SIMULATIONS OF HALF AND THIRD INTEGER RESONANT EXTRACTION FROM A ONE-GEV PULSE STRETCHER RING

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Introduction

In the past few years, the pulse stretcher ring (PSR) concept [1] has emerged as a viable method for extending the duty factor of pulsed linacs. The viability is a result of considerations such as cost and expected extracted beam quality. Table 1 compares some relevant parameters of PSR projects currently in various stages of design or construction. A few of the projects emphasize high average extracted beam intensity. This is accomplished through a high injection frequency (1 kHz) and relatively high injected peak currents (≥80mA). To enhance the injection efficiency, the number of turns of injection are minimized so the ring is long and there are fewer turns for extraction. There is considerable experience with beam extraction over large numbers of turns from synchrotrons, but time uniform extraction over fewer turns has not been treated as thoroughly. In this paper, some of the extraction calculations made for the proposed MIT-Bates One-GeV Pulse Stretcher Ring are discussed. The theory used for the simulation procedure will be reviewed, the results summarized, and some points of operational interest mentioned.

Table 1. Comparison of PSR Parameters

Place	Circumference (M)	Extraction (# turns)	RF (Y,N)
Synchrotron		100,000	
Tohoku	16	56,000	N
Bonn	165	36,400	Y
Lund	31	10,000	Y
Saskatoon	109	9,000	Y
CEBAF	360	1,000	Y
Saclay	360	1,000	Y
MIT-Bates	390	≤1,000	Y

Theory

The theory of resonant extraction has been well documented. To efficiently do particle tracking, a linearized formalism has been chosen both for its speed and the agreement obtained when compared to the results of other more accurate tracking codes such as DIMAT [2]. The formalism for third integer resonance has already been discussed [3], and we discuss its extension to the half integer case here. The symbols follow the definitions in reference 3 and will not be discussed here.

The governing equations for a particle in motion in a linear focussing system, represented by K(s), perturbed by a quadrupole E(s), and an octupole O(s) distribution are given in normalized phase space coordinate n and p by,

$$Q = \frac{1}{2\pi}\int\frac{ds}{\beta(s)}\ ,\qquad \frac{d^2n}{d\phi^2} + Q^2 n = -Q^2 E\beta^2 n - Q^2 O\beta^3 n^3 \tag{1}$$

The solution to (1) written in integral form is

$$n(\phi) = n_o\cos Q\phi + p_o\sin Q\phi - Q\int d\tau\,[\sin Q(\phi-\tau)]\,[E\beta^2 n + O\beta^3 n^3] \tag{2}$$

where n_o and p_o are the coordinates at $\phi = 0$, which we take to be the extraction point. Note that in the absense of perturbing terms on the right (E and O), the particle motion is simple harmonic and stable. If the particle coordinates are examined after every two turns then the change in n at extraction (after assuming the perturbing element lengths L to be small) is

$$\Delta n = \Sigma[-L_i E_i \beta_i 2\sin Q\phi_i]n_i + \Sigma[-L_j O_j \beta_j^2 2\sin Q\phi_j]n_j^3 + 4\pi\Delta Q p_o \tag{3}$$

Where ΔQ is the deviation of Q from the half integer resonance. This gives the pitch at the extraction location. In this formalism, the equation of the separatrix equipotential curves can be written as

$$p = \left[\frac{\sin[mQ(\phi_E-\phi_O)}{\pi\beta_E} \pm \frac{A(n^2-n_1^2)}{\sqrt{2}n_1}\right]/B\ ;\quad n_1 = \sqrt{\frac{A}{B}\frac{2\pi\beta}{O}}$$

$$A = \Delta Q(\Delta Q - E/2\pi\beta_E)\ ;\quad B = \Delta Q - \frac{E}{4\pi\beta_E}[1-\cos mQ(\phi_E-\phi_O)] \tag{4}$$

The relevant formula for the tracking problem is the impulse approximation version of equation 2.

Ring Design

The ring lattice is shown in Figure 1. The bend sections are second order achromats [4] and the short straight sections have symmetric first order optics such that the chromatic centroid shifting aberrations are corrected. The sextupoles in the bend are placed so that the chromaticity of the ring can be varied without affecting these aberrations. There is a high beta (32M) at the extraction point to minimize the interception of the extraction septum with the beam and to cause the extracted emittance to be mainly dependent upon the extraction pitch.

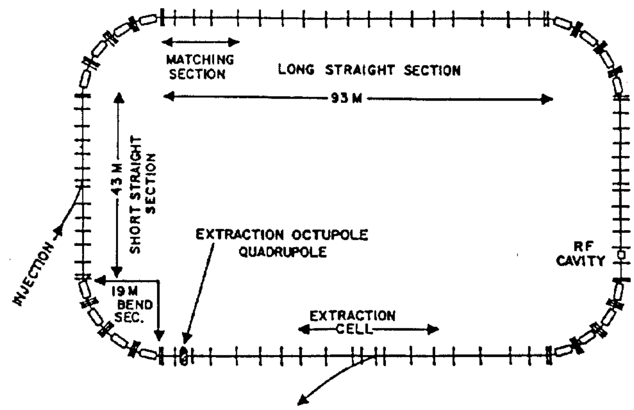

Figure 1. Layout of PSR components

Extraction Simulations

There are three main types of extraction mechanisms considered in this paper. Achromatic extraction uses a zero chromaticity and depends upon changing the tune of the ring to extract particles from the stable area. Monochromatic extraction uses a finite chromaticity coupled to a controlled loss of energy by synchrotron radiation or other means to effectively change particle tune and cause it to be unstable. Chromatic extraction uses a chromaticity also but relies on a tune change to bring the particles into the unstable region.

In general, there are two main features in common to all the above extraction methods. First, the shape of the injected particle phase space distribution does not usually match the shape of the stable area. It takes some number of turns, dependent on the tune and nonlinearities, for the stable particles to accommodate to that shape. During that time, the particles orbit in phase space, sampling a variety of tunes resulting from passing through different positions in the nonlinear elements and eventually approach the point in phase space known as the fixed point, where the tune is near resonance. In this way, the particles are slowed down and the stable phase space is filled up. This situation is common to all resonant extraction techniques when injection is inside the stable area. (See reference 5 for another method.) As will be seen below, the uniformity of the extraction is very dependent upon this process.

Second, the output beam quality must be optimized.

The extracted emittance (in the plane of extraction) will depend upon the location of the extraction septum, as well as the strength of the nonlinear element and the orientation of the separatrix. This must be optimized. A variety of cases were considered under different conditions and some parameters are summarized in Table 2 below.

Half-Integer Resonant Extraction

Single Turn Injection: The most straightforward technique involves centering the injected beam within the stable area and contracting the stable area so as to squeeze the particles into the unstable region optimized for the separatrix orientation and pitch at extraction. Using the parameters in Table 2, the resulting time dependence of extracted beam intensity is shown in Figure 2.

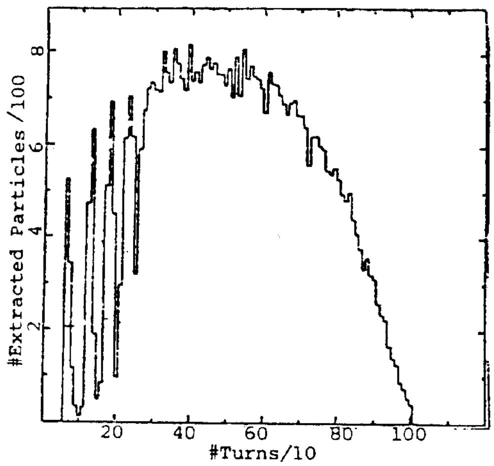

Fig. 2 . Time dependence of Extracted Beam

After about 300 turns, the intensity begins to become uniform. However, the earlier time dependent structure is evident. As the beam phase space ellipse rotates, the area at the intersection points of the curves of the separatrix gets filled and some beam is extracted. After the contraction of the stable area, the shape of the beam phase space is forced to match the stable area and the uniformity is improved. One can improve the initial uniformity by injecting in a phase space too small for the beam at the expense of an initial large extracted beam for the early turns. The overall duty factor is good and the other beam quality factors are excellent. However, for two reasons, we have decided not to propose this technique. First we would like to try to improve the overall uniformity of extraction, and secondly, one turn injection does not produce sufficient average output current for our purposes.

Two-Turn Injection: One injects off axis and after some number of turns a hollow phase space is filled with beam. The input beam and calculated separatrix after two turns of injection is shown in the left hand side of Figure 3. After 100 turns, the phase space filled up is displayed in right side of Figure 3.

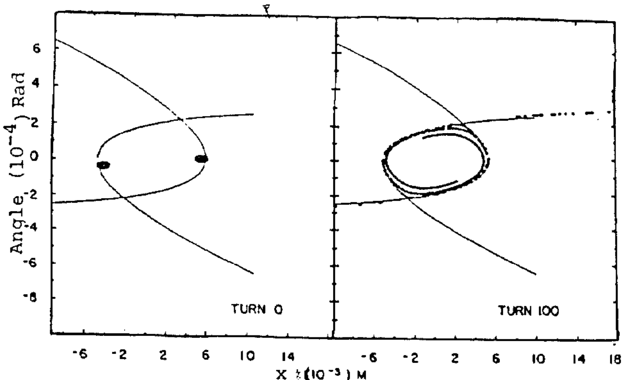

Fig 3 Phase space occupied during Achromatic Extraction

Note the spiral effect and gaps evident near the fixed point vertices. This results from the tune dependence of a particle's position in phase space. The resulting output intensity is shown in Figure 4. The time structure is due to the time it takes for a particle in phase space to travel to the fixed point, or roughly $1/(2\Delta Q)$ turns.

In order to optimize the resulting duty factor, it is beneficial to introduce an additional tune spread in the beam, in such a way as to not increase the nonlinearity of the extraction process. A useful method is to use the energy spread in the beam and couple it with a finite chromaticity producing a tune spread. With this technique, the output time dependence improves.

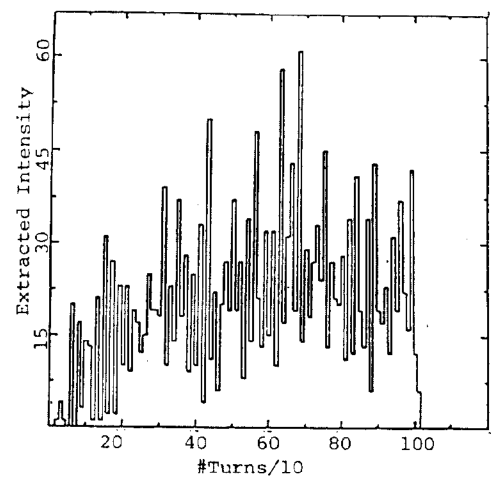

Fig. 4. Time dependence of Extracted Beam

This technique becomes difficult to implement with the inclusion of RF in the ring. With synchrotron

Table 2. Parameters for Some Cases of Resonant Extraction

Case	Half Integer				Third Integer	
	Single Turn Achromatic	Two Turn Achromatic	Chromatic	Monochromatic	Three Turn Achromatic	Three Turn Chromatic
Input Emittance π(mm-mr)	.01	.01	.01	.01	.01	.01
Input Energy Spread	2.(-4)	2.(-4)	2.(-4)	1.(-2)	2.(-4)	2.(-4)
Nominal ΔQ	.015	.04	.04	.04	.015	.015
Tune Period (Turns)	67	25	25	25	67	67
Chromaticity	0	0	10	20	0	5
RF Voltage (keV)	0	0	40	0	0	0
Synch. Period (Turns)	--	--	56	--	--	--
Septum Location (mm)	5	8	8	8	12	12
Output Emittance π(mm-mr)	0.01	0.01	0.01	0.06	0.03	0.03
Output Energy Spread	2.(-4)	2.(-4)	2.(-4)	4.(-4)	2.(-4)	2.(-4)
Output Duty Factor	80%	66%	87%	93%	64%	82%

oscillation, it is possible to oscillate in energy at such a rate that the average energy deviation of a particle from the central energy is zero in a time less than $1./2*\Delta Q$ turns. In this case, the results of extraction become identical to Figure 4. In order to solve this problem, it is necessary to both increase the synchrotron period as long as possible, and increase the velocity of the particles in phase space by increasing ΔQ. This is limited, however, by the necessary complementary increase in octupole strength for maintaining a given stable area which itself must be optimized given other constraints such as ring admittance and extraction pitch and emittance. The resulting optimization processes produce the beam phase space during extraction shown in Figure 5 and the output beam intensity shown in Figure 6.

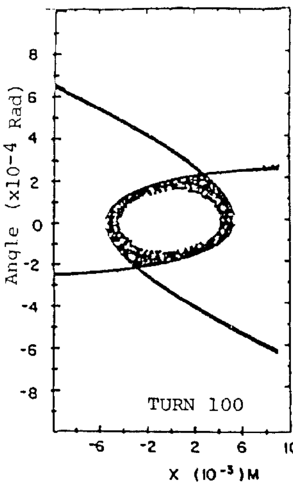

Fig. 5. Phase space for Chromatic Extraction

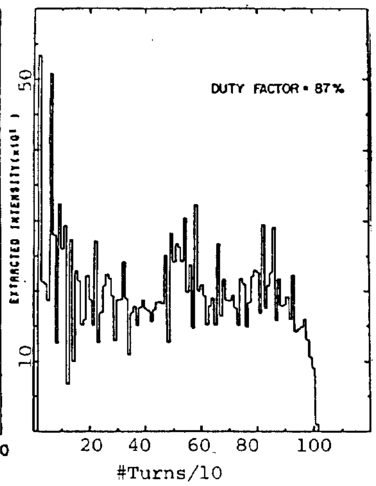

Fig. 6. Time dependence of Chromatic Extraction

Figure 7 shows the output beam phase space after correcting for an angular dependence with time and an angular dispersion. The former comes about as a result of the changing separatrix during the extraction and the orientation of the separatrix, and the latter from the chromaticity. This is the final situation chosen for the proposal.

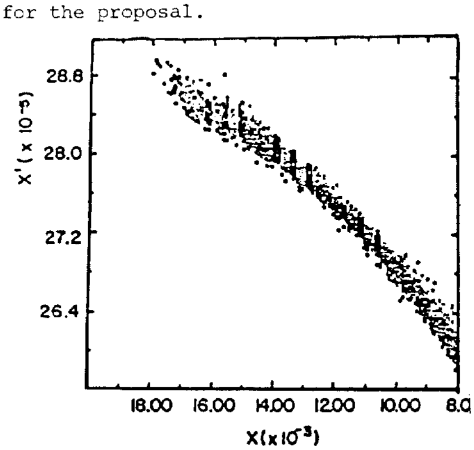

Fig. 7. CORRECTED EMMITANCE

Simulations of monochromatic extraction result in the best duty factor of all methods, however its use is limited in a range where the energy loss per turn is sufficient to extract the beam in the required time. Use of RF to decelerate the beam complicates the situation with synchrotron oscillations. In addition, the high degree of energy dependence of the output beam makes it difficult to correct the output emittance and results in an acceptable, but higher value than other methods.

Third Integer Resonant Extraction

The third integer case is essentially identical to the two-turn, half integer case. The time constants are slightly faster since the number of fixed points are increased. Simulation of this case does not result in an output emittance as good as the corresponding half-integer case.

Operational Considerations

The output beam parameters obtained in the above described simulations are dependent upon the particle's trajectories in phase space, the strength of the magnetic elements and thereby, the location of the circulating beam centroid relative to an extraction septum. The tolerances associated with the pitch, or the location of the septum relative to the beam are not critical. The most unintuitive factor is the tolerance associated with the position in the non-linear element through which the beam passes. This affects the particle tune. Simulations to determine this accuracy were done by examining the effects of this error on the output beam parameters, including the duty factor. Using a criteria of not reducing the duty factor by more than 10%, the necessary tolerances dictate that the beam centroid pass within .5mm from the octupole center.

Drifts in the ring magnetics or injection coordinates can produce non-uniformities in the extracted beam intensity. The response time to effect a change in the extraction mechanism is about 50 turns. Thus some feedback is possible to maintain a uniform extraction. However, given the stability of the MIT-Bates beam, we feel that a feed-forward system will be more important in correcting long term drifts.

Conclusions

A technique has been simulated for extraction from a pulse stretcher ring which indicates that it is possible to achieve a high duty factor beam of excellent quality from a one-GeV pulse stretcher ring. Care must be taken in choosing the nominal tune, RF system parameters, location of the extraction septum, and the perturbing element locations and strengths. Once this is done, the relevant tolerances and specifications indicate that one can realistically expect to achieve an optimized extracted beam.

References

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