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DEPOLARIZING 'BEAT' RESONANCES IN THE BROOKHAVEN AGS

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Abstract

While accelerating polarized protons in the Brookhaven AGS we found a variant of the standard imperfection and intrinsic depolarizing resonances which has some of the properties of both types. Imperfection resonances occur at $G\gamma = k$, when the number of spin precessions per revolution, $G\gamma$, equals a harmonic of the depolarizing field, k. Intrinsic resonances occur at $G\gamma = nP \pm \nu_z$, when the AGS gradient periodicities, nP, modulate free vertical betatron oscillations to create the sum and difference frequencies. The variant resonance is a beat between nP and an imperfection driven betatron oscillation of periodicity k'. These occur at $G\gamma = nP \pm k'$, and are strongest when the driven betatron oscillation is largest. The effect was most dramatic at the strong $G\gamma = 27$ resonance. Since $\nu_z = 8.8$ for the AGS, and there is a major nP = 36 AGS periodicity, a strong beat resonance should exist at $G\gamma = 36-9 = 27$. Applying a 27th harmonic correction directly was unsuccessful, but a 9th harmonic correction removed the depolarization.

Introduction

The achievement of a high energy polarized proton beam requires minimizing polarization losses during the acceleration process. These losses occur at resonance energies where the spin precession frequency equals that of a magnetic field component that can flip the particle spin. These resonances are generally classified into two types, imperfection and intrinsic. Imperfection resonances are driven by horizontal magnetic fields from misalignments which have a Fourier component at the spin precession frequency. Intrinsic resonances are driven by the horizontal magnetic fields seen by the particle due to its vertical free betatron oscillations in the intrinsic alternating gradient field structure of the machine. In the latter case the observed field is a wave at the vertical betatron oscillation frequency which is amplitude modulated by the alternating gradient machine structure, leading to sum and different frequencies as seen by the particle, and depolarization when these equal the spin precession frequency. It is evident that the above description is not the whole story because strong depolarizing resonances which involve both imperfection and intrinsic effects can occur due to imperfection driven betatron oscillations in the intrinsic alternating gradient fields, leading to another set of sum and difference frequencies. These imperfection-intrinsic 'beat' resonances should be strongest when the driven betatron oscillation is largest, which will occur when the driving imperfection frequency is near the free vertical betatron oscillation frequency, leading to strong 'beat' resonances close to the standard intrinsic resonances. Elimination of polarization loss from a strong 'beat' resonance is clearly most readily achieved by correcting the imperfection harmonic driving it, which will be close to the vertical betatron oscillation frequency.

In the polarized proton accelerated beam project at the AGS the capability was built in at each resonance to simultaneously control two imperfection harmonics, a driving harmonic for the 'beat' resonance, as well as the standard imperfection harmonic. During the commissioning phase of the AGS polarized proton program we performed studies which enabled us to see clear evidence of the 'beat' resonance phenomenon. The following sections will discuss depolarizing resonances in more detail and will present our 'beat' resonance results.

Depolarizing Resonances

General Discussion

The proton polarization vector can be rotated from its normal (assumed upward) direction by a horizontal magnetic field component, as illustrated in Fig. 1. The polarization vector, $\vec{P}$, is viewed in the particle rest frame. The guide field is downward for a counter-clockwise particle motion, particle velocity is assumed to be to the right. A component of magnetic field transverse to the particle motion precesses the polarization vector an angle[1,2] $[(g-2)/2]\ \gamma\delta\theta = G\gamma\delta\theta$ where $\delta\theta$ is the particle laboratory deflection angle due to the motion in the transverse field, $\gamma = E/M$ and the factor $G = (g-2)/2$, which equals 1.793 for a proton, includes the effect of the relativistic Thomas precession. Longitudinal fields also can cause precession but for a given field the effects are down by a factor of γ and will be ignored here in this simplified presentation. For a more complete general discussion see Ref. 2. The polarization vector is precessing about the vertical z axis due to the vertical guide field an amount $G\gamma\delta\theta$;

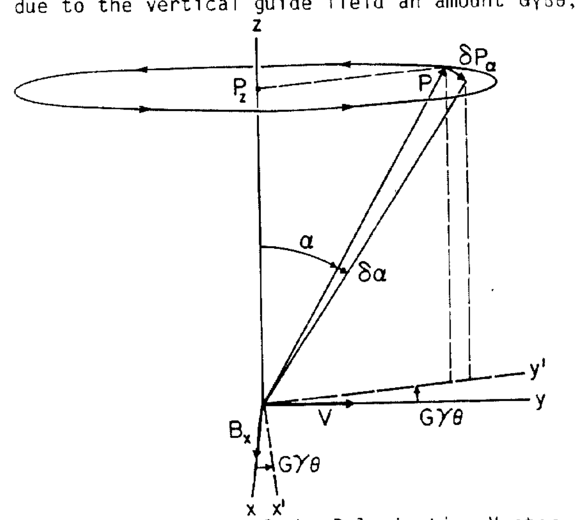

Fig. 1. Precession of the Polarization Vector

$\delta\theta$, the horizontal deflection angle, equals $\delta s/\rho$, where ρ is the local radius of curvature. There is a simultaneous precession about the transverse x axis which, for protons, and $\beta \cong 1$, is $.573B_x\delta s$, from any horizontal transverse (laboratory) field B_x (Tesla), distance s (in meters), at the particle position. The component of this precession which generates depolarization is $\delta\alpha = .573B_x \cos(G\gamma\theta)\delta s$. The strength of the depolarizing resonance is defined as $\varepsilon = \langle d\alpha/d\theta \rangle$ at the resonant condition where B_x has a frequency component in phase with the polarization precession. Forming the average by integrating over a complete revolution around the accelerator gives $\varepsilon = (.573/2\pi)\int B_x \cos(G\gamma\theta)\, ds$. The imperfection resonances are at $G\gamma = k$, k an integer, the intrinsic at $G\gamma = nP \pm \nu_z$, where P, an integer, is any periodicity of the machine gradient; n, also an integer, denotes the harmonic, and ν_z the number of vertical betatron oscillations per revolution. The depolarizing resonance strengths for the AGS have been estimated in a computer calculation by Courant and Ruth.[2] The misalignments driving the imperfection resonances were randomly generated (0.1 mm displacement rms). Their results are shown in Fig. 2. Also shown are lines indicating limiting polarization losses for the AGS acceleration rate. The general increase in ε with energy is expected since the fields are increasing with energy.

Imperfection-Intrinsic 'Beat' Resonances

It is evident from Fig. 2 that there is a clustering of large imperfection resonances near the intrinsic resonances. If the imperfection horizontal fields seen by the particles were simply due to the misalignments, this association would not be expected and ε should just increase uniformly with energy, with random statistical variations about the average. The observed horizontal field, however, is also coming from the imperfection driven vertical betatron oscillations traversing the intrinsic machine field gradients, a situation similar to the intrinsic resonances, where the betatron oscillations are free. Parallel to the intrinsic resonance case, the horizontal field components from this process are:

$$B_{xnPk'} = \frac{dB_x}{dz}\, z_{k'} \quad , \qquad (1)$$

where again the field gradient will have periodicity nP; $z_{k'}$ is the amplitude of the driven vertical betatron motion caused by the k' Fourier component of the magnetic imperfection fields. The field B_x will then have components as $B_{xnPk'} = \overset{\circ}{B}_{xnPk'} \cos(nP\theta)\cos(k'\theta)$ which has periodicities $nP \pm k'$, so resonances will occur at[3]

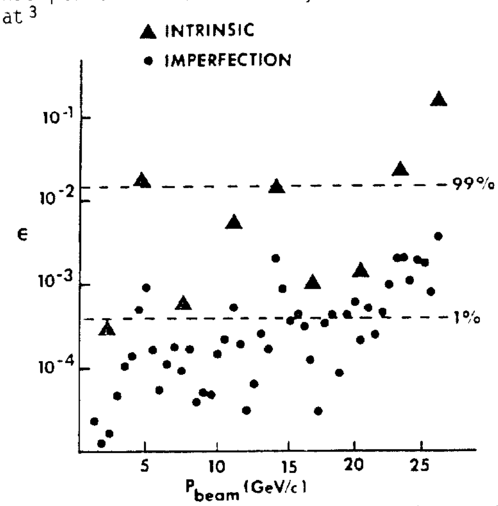

Fig. 2. AGS Depolarizing Resonance Strengths

$$G\gamma = nP \pm k' \quad . \qquad (2)$$

The strength of the resonances will be largest when the driven betatron oscillation is largest, i.e. when $k' \cong \nu_z$, the driving periodicity is close to the free vertical betatron oscillation frequency, as seen from the constant gradient 'smooth approximation' solution for a driven vertical betatron oscillation

$$z_{k'} = \frac{.3R^2}{P}\, \frac{\overset{\circ}{B}_{xk'}}{((k')^2 - \nu_z^2)} \cos(k'\theta) \quad , \qquad (3)$$

where R is the average machine radius, P the momentum in GeV/c, and $\overset{\circ}{B}_{xk'}$ the Fourier coefficient of the driving field. For the special case[4] $nP = 0$, dB_x/dz of Eq. (1) is the average machine gradient, which can be obtained from ν_z^2 using the smooth approximation. The net field seen by the particle is then

$$B_{x0k'} = \frac{1}{[1 - (\frac{\nu_z}{k'})^2]}\, \overset{\circ}{B}_{xk'} \cos(k'\theta) \quad . \qquad (4)$$

The average gradient amplifies the effect of the perturbing field alone.

Depolarizing Resonance Studies at the AGS

The AGS project for accelerating polarized protons has been described previously.[5] The recent polarized beam commissioning program is discussed in an accompanying paper for this Conference.[6] Of relevance to the topic of the present paper is the method employed to eliminate the depolarization losses due to imperfection perturbing fields, which will be discussed in the following section.

Applied Dipole Fields

The effects of the unknown perturbing fields are to be cancelled at each imperfection driven resonance, $G\gamma = k$, by applying, during the time of the resonance crossing, a correcting field with Fourier components $B_{xk} = a_k \sin(k\theta) + b_k \cos(k\theta)$ (for k or k'). The correction field coefficients a and b are experimentally determined by varying each to minimize the polarization loss at each resonance by applying a current in a set of horizontal field dipoles

$$I_k(\theta) = \overset{\circ}{I}_{ks} \sin(k\theta) + \overset{\circ}{I}_{kc} \cos(k\theta) \quad . \qquad (5)$$

For the set of AGS dipoles we calculate the non-amplified dipole strength $\varepsilon_k/I_k = .69\ 10^{-3}$/ampere, and the current required to go from zero polarization loss to 50% loss on a resonance crossing to be $\Delta\overset{\circ}{I}_{1/2} \cong 4.2$ amps. For $nP = 0$ where $k' = k$, including the amplifying factor of Eq. (4) the FWHM of the polarization vs. current curve is then

$$\overset{\circ}{I}_{FWHM} = [1 - (\frac{\nu_z}{k})^2]\, 8.4 \text{ amperes} \quad . \qquad (6)$$

Table I

Calculated widths of dipole driving current curve for 50% polarization loss, from Eq. (6), with $\nu_z = 8.8$.

k	6	7	8	9	10	11	27	28
$\overset{\circ}{I}_k$ FWHM Amperes	9.7	4.9	1.8	0.4	1.9	3.0	7.5	7.6

Experimental Results

Plots of the experimentally measured polarization remaining as a function of dipole current excitation at some selected resonances are shown in Fig. 3. In all of these curves the position of the maximum indicates the correction current required to cancel the effect of the accelerator depolarizing fields at the resonance. As discussed above, the width of a curve is a measure of the effect of the dipole excitation on the polarization vector rotation--the narrower the width, the more rotation and larger dipole generated ε for a given applied current. The top portion of Fig. 3 covers the range $G\gamma = k$ from 7 to 10, in the region of the vertical betatron resonance frequency $\nu_z = 8.8$, where the discussion of the previous section is directly applicable. The driven betatron oscillation, largest when k is near ν_z, is beating with the average gradient of the machine, the case nP = 0, which lead to Eq. (6) and the calculated estimates of curve widths of Table I. Comparison of the observed curve widths of Fig. 3 with the values of Table I shows quite good agreement. The width narrowing for k near ν_z indicated in the model calculation is dramatically evident in the data. Quantitatively, the observed widths are approximately 10% larger than calculated. The bottom portion of Fig. 3 shows results for the resonances $G\gamma = 27$ and 28. Since there is a strong nP = 36 harmonic component of the gradient in the AGS, Eq. (2) would suggest that perturbing fields of periodicity $k' \sim \nu_z$ should have a large effect at

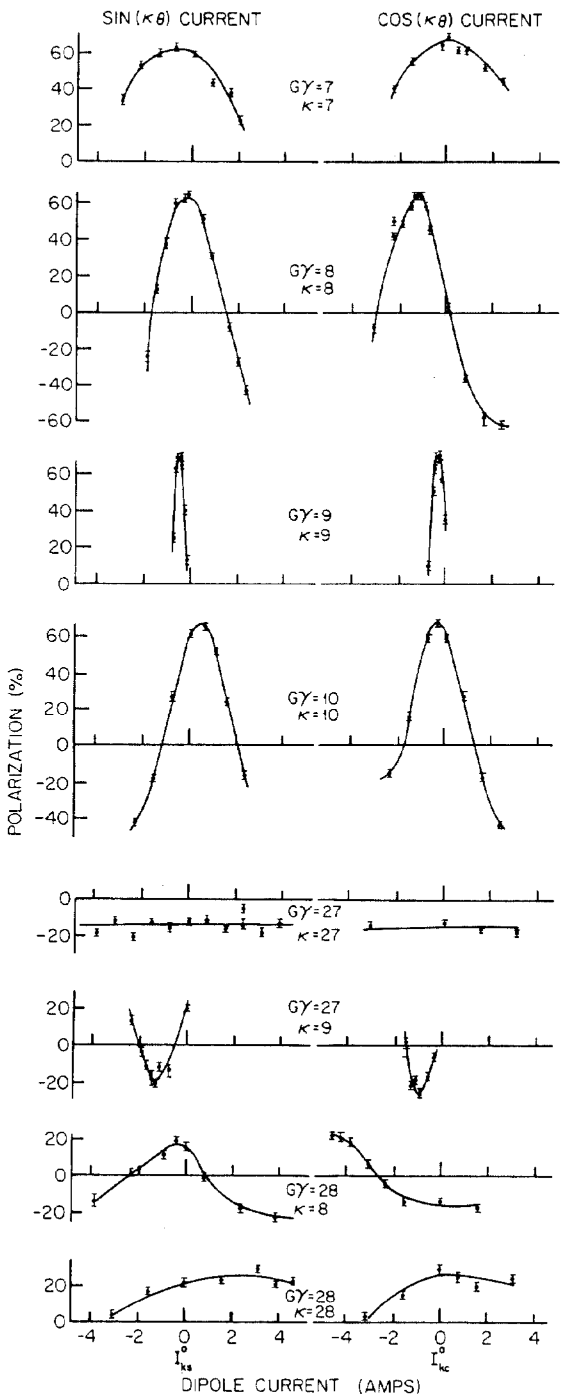

Fig. 3. Polarization Response to Dipole Currents

$$G\gamma = 36 - k' \quad , \tag{7}$$

leading to large depolarizing fields at $G\gamma = 27$ for $k' = 9$ and $G\gamma = 28$ for $k' = 8$, both k' being close to ν_z. At $G\gamma = 27$, direct application of the dipole harmonic k = 27 had no appreciable effect; presumably, the resonance was so strong that the required 27th correction perturbation was outside the range available. The 9th harmonic, however, was successful in eliminating the polarization loss, with a much narrower width curve (more effective) than estimated for the direct dipole excitation at k = 27 (see Table I), in agreement with the beat resonance model. Similarly, at $G\gamma = 28$, polarization maximization was just possible with the 8th harmonic, with a wider curve than with the 9th (though not quite by the ratio of 3.8, expected from Eq. (3)). Note that the required correction settings for k' = 8 and 9 at $G\gamma = 28$ and 27 approximately scale with momentum from the values needed at $G\gamma = 8$ and 9, consistent with the beat resonance idea that the machine depolarizations were predominately due to the same fixed 8th and 9th harmonic machine misalignments in both cases. The depolarizations were large there because of the beat amplification factors and correctable for the same reason. The last curve at $G\gamma = 28$ shows the polarization response to application of dipole excitations at k = 28 while the k' = 8 correction settings were simultaneously being pulsed. The curve width is large, as expected from Table I, so this k = 28 harmonic is serving as a less responsive vernier correction. In our picture the center of this curve would be at zero if the 8th harmonic correction was precisely determined and there was no shift in machine parameters.

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