

# DESIGN OF TE CAVITIES FOR RF ENERGY STORAGE

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**Abstract:** This paper deals with the design of high Q TE resonators for high efficiency RF pulse compression. This kind of resonator is used to double the energy gradient of linear accelerators. In order to study the influence of the geometry of the resonator on the  $Q_0$ , a computer code named OSCAR2D was developed. By means of this code an optimum geometry for TE storage cavities was found. The results of the computer simulation are shortly compared with the experimental measurements performed on a TE cavity prototype.

## 1. - Introduction.

As proved elsewhere <sup>1</sup>, it's possible to double the energy gradient of a linear accelerator, without modifications of accelerating sections and RF systems, by installing a pulse compressor after the klystron. The pulse compressor makes use of a high Q resonator to store the RF energy.

A feasibility study of a similar system started at INFN Frascati Laboratories with the goal to double the energy of the 400 MeV linac. The results of this investigations show that the storage cavities must exhibit a quality factor of, at least,  $10^5$ . Such very high values of Q can be achieved with TE mode resonators <sup>2</sup>.

Our report presents the various kinds of usable cavities, taking into account the problems given by the degeneration of TE modes into low Q TM modes.

A computer code is presented, which allows the calculation of the main parameters (frequency,  $Q_0$ , etc.) of the TE cavities with azimuthal symmetry for resonance modes independent from azimuth.

## 2. - Definition of the problem.

It has been demonstrated at SLAC, that it is possible to increase the energy gain of an electron linac for a given klystron peak power. The peak power fed to accelerating sections can be increased by means of a device consisting of two high Q resonant cavities, a 3 dB coupler and a fast phase shifter.

To achieve the best efficiency, the energy stored inside the cavity must be as much as possible constant during the length of RF pulse. The length of pulse fed to the accelerating section must be equal to the filling time of the section. The loaded Q of an accelerating section is nearly  $10^4$ , therefore, for a good operation of the system, the storage cavities need a  $Q_0$  of  $10^5$  at least; the working frequency must be, of course, the same of the accelerating section, i.e. 2856 MHz.

## 3. - Storage cavities design.

The TE modes azimuthally invariant in cylindrical resonators have been considered as resonance modes for the storage cavities of the pulse compression system; such resonators have very high quality factor so that they can satisfactorily be used to store the RF energy.

The analytical solution for the fields in TE resonators <sup>3</sup> (spherical or cylindrical shape) is well known and we can calculate the  $Q_0$  for a given material conductivity. Nevertheless, a real resonator is slightly different from an ideal one so that the analytical computation cannot be performed, in fact:

- a) the real resonator has a different geometry due to the coupling windows, the tuning devices, etc. so to change the field distribution and the quality factor;
- b) the TE <sub>on</sub> mode is normally degenerated with a low Q <sub>TM</sub> <sub>ln</sub> mode and one must introduce a perturbation inside the cavity in order to remove the coupling between the modes and to avoid the energy transfer from each to other.

The perturbation acts in a well known way on the frequency; its effect on the quality factor is less known. In order to take into account these effects, in designing TE storage cavities, the finite difference code OSCAR 2D <sup>4</sup> has been extended for the calculation of the first TE modes for resonators having azimuthal symmetry. The method used by the code is briefly described in ref. [4].

The program was modified in the following way for the solution of TE modes:

- i) the Helmholtz eigenvalue equation is used

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = k^2 \vec{E}, \text{ with } \vec{E} = (0, E_\varphi, 0), \quad k = \omega/c \quad (1)$$

which follows from Maxwell equations. If  $E_\varphi$  does not depend on  $\varphi$  (1) becomes the scalar equation:

$$\frac{d^2 F}{dr^2} - \frac{1}{r} \frac{dF}{dr} + \frac{d^2 F}{dz^2} + k^2 F = 0, \text{ where } F = r E_\varphi(r, z); \quad (2)$$

- ii) The boundary condition  $\vec{E} \times \vec{n} = 0$  on the resonator wall, can be expressed for F as the Dirichlet condition  $F = 0$ . Such a condition is imposed with high accuracy on the boundary of the resonator by using a second order bidimensional Taylor expansion. Our approximation for the Dirichlet condition works equally well whether the resonator boundary is coincident with lines of the discretization grid or not.

## 4. - Choosing the resonator geometry.

The OSCAR2D code allows for the optimization of the resonator geometry. Two different shapes can be used: the sphere and the cylinder.

The spherical shaped resonators have a quality factor 20% higher than cylindrical cavities, but the

$TE_{011}$  degenerates with  $TM_{111}$  and three times with itself because the orientation of the field is not fixed in a sphere. This is a disadvantage, in fact it's necessary to know where the maximum and minimum of the field are located in order to determine the position of the coupling (loop or window) and tuning devices.

A perturbation of the geometry is needed to remove the degeneration of the modes and to provide an orientation of the fields to position the tuning and coupling devices.

The perturbation modifies the resonator geometry decreasing the  $Q_0$  of 10%. The advantage of a very high quality factor is lost.

A cylindrical cavity has then been considered. In this case, the only degeneration, which can be easily removed, is degeneration between  $TE_{0nm}$  and  $TM_{1nm}$ . From the perturbation theory one has:

$$\frac{\Delta f}{f_0} = \alpha \frac{\Delta V}{V_0} \quad (3)$$

where  $f_0$  and  $V_0$  are respectively the resonant frequency and volume of the unperturbed resonator and  $\alpha$  is a coefficient proportional to the energy stored in the removed volume.

In fig. 1, a section of the selected resonator, in the  $r, z$  plane, is presented.

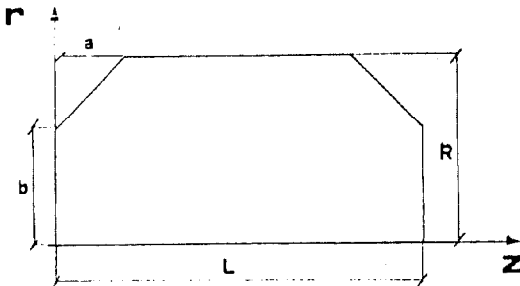


Fig. 1 - Axial section of resonator.

A suitable perturbation of the resonator is carried out by removing the corners as shown in the figure. Being the  $TM_{1n}$  H field close to its maximum value in the removed volume, the frequency shift of degenerate  $TM_{1n}$  is high.

#### 5. - Optimization of the mode.

Once the geometry of the resonator has been chosen, the next step is the calculation of the parameters  $a$  and  $b$  by the OSCAR2D code, keeping the ratio  $2R/L = 1$ .

The previous condition guarantees the best  $Q_0$  for the unperturbed resonator. We define the magnetic geometry factor:

$$G = 2\pi f_0 \mu_0 \frac{\iiint_V H^2 dV}{\iint_S H_{\parallel}^2 dS} \quad (5)$$

$G$  is related to the shape and the cavity mode to be used.

We also have:

$$Q_0 = \frac{G}{R_S} \quad (6)$$

where

$$R_S = \sqrt{\frac{\rho}{\sigma}} \quad (7)$$

is the surface resistance of a conductor having the conductivity  $\sigma$  at the frequency  $f$ ;  $\mu_0$  is the permeability of vacuum.

In Figs. 2, 3, 4, 5 we show the variation of  $G$  with the parameters  $a$  and  $b$  for first four  $TE_{01n}$  modes. It can be seen from these figures, that the geometry factor  $G$  reaches a maximum for each mode and this maximum value is greater than the factor  $G_0$  of the unperturbed cavity ( $a=0$ ). Now taking  $R_S = 14 \times 10^{-3}$  and  $f = 2856$  MHz, the value of  $TE_{014}$  mode corresponding to the maximum of  $G$  is

$$Q_0 = \frac{G}{R_S} = \frac{1860}{14 \times 10^{-3}} \approx 140.000$$

This value satisfies the requirements of a cavity for energy storage.

#### 6. - Conclusions.

The results of simulation have been verified by means of a 2856 MHz brass modified resonator operated on the  $TE_{013}$  mode with  $a = 6$  cm and  $b = 6.5$  cm (see fig. 4). The difference between the calculated frequency and the measured one is within 1 MHz. The geometric factor  $G = 1300 \Omega$  is 10% lower than the value we expected. The difference is probably due to the imperfect smoothness of the surfaces and to the imperfect knowledge of the conductivity of the used brass alloy.

We finally conclude that OSCAR2D code can be successfully used to design energy storage cavities and to demonstrate that a carefully optimization of the geometry of a cylindrical resonator allows both the removal of degenerations and an improvement of the quality factor by about 20%.

These performances permit the  $TE$  cavities to be used as storage cavities in pulse compressor with a lower mode ( $TE_{013}$ ) instead of higher modes ( $TE_{015}$ ) used elsewhere. As an example, if a  $Q_0$  of  $10^5$  is needed a  $TE_{013}$  can be used, allowing for a wider separation of unwanted modes and a less critical mechanical construction.

The use of modified resonators on modes higher than the  $TE_{013}$  will guarantee higher  $Q_0$  allowing for wider safety margins against unwanted losses, due to bad processing or contamination of the cavity surface during operation.

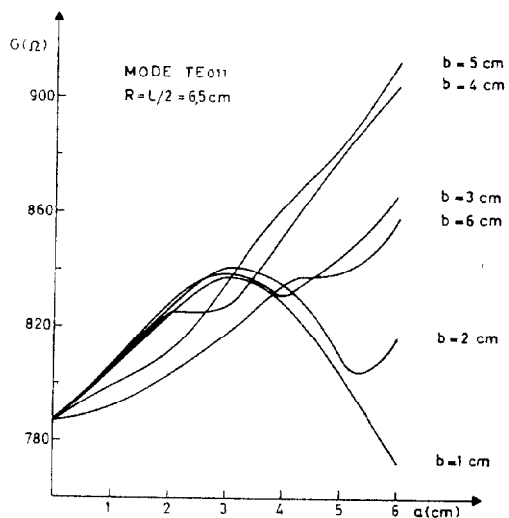


Fig. 2 - Variation of  $G$  with  $a$ ,  $b$  for  $TE_{011}$  mode.

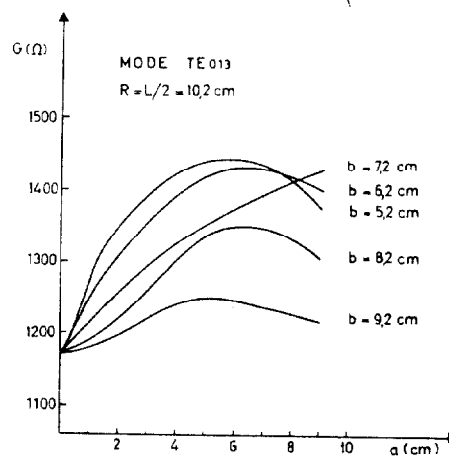


Fig. 4 - Variation of  $G$  with  $a$ ,  $b$  for  $TE_{013}$  mode.

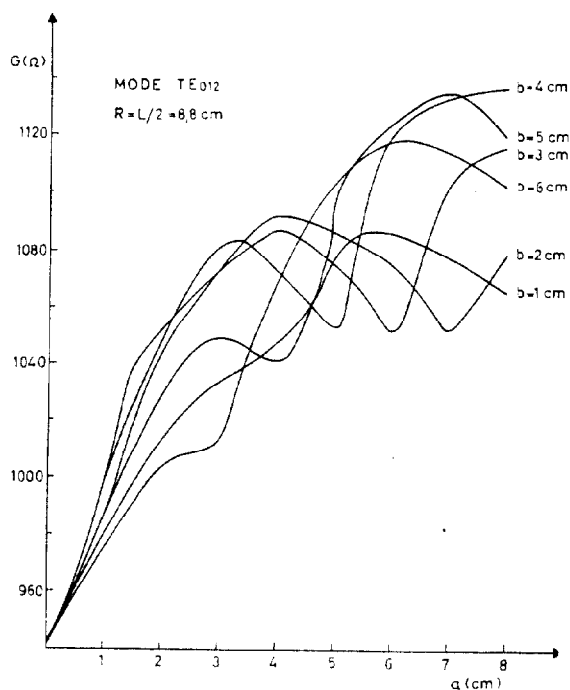


Fig. 3 - Variation of  $G$  with  $a$ ,  $b$  for  $TE_{012}$  mode.

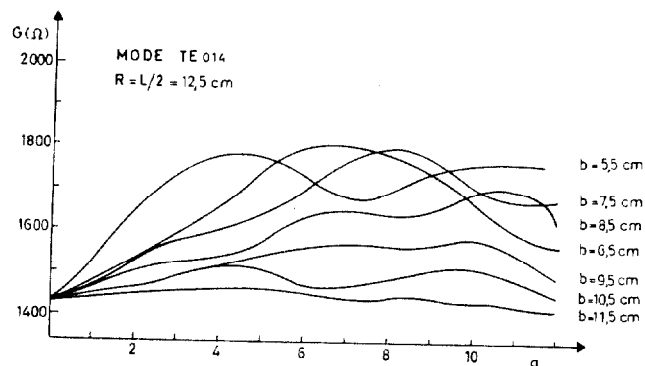


Fig. 5 - Variation of  $G$  with  $a$ ,  $b$  for  $TE_{014}$  mode.

#### REFERENCES

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