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AN RFQ RESONATOR MODELING COMPUTER PROGRAM*

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Summary

The mathematical background for a multiport-network-solving program is described. A method for accurately numerically modeling an arbitrary, continuous, multiport transmission line is discussed. A modification to the transmission-line equations to accommodate multiple rf drives is presented. An improved model for the radio-frequency quadrupole (RFQ) accelerator that corrects previous errors is given. This model permits treating the RFQ as a true eight-port network for simplicity in interpreting the field distribution and ensures that all modes propagate at the same velocity in the high-frequency limit. The flexibility of the multiport model is illustrated by simple modifications to otherwise two-dimensional systems that permit modeling them as linear chains of multiport networks.

Introduction

The transmission-line model has contributed much toward understanding the azimuthal and longitudinal field-distribution properties of the RFQ accelerator and has been a useful guide for developing tuning procedures. Until the appearance of a program by Ron Hutcheon of Chalk River,[1] much of the effort toward understanding RFQ rf properties was analytical and qualitative. Hutcheon's program calculates the field distribution for an RFQ with parameters that are arbitrary continuous functions of the longitudinal coordinate by numerically integrating the transmission-line equations for a six-port transmission line.

The program described here uses the 2N-port chain-matrix accelerator-modeling technique developed at Los Alamos.[2] This method facilitates the inclusion of discrete components, multiple rf drives, and rf power losses. In addition, the code is not limited to a particular model or to the RFQ structure. It can be used for any structure that may be modeled as a chain of 2N-port networks and transmission lines. One restriction is that transmission lines with continuously varying parameters must be represented by a series of uniform line segments, approximating the desired variation in parameters.

The Matrix Representation

In the chain-matrix representation, a network with 2N ports is considered to have N input ports and N output ports. The matrix M that relates the voltages and currents at the output ports to the voltages and currents at the input ports is the chain matrix of the network. In the notation used here, the voltages and currents are represented by a 2N-vector amplitude. The first N elements of the vector are the voltages at a set of N ports and the second N elements are the corresponding currents. Thus, for example, the i-th amplitude vector will be

$$\begin{pmatrix} V_i \\ I_i \end{pmatrix} . \qquad (1)$$

It is convenient to consider the matrix M to be composed of the NXN submatrices A, B, C, and D:

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} . \qquad (2)$$

These conventions result in the equations for a 2N-port network being analogous to the two-port case with the individual variables and parameters replaced by vectors and matrices.

With this analogy in mind, we will write, without further justification, the matrix transmission-line equations for a 2N-port network: $dV/dx = ZI$ and $dI/dx = YV$. Here Z is the series impedance per unit length, and Y is the shunt admittance per unit length. Both Z and Y are 2NX2N symmetric matrices.

Our conventions are that the input ports are on the right, the output ports on the left, the transmission line extends in the -x direction, and the current is positive when it flows to the right (the +x direction).

To develop the chain-matrix method for a continuous transmission line, we consider the difference equations corresponding to the transmission-line equations. If we write these equations in the form that relates the vector amplitude at $x + \delta x$ to the amplitude at x, we get the chain-matrix representation for an increment of line of length δx.

$$\begin{pmatrix} 1 + ZY\delta x^2/2 & Z\delta x + ZYZ\delta x^3/4 \\ Y\delta x & 1 + YZ\delta x^2/2 \end{pmatrix} \qquad (3)$$

The order of the terms, in powers of δx, has been chosen so that the determinant of the matrix is equal to 1, as required for conservation of energy. The matrix above is equivalent to a T-network with series impedance 0.5 $Z\delta x$ and shunt admittance $Y\delta x$.

Although the transmission-line equations are formally integrable, it is more useful for numerical analysis to let δx be small but finite. If we choose $\delta x = \ell/n$, we can approximate the matrix for a continuous line of length ℓ by calculating $M(\ell/n)^n$.

For a sufficiently small δx, this expression will yield a satisfactory result for the matrix of the finite-length transmission line. However, there will be a lower limit on the size of δx because of truncation errors in computer arithmetic. In addition, for small δx, the number of matrix multiplications can become impractically large. To improve the speed of calculation while allowing a small increment δx, we have chosen n to be an integer power of 2. Then, the matrix for the finite-length transmission line can be calculated by a series of squaring operations and the number of matrix multiplications becomes $\log_2(n)$.

Other components of the accelerator structure, such as shorting rings and end tuners, can be represented by combinations of discrete elements of series impedance and shunt admittance. The discrete model is useful for components with lengths short compared to a wavelength. Any linear combination of discrete and distributed elements can be combined by matrix multiplication to determine the matrix for a complete rf structure. For example, the combination of elements illustrated in Fig. 1 has an overall matrix.

To solve the amplitude distribution problem, boundary conditions, including an rf drive if losses are present, are applied and the matrix equation is solved for the unknowns. Take the example of Fig. 1 with short-circuit boundaries at both ends and a voltage drive V_D at the left end. The matrix equation is

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$$\begin{pmatrix} V_D \\ I_n \end{pmatrix} = M \cdot \begin{pmatrix} 0 \\ I_0 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} 0 \\ I_0 \end{pmatrix} = \begin{pmatrix} B \cdot I_0 \\ D \cdot I_0 \end{pmatrix} \quad . \qquad (4)$$

Solving for the unknown current vectors yields

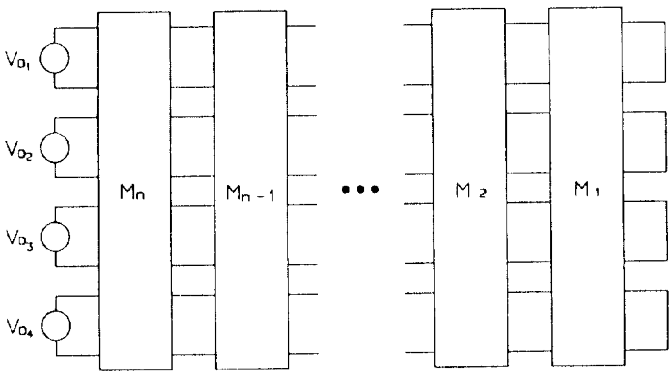

Fig. 1. Chain of eight-ports with short circuit boundaries and voltage drive.

$$I_0 = B^{-1} \cdot V_D \text{ , and } I_n = D \cdot I_0 = D \cdot B^{-1} \cdot V_D \quad . \qquad (5)$$

$$M = M_n \cdot M_{n-1} \cdots M_2 \cdot M_1 \quad .$$

where B^{-1} is guaranteed to exist if the input ports are actually connected to the output ports through finite impedances and nonzero admittances. After solving for the boundary currents, the intermediate amplitudes may be calculated:

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = M_1 \begin{pmatrix} 0 \\ I_0 \end{pmatrix} \qquad \begin{pmatrix} V_2 \\ I_2 \end{pmatrix} = M_2 \cdot M_1 \begin{pmatrix} 0 \\ I_0 \end{pmatrix} \quad \text{etc.} \qquad (6)$$

For a lossless network, the voltage drive V_D must be zero. The equations are then solved for the eigenfrequencies (modes) of the network before the relative amplitudes can be calculated. The 2N-port program assumes that there will always be losses that are due to resistive components in some or all of the network's elements. Thus, for any finite drive, there will always be a finite response whose phase with respect to the drive depends on the drive frequency relative to the modes of the structure. The program requires some definition of resonance to find a mode. The usual method for one drive is to require that the response at the drive port be in phase with the drive. For multiple drives, a more sophisticated definition of resonance is required. This problem has not been resolved yet.

For the single-drive case, Newton's method is used to find the zero in the phase difference between the drive and the response to a specified accuracy. If the initial guess for the mode frequency is reasonable, the program will converge rapidly because the phase is a nearly linear function of frequency near resonance. Alternatively, if no good estimate of the frequency exists, the program can solve for the drive response as a function of frequency over a range of frequencies with a specified step size. The latter mode of operation is called the scan mode, the former is the lock mode.

In the case of multiple drives, not only is the resonance criterion more difficult to specify, but the matrix equations become considerably more complex. Consider the general network shown schematically in Fig. 2. Here, there is the possibility of a voltage and/or current drive at every junction. Drives at the ends are treated as drives at the zero-th and n-th junctions. The matrix equation is written

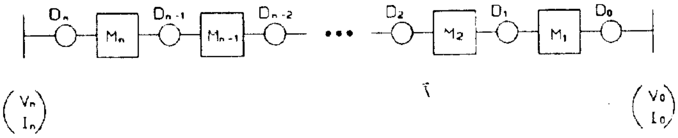

Fig. 2. Chain of 2N-ports with drives at every junction.

$$\begin{pmatrix} V_n \\ I_n \end{pmatrix} = D_n + M_n \cdot \Bigg[D_{n-1} + M_{n-1} \cdot \Big[D_{n-2} + \cdots + M_2 \cdot \Big[D_1 + M_1 \cdot \Big[D_0 + \begin{pmatrix} V_0 \\ I_0 \end{pmatrix} \Big] \Big] \Big] \Bigg] \cdots \qquad (7)$$

where D_k is the k-th drive vector. Expanding the products to remove all parentheses yields

$$\begin{pmatrix} V_n \\ I_n \end{pmatrix} = D_n + M_n D_{n-1} + M_n M_{n-1} D_{n-2} + \cdots + M_n M_{n-1} \cdots M_2 D_1 + M_n M_{n-1} \cdots M_2 M_1 D_0 + M_n M_{n-1} \cdots M_2 M_1 \begin{pmatrix} V_0 \\ I_0 \end{pmatrix} \quad , \qquad (8)$$

which can be abbreviated to

$$\begin{pmatrix} V_n \\ I_n \end{pmatrix} = \begin{pmatrix} V_D \\ I_D \end{pmatrix} + M_n M_{n-1} \cdots M_2 M_1 \begin{pmatrix} V_0 \\ I_0 \end{pmatrix}$$

$$\text{or} \quad \begin{pmatrix} V_n \\ I_n \end{pmatrix} = \begin{matrix} V_D \\ I_D \end{matrix} + \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_0 \\ I_0 \end{pmatrix} \quad , \qquad (9)$$

where V_D, I_D is a net drive vector consisting of all but the rightmost term of the above equation. The drive vectors are considered to be either shunt-current sources or series-voltage sources. Thus, an open boundary should be associated with each current source on either end, and a short-circuit boundary should be associated with voltage sources on the ends.

If the boundaries are shorts at both ends, the new equation can be solved:

$$I_0 = -B^{-1} \cdot V_D \quad \text{, and } I_n = I_D - D \cdot B^{-1} \cdot V_D \quad .$$

If the boundary conditions differ from this assumption, the problem is reduced to the same form by rearranging the vectors so that the amplitude components that are forced to zero are the top N elements of the vectors. The rows and columns of the matrix are then transformed accordingly. After the solution for the unknowns on the ends has been found, the vectors and matrix are restored to their normal order and the intermediate amplitudes are calculated.

The Program

At this time the program only exists conceptually. There will be a front-end program that will format the input data from specific models that will be developed as needed. The data for the network will consist of lists of matrix elements for several combinations of series and shunt resistance, capacitance, and inductance. Elements of length zero will be taken to be discrete. Nonzero length elements will be considered to define an infinitesimal element of a continuous line. The continuous line will be calculated as though it consisted of a finite number of symmetric T-networks as discussed above.

The series and shunt elements will be thought of as either normal or "inverted." A normal element would be a series RLC combination for a series impedance, or a parallel RLC for a shunt admittance. The inverted form would assume a parallel RLC for a series element, or a series RLC for a shunt element. Series elements will be specified by matrices of R, L, and S (= C^{-1}), whereas shunt elements will be specified by matrices of G (= R^{-1}), B (= L^{-1}), and C.

A discrete element may consist of one matrix each of either R, L, and S, or one each of G, B, and C. A distributed network requires at least one of R, L, and S and at least one of G, B, and C. Ordinarily there will not be resistive components alone in a distributed network. Not all matrices need be present if the above conditions hold. All matrices will be saved in a large multidimensional array. Each element will have a pointer array that locates the data making up the network element. The data set consists of a list of all the values, along with identification words to define the data. Each element of the network follows in succession.

Interspersed between the elements will be drive vectors and output flag vectors. These will also be pointed to by the pointer array for each element. The output flag vector will be used to determine if a plot or listing of a particular amplitude component is desired at the end of the calculation. If amplitudes at points within a continuous transmission line are desired, the line will have to be broken into segments with a junction at each desired point.

Transmission Line Models

The shunt element for the RFQ is similar to the one proposed in earlier papers.[3] However, to treat the RFQ as an eight-port network for retention of the structure symmetry in the calculations, it is necessary to modify the model by adding a small shunt capacitance from each vane tip to the ground reference. This introduces a zero mode in the azimuthal direction. There are then four azimuthal modes described by the model, the quadrupole, two dipole, and the zero as required for a general eight-port network. If the capacitance is small enough, there will be no significant effect on the calculated field distribution. If there is no shunt capacitance, the problem is incalculable without reverting to the unsymmetrical six-port representation.

For a TE-mode transmission line, the series impedance is determined completely by the shunt admittance and the requirement that all modes propagate at the same velocity in the infinite frequency limit. In the case of the RFQ, we require that the series-inductance matrix L be given by $L = c^{-2}\,C^{-1}$. In the case of a TM mode, an analogous expression relates the shunt capacitance to the series inductance.

The calculation of the series impedance must be performed by the front-end program because it depends on the exact model being used. Figure 3a shows the general RFQ shunt element. Figure 3b shows the corresponding series element. The mutual inductances arise from the requirement that all modes propagate at the same velocity in the high-frequency limit.

The flexibility of the 2N-port method is illustrated in Fig. 4 showing an RFQ with a resonantly coupled manifold. The coupling resonators are accommodated by treating them as a chain of coupled resonators that are also coupled to the manifold and an RFQ quadrant. By making the coupling between adjacent resonators weak enough, the model approaches the case of uncoupled resonators, which cannot otherwise be handled in the context of a pure chain of 2N-ports.

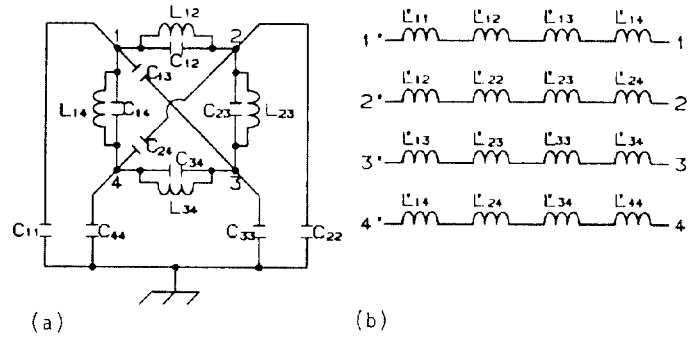

Fig. 3a. Shunt admittance element for RFQ structures.
Fig. 3b. Series impedance element for RFQ structure.

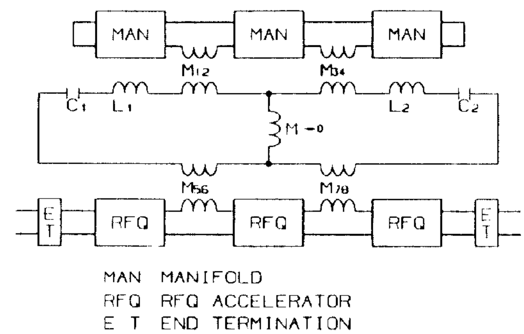

Fig. 4. RFQ with resonantly coupled manifold.

It is even possible to accommodate the coupled accelerator-decelerator as used in the free-electron laser energy-recovery experiment by a suitable contortion of the topology as shown in Fig. 5.

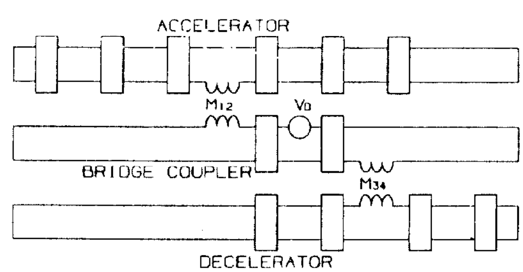

Fig. 5. Resonantly coupled accelerator-decelerator model.

Conclusion

The principles of a program for calculating the field distributions and tuning sensitivities for models of accelerator structures that can be expressed as linear chains of 2N-port networks has been described.

References

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