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MEASUREMENTS OF STABILITY LIMITS FOR A SPACE-CHARGE-DOMINATED ION BEAM IN A LONG A.G. TRANSPORT CHANNEL\*

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# Abstract

The Single Beam Transport Experiment at LBL consists of 82 electrostatic quadrupole lenses arranged in a FODO lattice. Five further lenses provide a matched beam from a high-current high-brightness cesium source for injection into the FODO channel. We call the transport conditions <u>stable</u> if both the emittance and current remain unchanged between the beginning and end of the channel, and <u>unstable</u> if either the emittance grows or the current decreases because of collective effects. We have explored the range of single-particle betatron phase advance per period from  $\sigma_0$  = 45° to 150° to determine the stability limits for the space-charge depressed phase advance,  $\sigma$ . No lower limit for  $\sigma_0$  above 90°.

#### Introduction

Our practical motivation for the experiments reported here is the possibility of driving inertial confinement fusion via a heavy ion linear induction accelerator. In such a device, efficient acceleration requires very high beam current. For the fusion application, both transverse and longitudinal emittance must be kept very small to allow focussing of the beam onto the fusion fuel target. This experiment was designed to investigate the transverse stability limits of a beam in an A.G. lattice for high current and low emittance.

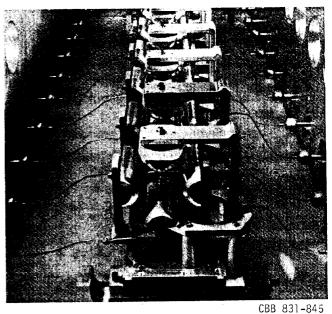
#### Apparatus

The Single Beam Transport Experiment (SBTE) comprises five matching quadrupoles (MI-M5) with independent voltage controls, followed by 82 quadrupoles (Q1-Q82) with equal voltages alternating in sign to form a long FODO transport lattice [1]. The quadrupole aperture is 25.4 mm in radius. A portion of the SBTE transport lattice is shown in Fig. I below. Cesium ions are produced from a heated aluminosilicate button 12.5 mm in radius, and are accelerated through a four-electrode injector to an energy that could be varied from 120 to 200 kV. Downstream of the injector a set of three biassed grids could be introduced to increase the emittance, as desired, in a controlled way. Also at this location, a set of attenuators mounted on a large rotating wheel provides the capability of selecting a beam current with various values down to 1% of the unattenuated value. Two deep Faraday cups, each with a biassed ring electrode (but no grid) at its entrance, could be introduced after M5 and after Q82 to give an absolute measurement of beam current at the beginning and end of the transport lattice. Additional current monitors - gridded Faraday cups that were shallow enough to slide between adjacent quadrupoles - were available for use at Q36 and Q60. Emittance measurements in both the (x,x') and (y,y')planes could be made by scanning with pairs of displaced slit-apertures at Q4-5, Q35-36, Q59-60, and Q80-81.

# Experimental Method

The experimental procedure is straightforward. A value of  $\sigma$  is chosen by selecting the appropriate voltage for

Q1-Q82. The desired current and emittance are set by choice of the attenuator and the voltage applied to the emittance-spoiling control grids. With guidance from numerical integration of the Kapchinskij-Vladimirskij (K-V) envelope equations, the voltages of the matching quadrupoles M1-M5 are tuned until the emittance ellipses in (x,x') and (y,y') phase-space have the correct size and orientation to deliver a matched beam to the transport system. (A beam is considered "well-matched" if the residual envelope oscillation amplitude is less than  $\pm 10\%$  of the beam radius.) If the current and emittance are found to be unchanged in passing through the entire transport system, it is empirically labeled as "stable". If either has changed, however, the additional diagnostics at Q35 and Q59 are activated to provide more information on the evolution of the unstable behavior.



# Fig.1. A portion of the SBTE transport lattice

#### Results

The quantities  $\sigma_0$  and  $\sigma$  are, resp., the betatron phase advance of a particle in the lattice with and without space-charge. Measurements were made for 13 values of  $\sigma_0$ from 45° to 150°. Given the uniformly placid behavior of the beam observed for 60°  $\leq \sigma_0 <$  85°, we made only one brief measurement for  $\sigma_0 =$  45° and none for  $\sigma_0 <$  45°. For  $\sigma_0$ > 120°, the closed orbit deviations due to misalignments become large, precluding meaningful measurements for  $\sigma_0 >$ 150°.

In summary, beam behavior was always found to be stable below  $\sigma_0 = 90^\circ$  within the limits of current and emittance accessible to us, and unstable above  $\sigma_0 = 90^\circ$  if a sufficiently high current was injected.

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The emittance at injection into the periodic lattice began to increase above the baseline value for  $\sigma_0 = 85^\circ$  rather than  $\sigma_0 = 90^\circ$ , but we attribute this to aberrations from the matching section, which became visibly more pronounced as we attempted to match into stronger lattices. This might also be due to exceeding the stability limits locally in the matching section, as the beam disruption at even slightly higher  $\sigma_0$  is very rapid.

Figures 2 and 3 are representative of how differently the beam acts depending on whether  $\sigma_0$  is less than or greater than 90°. To obtain these graphs, we have chosen contours of equal density in the measured (x,x') distribution and have displayed the total inscribed current versus the phase-area (divided by  $\pi$ ) within the contour. We have chosen to estimate this phase area by calculating the root-mean square emittance of the (x,x') distribution within the contour, and then multiplying by 4 $\beta\gamma$  to give a corresponding normalized K-V emittance. Thus

$$c = 4\beta \gamma (\langle \chi^2 \rangle \langle \chi'^2 \rangle - \langle \chi \chi' \rangle^2)^{\frac{1}{2}},$$

with x and x' measured from the centroid of the figure. This definition of area/ $\pi$  is not far from the circumscribed area/ $\pi$  as directly measured, but is more stable against fluctuations in the data, and is of use in estimating  $\sigma$  as described below.

The  $[\varepsilon, I(\varepsilon)]$  presentation was chosen because it graphically displays the ratio  $\varepsilon/I$ , which is of great importance in parameterizing the depressed tune. We will illustrate a scaling possible in a linear lattice via an envelope equation, but the basic idea holds for general Vlasov equilibria by scaling the distribution function. Consider the smooth approximation envelope equation for a K-V distribution:

$$a'' + Ka - \frac{Q}{a} - \frac{\varepsilon^2}{a^3} = 0$$
.

While maintaining the same beam dynamics, including  $(\sigma_0, \sigma)$ , we may re-scale the radius a  $\Rightarrow$  ka, and scale Q  $\Rightarrow$  k<sup>2</sup>Q and  $\epsilon \Rightarrow$ k  $\epsilon$ . The envelope equation is satisfied, and the ratio  $\epsilon/I$  is the same as for the original solution. It is this ratio, in general, that determines the  $\sigma$ , given  $\sigma_0$ . The real brightness, proportional to  $I/\epsilon^2$ , is not the determining parameter. It is true that for real beams  $\sigma$  is not uniform for all particles, but we make contact with the RMS

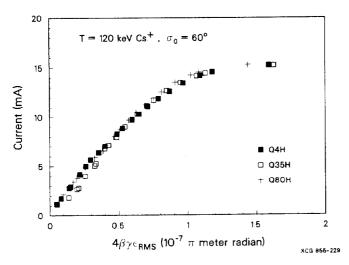
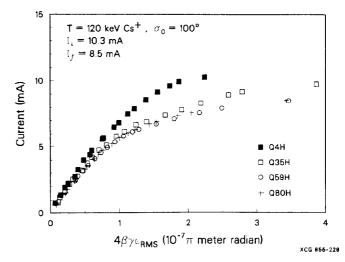
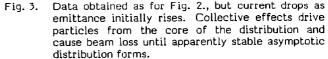


Fig. 2. From the measured (x,x') distribution of the beam at various lattice locations we calculate the RMS emittance and enclosed current of subdistributions bounded by various intensity contours. Beam current is 15.2 mA, and emittance is well maintained through the lattice.





envelope equations by noting that with the RMS emittance constant, the K-V equations are identical in form to the RMS equations [2]. We then use the K-V equivalent  $\sigma$  as a measure of beam intensity, some average of all particles' oscillation frequencies.

We define two slightly different quantities  $\sigma$  and  $\sigma'$ according to the following recipe. The three measured quantities are  $\varepsilon$  , I( $\varepsilon$ ), and the beam radius at a point of antisymmetry; inserting  $\epsilon$  and I( $\epsilon$ ) into the K-V equations allows one to derive a phase advance per period for a single particle moving in the combined fields (both assumed linear) of the space-charge and quadrupole forces. Inserting the full beam values ( $\epsilon_0$ ,  $I_0$ ) into the K-V equations, we derive a value  $\sigma$  for the depressed tune. Using the core 95% of beam this current to calculate  $\epsilon$ ', the emittance of sub-distribution, we calculate  $\sigma'$  from ( $\epsilon'$ , .95 x  $I_0$ ). The values are typically close to each other, but the difference is a measure of the extent of the halo of the beam in phase space. In practice, we have found that the radius derived using  $(\varepsilon,I)$  is in very close agreement with the measured value. Because the 95% emittance was more immune to fluctuations in value from run to run, emittances quoted in the following text will be 95% core values. (The intuitive value of thinking of  $\sigma$  in terms of defining a smoothed slow sinusoidal motion of a single particle may become of questionable value, however, when the space charge and average external fields are virtually cancelling each other. In that case, the nonlinear forces can predominate in determining the particle motion.)

# The Region $\sigma_0 < 90^{\circ}$

To obtain the lowest possible value of  $\sigma$ , the emittance grids were removed to minimize the emittance. Figure 1 shows the results for one example run  $\sigma_0 = 60^{\circ}$  and  $\sigma = 8^{\circ}$ . The emittance distribution can be seen to be unchanged, as measured at several points throughout the transport system. The behavior is the same for all other values of  $\sigma_0$  less than 85° for the same injected beam emittance and current. At such low  $\sigma$  values, the emittance the space-charge term.

For a K-V distribution, the graph of I vs.  $\epsilon$  is a straight line segment; the data have close to a linear form, too, except for the contribution from the tails of the

distribution. In reality, the actual beam is not well represented by a K-V distribution; while it is approximately uniform in configuration space, it has a nearly Gaussian distribution in transverse velocity. (We refer to this as "semi-Gaussian".) For beams with significantly lower current or larger emittance, when the emittance term in the envelope equation is not negligible, the data points lie to the right of the curve shown and tend to be convex upwards.

# The Region $\sigma_0 > 90^{\circ}$

In all cases studied, unstable beam behavior could be observed and the properties of the injected beam did not set a limitation. For  $\sigma_0 \ge 120^\circ$ , characterization of the beam limits was straightforward. We found that too low a value of  $\sigma$  at injection resulted in a degraded, higher value at Q80. As we raised the  $\epsilon/I$  of the injected beam, the output  $\epsilon/I$  fell until the two coincided, then both rose together. For cases in which the current and emittance were preserved over the full lattice, the data are plotted in the summary Figure 4 with filled-in symbols.

Figure 3 illustrates beam evolution for one case, for  $\sigma_0 = 100^\circ$ , injection current of 10.3 mA, and 95% emittance  $1.8 \times 10^{-7} \pi$  meter radian. In the 10% periods between Q4 and Q35 the beam evolves significantly, with somewhat more change occurring by Q59. No change is detectable afterward to Q80.

Injection for these current and emittance values was chosen because an earlier 15 mA,  $1.4 \times 10^{-7} \pi$  emittance beam showed this same pattern, degrading to 10 mA with emittance  $2.2 \times 10^{-7} \pi$  at Q59, but seemingly stable with those parameters from then on to Q80. For  $\sigma_0 = 100^\circ$  we have plotted the optimal results from initially unstable conditions for which the beam showed no change from Q59 to Q80. The same is true for the 93° and 97° lattice. We plan further investigation in the region 90° <  $\sigma_0 < 120^\circ$ .

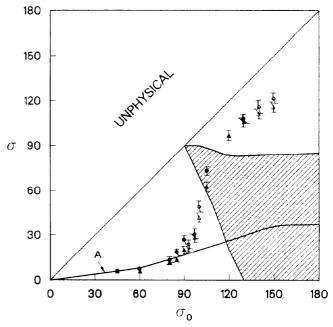
While the data analysis is not complete and a theoretical basis for understanding not yet in hand, we report these preliminary results as well-documented experimental data. The spontaneously organized, apparently stable asymptotic 10 mA distribution observed for  $\sigma_0 = 100^\circ$  appears to allow for more current at a lower emittance than does the semi-Gaussian distribution with the same current and RMS emittance that we can obtain directly from our source. One cannot, of course, be certain that evolution to a truly stable distribution (i.e., with exact current and emittance conservation) is being observed or whether emittance growth is still proceeding but at an undetectably low rate.

# Discussion of Results

Figure 4 shows a summary of results to date. Source properties set limits on how small a value for  $\sigma/\sigma_0$  can be explored. Below  $\sigma_0 = 90^\circ$ , and for strongly depressed betatron frequency,  $\sigma$  varies as  $\sigma_0(\varepsilon/I)$  and so is limited by the transverse temperature and the emission of the source. It has been found that the lowest values of  $\sigma$  can be explored if we operate the injector in a high perveance configuration, limited to 120 kV by sparking.

For  $\sigma_0 < 85^\circ$ , the data points represent the lowest values of  $\sigma$  reached; in all cases, beam propagation was found to be stable and well represented by the data in Fig. 2. How much further down in  $\sigma$  the beams remain stable is not known; simulations suggest that the limit will be set by nonlinearities due to the vacuum field and due to image charges when the effects of misalignment are taken into account [3].

In the right hand part of the diagram are plotted the values of  $\sigma$  corresponding to the values of emittance



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Fig. 4. Plotted are calculated  $\sigma$  values for stable and apparently stable beams for various  $\sigma_0$ . Filled-in symbols represent beams with the same current and emittance at the beginning and end of the lattice. Hollow symbols mark  $\sigma$  values derived from beams reproducing  $\epsilon$  and current over at least the last 10 periods, as illustrated in Fig. 3 for  $\sigma_0$  =100°. Circles mark  $\sigma$  values derived using full beam distribution RMS emittance. Triangles mark calculations using central 95% current of the phase space distribution. The shaded region marks the calculated instability of the envelope equations. Curve A marks the region of equivalent  $\sigma$  attainable at injection with our limited source emittance.

and current representing stable propagation through the full SBTE lattice or asymptotically stable propagation through at least the last 10 periods, as for the example of Figure 3.

More data points can be added below  $\sigma_0 = 45^{\circ}$  by stopping down the radius of the beam; it has not been thought urgent to do so because such weak focussing seems inapplicable to practical accelerator applications, and no different physics is expected.

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