E_+/E_t$.

With $\tau_i$ given by eq. (12), one may solve for $\tau_c$ according to eq. (7) and calculate the ratio of $\bar{L}_{max}/\hat{L}_\circ$ by:

$$\bar{L}_{max}/\hat{L}_\circ = e^{-2\tau_c} \left(\frac{E_{opt}}{E_\circ}\right)^2 \tag{15}$$

$\hat{L}_\circ$ stands for peak luminosity as the designed energy.

The results obtained are under the assumption that there is no single beam instability. In most cases, the storable current is injection energy dependent. This will be treated in the following section.

## Optimization of Average Luminosity When Beam Currents are Limited by Fast Head-tail Instability

The fast head-tail instability is generally considered as the limiting factor for storable currents especially when injection energy is low. The threshold of fast head-tail instability is difficult to calculate accurately. In the following, an approximate treatment based upon scaling from spear's result[8][9] will be discussed.

For machine similar to SPEAR, say BEPC, one can write

$$I_{th}(\text{BEPC}) = \left(\frac{E_i Q_s}{\beta_1 R_1 + \beta_2 R_2}\right)_{\text{BEPC}} \times \left(\frac{\beta_1 R_1 + \beta_2 R_2}{E_i Q_s}\right)_{\text{SPRAR}} \times I_{th}(\text{SPEAR}) = kE_i \tag{16}$$

where $E_i$ is the injection energy, $Q_s$ is the phase oscillation frequency assuming kept constant, $\beta_1 \beta_2$ the average $\beta$ value at R.F. cavity and the average vertical $\beta$ repectively, $R_{1,2}$ the transverse impedance of R.F. cavity and vacuum chamber respectively.

the injection time expression (12) now becomes:

$$\tau_i = \frac{N_b K}{T_\circ n_- \eta_+ Y E_t^3} \frac{k}{I_\circ} \frac{1}{x(1-x)^2} + \tau_r \tag{17}$$

Where x is used to denote $E_+/E_t$.

By differentiation, one obtains the condition for minimum injection time with fast head-tail instability limitation:

$$E_+ = E_t/3 \tag{18}$$

and

$$\frac{(\tau_i)_{min} - \tau_r}{\frac{K}{I_\circ}\left(\frac{N_b K}{T\, n_- \eta_+ Y E_t^3}\right)} = \frac{3^3}{2^2} \tag{19}$$

Fig.1 (b) shows the variation of $\tau_i$ vs $E_+/E_t$ for this case. Now, the peak luminosity corresponding to different injection energy can be expressed as :

$$\hat{L}_{f.e.} = \hat{L}_\circ \left(\frac{kE_i}{I_\circ}\right)^2 \tag{20}$$

Substitute the above into eq. (6), one obtains the expressions of average luminosity for fixed emittance as:

$$\bar{L}_{f.e.} = \hat{L}_\circ \left(\frac{kE_t}{I_\circ}\right)^2 (1-x)^2 \left(\frac{\tau_c}{\tau_i + \tau_c}\right)\left(\frac{1-e^{-2\tau_c}}{2\tau_c}\right) \tag{21}$$

Give x value, one may calculate the corresponding $\tau_i$ when machine parameters are given. And by assuming different $\tau_c$ values, one can obtain the maxmum average luminosity as a function of $E_+/E_t$.

## Numerical Computations

Taking BEPC design parameters for example: T=6.7 hrs, $T_r$=24 min, $\tau_r$=0.06 , K=0.187 (sec Gev$^3$), $N_b$=3.3x$10^{11}$ x $E_{opt}$/2.8 , $n_-$=1x0.5x6.2x$10^{18}$ x2.5x$10^{-9}$=7.8x$10^9$ , $\eta_+$=10% , Y=0.025 $e^+$/$e^-$ Gev, $N_b K/T_\circ Y n_- \eta_+$=0.131 $E_{opt}$/2.8 (Gev$^4$).

The total linac energy is related to the klystron out-put power approximately by

$$E_t(\text{Gev}) = 0.405\sqrt{P(\text{MW})} \tag{22}$$

Substitute the above numbers in (12), one can find $\tau_i$ corresponding to different $E_+/E_t$ and then from eq. (7), one may solve for $\tau_c$. If we use $\hat{L}_\circ$ to denote 1.7 x$10^{31}$ cm$^{-2}$s$^{-1}$, the designed luminosity at 2.8 Gev, one can finally get $\bar{L}_{max}/\hat{L}_\circ$ according to eq.(15) as a function of $E_+/E_t$ for the case of no single beam instability as given in fig. 2. It can be seen at $E_+/E_t$=1/4, all curves have a maximum. However, one should note the variation of these curves are generally rather slow and $\bar{L}_{max}/\hat{L}_\circ$ decreases only slightly for $E_+/E_t$ values lower than 1/4. For example, if $E_+/E_t$ is chosen as 1/10 instead of 1/4, the reduction of $\bar{L}_{max}/\hat{L}_\circ$ is only a few percent for most cases while the gain in injection energy is 20%. What ratio $E_+/E_t$ we should chose is indeed worth of careful consederation even for the case of no single beam instability. Taking fast head-tail instability into consideration, one may calculate the current threshold according to eg.(16) by substituting the following numbers. For SPEAR[8][10]: E=1.5 Gev, $Q_s$=0.035, $\beta_1$=14m, $\beta_2$=20m, $R_1$=0.3R, $R_2$=0.7 R (R is the total transverse impedance) and the measured $I_{th}$ (SPEAR)=5ma. For BEPC[7] : $Q_s$=0.02, $\beta_1$=8m, $\beta_2$=12m, and vacuum chamber impedance can be assumed to be 1/7 of Spear's, R.F. cavity impedance, 1/10 of Spear's. Here $Q_s$ is assumed to be practically a constant through computer control to avoid synchro-beta resonance.

Then one obtains k=24. Substituting 66ma as the designed current $I_\circ$ at 2.8 Gev. in eq.(17), one may calculate $\tau_i$ as fuction of $E_+/E_t$ and finally find maximum average luminosity for f.e. case from eq. (21) after $\tau_c$ is optimized. The results of these calculations are given in Fig. 3. It can be seen for the case that beam currents are limited by head-tail instability, the maximum of $\bar{L}_{max}$ occurs near $E_+/E_t$=1/10 for most cases with klystron output power varies between 16 MW and 32MW.

The above result might be explained by the following argument. When not limited by single beam instability, two competing processes take place to determine the optimum positron producing energy. The number of positron produced favers high positron producing energy $E_+$ but the damping time consideration perfers lower $E_+$, i.e. higher injection energy. When stored currents are limited by fast head-tail instability, another factor comes into play, i.e. lower $E_+$ given higher current threshold. Therefore, the maximum average luminosity moves to lower energy region.

## Variation of Parameters

Results of the above analysis depend on the mumerical values assumed for various parameters and it is not at all sure that these numbers represent the real operating conditions. Therefore, the assumed parameters are varied from typical situation in order to investigate the range of validity of the above analysis.

The injection time eq.(17) can then be written as

$$\tau_i = \frac{N_b K}{T_\circ n_- \eta_+ Y E_t^3} \left(\frac{k}{I_\circ}\right) \frac{1}{x(1-x)^2} \frac{(1+a)}{(1+b)(1+c)} + \frac{T_r(1+d)}{T_\circ(1+c)} \tag{23}$$

with a,b,c and d to take care of the variations in fast head-tail threshold, $Y\eta_- \eta_+$ , $T_o$ and $T_r$ respestively by assuming values of -0.5 and 1.

The result of computation for each variation of those parameters for a typical operating condition with 16MW power input to the accelerator are given in Fig. 3 and Fig.4. It can be seen from these curves the variation of parameters does not chang the general behavior because the functional dependence remains unchanged. Furthermore, according to these figures. The effect of different parameters can be readily evaluated.

## Conclusions

Admitted the above is a simplified analysis of a complicated situation, however it seems to indicate the advisability of choosing $E_+/E_t \sim 1/10$ for BEPC for the following reasons:

1) When stored current is not limited by single beam instability, the minimum injection time and maximum average luminosity occur at $E_+/E_t=1/4$ with flat variation. Thus if one choose $E_+/E_t=1/10$, the loss in average luminosity is insignificant while one gains 20% in injection energy.

2) When stored current is limited by fast headtail instability, then minimum injection time occurs at $E_+/E_t=1/3$. However, because of the fact that stored currents depend on injection energy, the maximum averge luminosity occurs around $E_+/E_t \sim 1/10$. In BEPC, for klystron power output of 32MW, the choice of $E_+/E_t \sim 1/10$ represents an improvement in average luminosity over $E_+/E_t=1/4$ by as much as 36% for f.e. case and at the same time the injection energy will increase by 20% and one can have full energy injection over wider range.

3) The beam power at the positron-producing target is reduced and thus radiation demage,cooling and radiation shielding problems, will also be alleviated.

4) Most instabilities are more readily damped because of higher injection energy.

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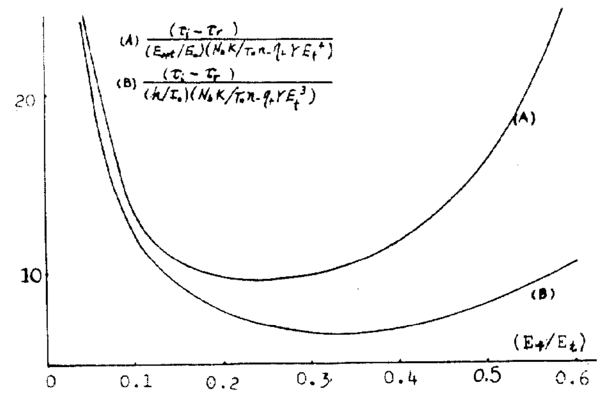


Fig.1 Universal injection time VS $E_+/E_t$ curve

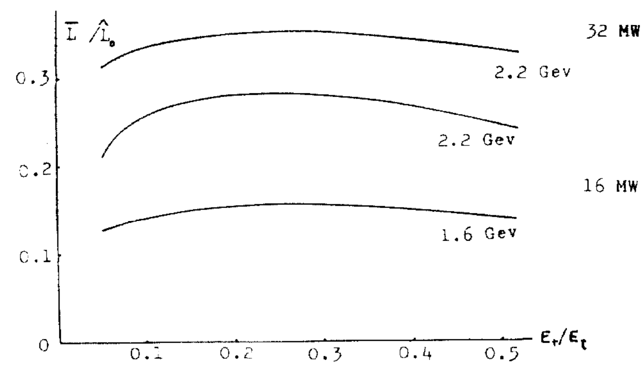


Fig.2 Variation of $\bar{L}/\hat{L}_o$ for BEPC when currents are not limited by single beam instability

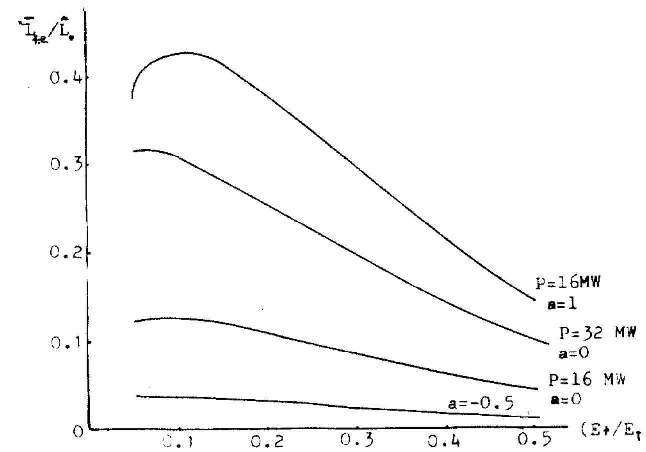


Fig. 3 Variation of $\bar{L}_{f.e.}/\hat{L}_o$ VS $E_+/E_t$ for different fast head-tail instability threshold

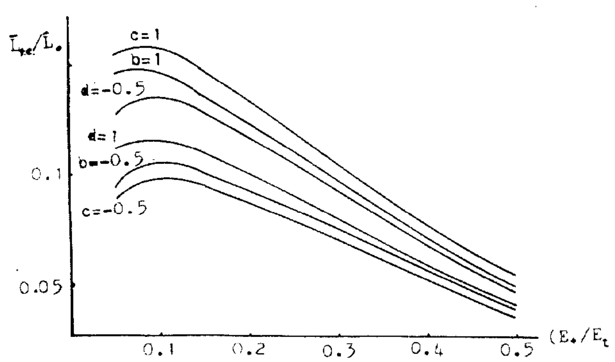


Fig. 4 Sensitivity of $\bar{L}_{f.e.}/\hat{L}_o$ to parameter variations (P=16 MW)