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BANDWIDTH BROADENING IN RF-STRUCTURE

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Summary

Broad bandwidth represents one of the features the most sought after in electron tubes. In linear accelerators, it ensures the insensitiveness of the RF-structure to mechanical error or beam loading. This is commonly achieved by increasing the coupling between accelerating cells. To go further however, coalescence between two passbands or more is being used. The post-coupled Alvarez structure, the side-coupled or resonant slot-coupled cavity chain, as well as the dish-and-washer structure, among others, are well known examples. More generally, coalescence can take place in complex RF-structures such as multiperiodic systems. Presently, the use of mode coalescence is extended to certain types of RF-structures in TWTs in order to avoid band-edge oscillations. The purpose of this paper is to derive rules that mode coalescence obeys. Examples will be given for illustrations.

Introduction

In this paper we will discuss the coalescence mechanism in RF-structures for Linacs or broadband electron tubes. By definition, coalescence appears when there is degeneracy between two orthogonal modes.

It may occur that coalescence is an intrinsic property of a periodic structure as in the interdigital structure in the symmetrical case, which can be seen as an interlaced two-circuit system. Sometimes, in cross-field tubes, by a design need such as cooling requirements, the interdigital structure must be built assymmetrical ; coalescence is then realized by appropriate dimensioning.

In general, coalescence is a situation mostly sought after for the following purposes :

- field flattening in long accelerating cavities
- bandwidth broadening in electron tubes
- oscillation damping in broad-band amplifiers.

The first two purposes are well known, the third is only recently applied to very broad band circuits.

It is known that amplification can only take place when the velocity of the RF-wave exhibits a certain drag compared to the electron beam velocity (there is then transfer of energy from beam to circuit). If the bandwidth of an amplifier is wide enough, the straight line representing electron velocity in the (ω, β) plane (figure 1a) and an edge of the pass-band may come close enough to favor oscillations. There are two reasons for that : i) at a band edge, the dispersion curve is usually very flat, making the velocity condition of oscillation very easy to satisfy ; ii) because coupling impedance increases as the inverse of the group velocity, which is null at the edge, losses are insufficient to damp oscillation growth. Coalescence will improve this situation, figure 1b.

The coalescent modes are orthogonal, in the sense that if one mode is well coupled to the electron beam and hence is dangerous, the other has no effect on the beam. Therefore, if coalescence is surpassed so that dangerous modes are thown over the upper side of the velocity straight line, oscillations are suppressed, figure 1c.

We will first try to derive the rules coalescence must obey and then discuss how to use them to synthetize circuits.

Two Circuit System

Coalescence takes place only when at least two circuits are coupled together. We consider this simple case first. The equations of propagation can be written as :

$$\begin{cases} \Delta\, \psi_1 + k_1^2\, \psi_1 + D_{12}\, \psi_2 = 0 \\ \Delta\, \psi_2 + k_2^2\, \psi_2 + D_{21}\, \psi_1 = 0 \end{cases} \qquad [1]$$

The scalars k_1^2 and k_2^2 are the eigenvalues of the uncoupled circuits, Δ is a second order operator, and D_{12} and D_{21} are two first order operators representing coupling between circuits. If these latter are supposed to be made of discrete resonating cells, separated by equally discrete coupling elements, i.e when the phase shifts are assumed to vary stepwise through the coupling hole, the coupled system can be represented by two interlaced systems as shown in figure 2. The graph has either a square- or a treillis-shape depending on the field symmetry, or parity, in the cell at the considered mode . ε, equal to + 1 or -1, accounts for the sign of the coupling coefficient (-1 for coupling between graphs of different parity and + 1 for same parity) and g, for the dissymmetry of the coupling on each side of the cell (g = 1, when coupling to the left is identical to coupling to the right and $g \neq 1$ otherwise). Coupling coefficients are represented by the scalars f_{12} and f_{21} which, in general, are neither equal nor of the same sign. In these conditions, Δ, D_{12} and D_{21} can be written in the following finite difference forms, where n is the cell number :

$$\begin{cases} \Delta\psi(n) = \frac{1}{2}\,[\psi(n-1) + \psi(n+1) - 2\psi(n)] \\ D_{12}\,\psi_2(n) = f_{12}\,[\varepsilon g\, \psi_2(n-1) + \psi_2(n)] \\ D_{21}\,\psi_1(n) = f_{21}\,[\psi_1(n) + \varepsilon g\, \psi_1(n+1)] \end{cases} \qquad [2]$$

Note that if instead of being interlaced the circuits are coupled in parallel, the D's are reduced to pure scalars.

Before solving Eqs. [1] and [2], let's consider the uncoupled system, i.e., $f_{12} = f_{21} \equiv 0$. Applying Floquet's theorem, one has :

$$\psi(n \pm 1) = \psi(n) \exp(\mp j\beta L), \qquad [3]$$

where βL is the phase shift of one period. Eq. [1] gives two uncoupled solutions ψ_1 and ψ_2 with :

$$\begin{cases} F_1(\omega) \equiv \cos \beta_1 L = 1 - k_1^2\ (\omega) \\ F_2(\omega) \equiv \cos \beta_2 L = 1 - k_2^2\ (\omega) \end{cases} \qquad [4]$$

$F_1(\omega)$ and $F_2(\omega)$ are defined as the dispersion functions of the uncoupled system and are assummed to be known.

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If the dispersion function of the coupled system is defined as $F(\omega) \equiv \cos\beta L$, then by using definition (4), Eq. [1] gives the following homogenous system :

$$\begin{pmatrix} F(\omega)-F_1(\omega) & f_{12}\,[\varepsilon g \exp(j\beta L)+1] \\ f_{21}\,[1+\varepsilon g \exp(-j\beta L)] & F(\omega)-F_2(\omega) \end{pmatrix} \begin{pmatrix} \psi_1(n) \\ \psi_2(n) \end{pmatrix} = 0 \quad [5]$$

where $F(\omega)$, ψ_1 and ψ_2 are unknown. By cancelling the determinant of Eq. [5], a second degree equation in $F(\omega)$ is obtained, giving two solutions F_+ and F_- :

$$F_\pm(\omega) = \frac{1}{2}(F_1+F_2)+\varepsilon g f \pm \sqrt{\frac{(F_1-F_2)^2}{4}+f[fg^2+\varepsilon g(F_1+F_2)+1+g^2)} \quad [6]$$

where $f = f_{12}\, f_{21}$. Thus the coupling coefficients appear only in the form of their product. The ratio of the amplitudes ψ_1/ψ_2 can be calculated easily.

Relation [6] gives the composition rule. Without further precisions on the parameters, certain conclusions can be drawn, in particular, on the coalescence. It is known that coalescence occurs when the dispersion curves join each other at zero- or π-mode with nonvanishing group velocity.

After the definition of $F(\omega)$, one has :

$$\begin{cases} \dfrac{dF}{d\omega} = -\sin\beta L \dfrac{L d\beta}{d\omega} \\ \dfrac{d^2F}{d\omega^2} = -\cos\beta L \left(L\dfrac{d\beta}{d\omega}\right)^2 - \sin\beta L\, L \dfrac{d^2\beta}{d\omega^2} \end{cases} \quad [7]$$

hence, at the coalescent point,

$$\begin{cases} \dfrac{dF}{d\omega} = 0 \\ F = \pm 1 \\ V_g = \dfrac{d\omega}{d\beta} = \pm L/\sqrt{\left|\dfrac{d^2F}{d\omega^2}\right|} \end{cases} \quad [8]$$

By differentiating [6] one has :

$$\frac{dF_\pm}{d\omega} = \frac{1}{2}\left[1 \pm \frac{\varepsilon f g}{\sqrt{\sim}}\right]\frac{d}{d\omega}(F_1+F_2) + \left[\varepsilon g \pm \frac{2fg^2+1+g^2+\varepsilon g(F_1+F_2)}{2\sqrt{\sim}}\right]\frac{df}{d\omega} \pm \left[\frac{F_1-F_2}{2\sqrt{\sim}}\right]\frac{d}{d\omega}\left(\frac{F_1-F_2}{2}\right) \quad [9]$$

where $\sqrt{\sim}$ stands for the term under square root sign of Eq.[6].

Eq. [8] are satisfied if the three terms in brackets of Eq. [9] vanish simultaneously, giving a sufficient condition :

$$\begin{aligned} & g = 1 \\ & F_1 = F_2 = F = -\varepsilon \end{aligned} \quad [10]$$

As a result, if the coupling is identical at both sides of the cell, i) coalescence occurs at π-mode ($F_- = -1$) when fields in the cells are of the same parity and ii) at zero-mode ($F_+ = 1$) when fields in the cells are of different parity.

Other interesting properties of the dispersion curves can be derived from Eq. [9]. In particular, at the frequency that cancels the term under the square root sign, the branches $F_+(\omega)$ and $F_-(\omega)$ join each other with vertical slope. According to whether the common value of $F_\pm$ at this point lies in the interval [-1, + 1] or not, the dispersion curve has or not a zero group velocity in the pass-band.

Different situations are shown in figure 3a and 3b. The case of zero group velocity lying in the pass-band is observed in many cases, particularly in interdigital structures[1], in iris-loaded deflecting structures[1] and also in disk and washer (DAW) structures[2]. This phenomenon depends mainly on the value of the coupling coefficient. Let's consider a simple case where $F_1(\omega) \equiv F_2(\omega)$ (case of the interdigital structure or of the DAW structure, when the uncoupled disk- and iris- structures have, for certain disk and iris dimensions, identical dispersion curves). The common value of $F_\pm$ is $(-1+\frac{1}{4}f)\varepsilon$, where f is positive, as $f_{12} = f_{21}$ x ; therefore, if the coupling is made small enough ($f<4$), the group velocity can be zero inside the pass-band. Of course, this situation must be avoided because of the poor mode separation and, more important, because of the high risk of band-edge oscillations in amplifier electron tubes.

System of More Than Two Circuits

Though, in general, analytical solutions cannot be easily obtained, the coupled equations have the same forms :

$$\begin{pmatrix} L & D_{12} & D_{13} & \cdots \\ D_{21} & L & D_{23} & \cdots \\ D_{31} & D_{32} & L & \cdots \end{pmatrix} \psi = 0 \quad [11]$$

where $L = \Delta + k^2$. If circuits are ranked following the frequency range of their pass-bands, coupling can be neglected between non adjacent bands and the matrix becomes sparse enough to be treated easily. Anyhow, locally, the rule stated for a two-circuit system remains valid.

Let's consider the three circuit case. Figure 4 shows how the bandwidth can be broadened by coalescence between the main circuit no. 2 with the two coupling circuits no.1 and no. 3. The first coalescence occurs at π-mode and the second, at zero- or 2π-mode. From these simple rules, methods can be derived for the design of broad-band circuits.

Some Examples of Mode Coalescence

Two well known examples are given by the biperiodic side-coupled structure and the post-coupled drift tube structure. In the first case, the modes in the accelerating cell and in the coupling cell are of the same TM01-type, hence, the coalescence will take place at π-mode as one knows. In the second case where coalescence takes place at zero-mode, it can be verified that the field excited by two adjacent stems has an odd pattern (longitudinal electric field is zero at the middle of the cell) whereas the TM01-field in the drift tube cell is obviously even.

Another example is given by a cavity chain coupled with resonant slots. It is known that coalescence can take place only at zero-mode and if adjacent slots are alternating with 180° rotation. Note that condition [10] is only sufficient, that means, it ensures coalescence only when coalescence is possible or, in other words, when it is possible to excite orthogonal modes in the circuits. In the case of aligned resonant slots for example, any kind of coalescence can be realized.

We now consider the coalescence of the DAW-structure shown in figure 5 in is simplest shape. It can be seen as an interlacing of an iris-loaded structure and its dual disk-loaded structure. Patterns of the two π-mode orthogonal fields are shown. One can see that the broken line field is excited by the irises only and is not perturbed by the disks, whereas the solid line field is excited by the disks only and is not perturbed by the irises, provided of course that the disks and irises are infinitely thin. Note that each field is built up by a couple of orthogonal boundary conditions, as better shown in figure 6a. In this case, the two TM01 fields are excited either with Neuman conditions on the left with Derichlet conditions on the right (ND) or vice versa (DN). Of course coalescence occurs when the two frequencies are equal. This example is chosen to show that the orthogonal modes need not necessarily be of the same type. If indeed the cell length is small enough (figure 6b), the frequency of the DN-TM01 becomes so high that coalescence is no longer realized between two TM01 fields but between a ND-TM01 and a DN-TM02.

The last example is intended to show how to synthesize a periodic structure for a given dispersion curve using the coalescence mechanism. Suppose one has to realize a dispersion curve as shown in figure 4, curve no. 4, such that it remains close to a straight line in a frequency range as large as possible. The solution could be a three-circuit system. As stated by the rule, if the main circuit has an accelerating even field, the high frequency coupling circuit must have an odd field and the low frequency one, an even field as the main circuit. In the structure shown in figure 7, high frequency coupling is achieved by resonant slots and low frequency coupling by side cavities. Curves 1, 2 and 3 in figure 4 show three dispersion curves corresponding to the three circuits before and after coalescence.

Note that, to convert the forward-wave dispersion curve no. 4 of figure 4 into a backward-wave curve, one has only to interchange the frequencies of the coupling circuits, by opening the slots and decreasing the volume of the side cavities. The slope of the dispersion curve no. 2 of the main circuit also changes its sign due to the enhanced coupling of the enlarged slots.

References

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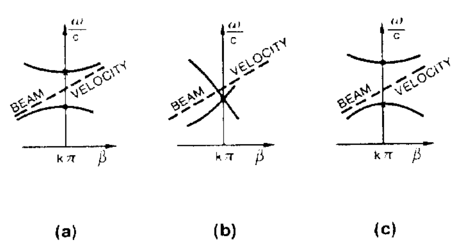

Figure 1 - Band-edge oscillation suppression by mode coalescence

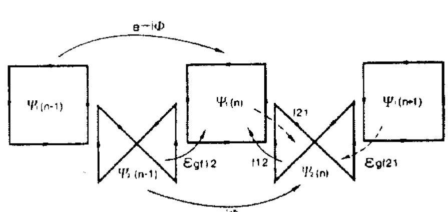

Figure 2 - Two-circuit system

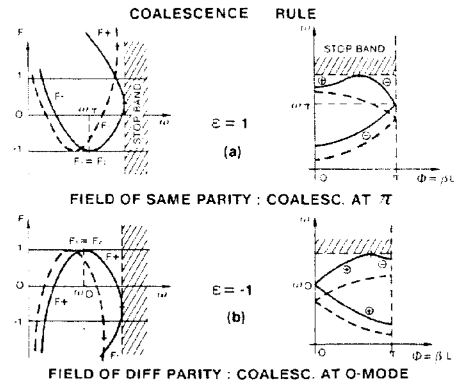

Figure 3 - Coalescence rule .
Field of same parity : π-mode coalescence
Field of different parity : 0-mode coalescence

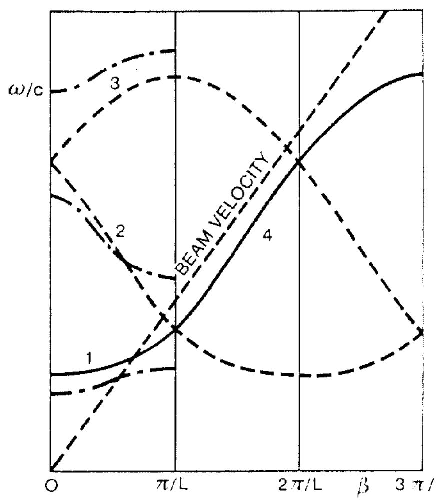

Figure 4 - Dispersion curve of a three-circuit system at coalescence

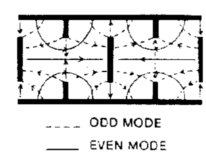

Figure 5 - Odd and even π-modes of DAW

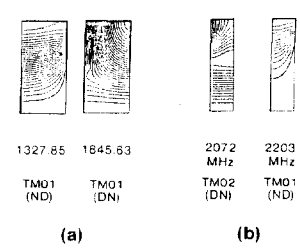

Figure 6 - Mode coalescence in DAW

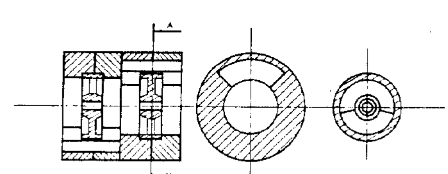

Figure 7 - Three-circuit broad-band structure