## SLOW EXTRACTION AT LAMPF II

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Half-integer resonant extraction will be used to slow extract the 45 GeV proton beam from the LAMPF II main ring during a time spread of $1 / 6 \mathrm{sec}$. High extraction efficiency is obtained by performing the extraction in a high-beta long straight section and by utilizing an electrostatic wire septum and iron septum.

## I. Introduction

The current reference design for the LAMPF II project includes a 60 Hz booster and 3 Hz main ring. The main ring accelerates $6.0 \times 10^{13}$ protons from 7.0-45.0 GeV. Following acceleration, we intend to slow extract the 45 GeV protons for $1 / 6 \mathrm{~s}$ - this corresponds to a $50 \%$ duty factor. Figure 1 shows the planned layout of the accelerators. The main ring is racetrack shaped with a circumference of 1323.3 m . The extraction line is shown at the top of Fig. 1, splitting off from the upper long straight section (LSS). The LSS are each 160 m in length and are dispersion free. In Sec. II we discuss the extraction dynamics and in Secs. III and IV we present simulations of the extraction process.

## II. Extraction Dynamics

Figure 2 shows the behavior of the monoenergetic amplitude functions $\beta_{x}$ and $\beta_{y}$, and the momentum dispersion $n_{x}$, all plotted for one-half the machine length. The magnetic elements are indicated at the top of the diagram. The sections are one LSS, two dispersion suppressor cells, eight standard cells, and two dispersion suppressor cells, respectively. The electrostatic extraction septum $S$ is positioned at the center of the LSS where $B_{x}=350 \mathrm{~m}$. The iron extraction septum $L$ is located $90^{\circ}$ downstream of $S$ in horizontal betatron phase with $\beta_{x}=25 \mathrm{~m}$. The horizontal tunc $v_{x}=7.45$, thus we plan to slow-extract the 45 GeV protons on the $2 v_{x}=15$ half-integer resonance. Quadrupoles supplying a $15^{\text {th }}$ harmonic perturbation can drive this resonance. With two quadrupoles we can create a $15^{\text {th }}$ harmonic perturbation by choosing $15\left(\theta_{2}-\theta_{1}\right)=j \pi$ with $j$ equal to an odd integer and by powering the quadrupoles with equal and opposite strengths; this arrangement creates no $0^{\text {th }} s_{1}$ or $30^{\text {th }}$ harmonic contributions. The angle $\theta_{i}=\delta^{1} d s / \nu \beta$ and represents the location of the $i^{\text {th }}$ quadrupole. We also utilize an octupole pair to introduce an amplitudedependeat tune spread on the $15^{\text {th }}$ harmonic. The horizontal $x-x^{\prime}$ phase space is split into stable and unstable regions, thus facilitating smooth extraction.

We chose to separate the elements of a pair by one-half the machine, i.e., with $j=15$. Figure 2 indicates the positions of the first quadrupole $Q$ and octupole 0 ; these are in the third and seventh regular cells in the arc, respectively. The second quadrupole and octupole reside in the corresponding locations in the second arc. Physically the elements are in the centers of the horizontal sextupoles in the arcs, and would be implemented as correction windings. The lattice functions at these four perturbing elements are $B_{x}=59.15 \mathrm{~m}, \quad \alpha_{\mathrm{x}}=0, \quad 3_{\mathrm{y}}=16.99 \mathrm{~m}, \quad \alpha_{\mathrm{y}}=0, \quad \eta_{\mathrm{x}}=$ 8.153 m , and $\eta_{x}^{x}=0$. Relative to the septum $S$ position, we have for the quadrupoles $\partial_{1}=0.4 \pi$ and $\theta_{2}=1.4 \pi$, and for the octupoles we have $e_{1}=0.668 \pi$ and $\theta_{2}=1.668 \pi$. These locations put the quadrupoles and octupoles nearly in phase on the $15^{\text {th }}$ harmonic.

The extraction process is studied at the septum $S$ using horizontal $x-x^{\prime}$ phase space diagrams. The matched betatron function $\beta_{s}$ increases as the quadrupole power $P_{q}$ is increased. When instability occurs particles arriving at the septum $S$ increase their $x$ amplitude every second turn by the step size $\Delta_{s}$. The septum wires are positioned at $x=x_{s}$; we assume a horizontally applied electric field in the region with $x>x_{s}$. Thus, when a particle arrives at the septum with $\mathrm{x}>\mathrm{x}_{\mathrm{s}}$ it recelves a deflection and later arrives at the iron septum $L$ where it can be extracted out of the machine.

We chose an octupole strength $\mathrm{P}_{0}=3.0 \mathrm{~m}^{-3}$ ( $\equiv \mathrm{B}^{\prime \prime \prime} \ell / \mathrm{Bp}$ ) ; this is the minimum value consistent with proper shaping of the extraction separatrices and achieving the desired step size at $x=x_{s}$. At 45 GeV we expect a transverse emittance area near $1.2 \pi \mathrm{~mm}-\mathrm{mr}$ and $a$ full relative momentum spread $|d p / p| \leqslant 0.07 \%$. Particles near the periphery of this phase space begin to go unstable for $\mathrm{P}_{\mathrm{q}}>0.004 \mathrm{~m}^{-1}$. The machine chromaticity is experter to be near zero so the tunes are not directly dependent upon $d p / p$. During the extraction process we expect to increase $\mathrm{P}_{\mathrm{q}}$ so that after $1 / 6 \mathrm{~s}$ all the beam is extracted.

Since LAMPF II is to be a high-intensity machine, we are especially concerned about maintaining a high extraction efficiency in order to minimize losses. During extraction we define the inefficiency $r$ as the fraction of beam which strikes the septum wires located at $x=x_{s}$. We would like to keep this quantity to less than $1 \%$. It can be expressed by ${ }^{2}$

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\begin{equation*}
r=\frac{t_{s}}{(d x / d n)_{x_{s}}}\left[\int_{x_{s}}^{x_{s}+\Delta_{s}} \frac{d x}{d x / d n}\right]^{-1} \tag{1}
\end{equation*}
$$

where $\mathrm{t}_{\mathrm{s}}$ is the wire thickness ( $\sim 0.1 \mathrm{~mm}$ ) and n is the turn number; the step size at $x=x_{s}$ is given by $\Delta_{s}=2(\mathrm{dx} / \mathrm{dn})_{\mathrm{x}_{\mathrm{s}}}$.

## III. Monoenergetic Particle Tracking

Figures 3(a)-3(d) show the $x-x^{\prime}$ phase spaces at the septum $S$ for the indicated perturbing quadrupole strengths $P_{q}$. The three curves represent the trajectories of ${ }^{q}$ three particles whose starting coordinates are $x=0, x^{\prime}=-\sqrt{\varepsilon / \beta_{S}}$ where the emittances $c$ are noted and $\beta_{s}$ is the matched betatron function for the perturbed system. The particles are tracked for 500 turns and plotted once per turn. The machine consists of five linear transformations, each separated by a perturbing thin lens. The two quadrupoles have focussing power $+\mathrm{P}_{\mathrm{q}}$, and $-\mathrm{P}_{\mathrm{q}}$, respectively; the octupoles have focussing power $+P_{o} x^{2}$, and $-p_{o} x^{2}$, respectively, where $P_{0}=3.0 \mathrm{~m}^{-3}$. Figure 3 (a) shows the machine is whill stable for $P_{q}=0.004 \mathrm{~m}^{-1}$ for the entire expected emittance of $1.2 \pi \mathrm{~mm}-\mathrm{mr}$; Fig. 3(b) shows the same particle rrajactories for $P_{g}=0.00425 \mathrm{~m}^{-1}$. The outer particle is now unstable and "walks" out of the machine. The stable phase space is further squeezed as $P_{q}$ is increased, as is evident in Figs. 3(c) and 3(d); for $\mathrm{P}_{\mathrm{q}}=0.00525 \mathrm{~m}^{-1}$ only particles within $\varepsilon \leqslant 0.048 \mathrm{p} m \mathrm{mmr}$ are still stable. The data for the extracting particles in Figs. 3(b)-3(d) can be used to obtain the inefficiency using Eq. (1). We find empirically $x=\exp \left(a+b n+c n^{2}\right)$ for large $x$.

The coefficients are obtained from a fit to the data in Figs. $3(b)-3(d)$. We derive the step size $\Delta_{s}$ at $x=x_{s}$; then the expression (1) is evaluated. These results are tabulated in Table I.

TABLE I. Extraction Parameters

|  | $\mathrm{P}_{\mathrm{q}}\left(\mathrm{m}^{-1}\right)$ |  | $\Delta_{\mathrm{s}}(\mathrm{mm})$ | r |
| :--- | :---: | :---: | :---: | :---: |
|  |  | $\Delta_{\mathrm{s}}=60 \mathrm{~mm}(\mathrm{~mm})$ |  | r |
| 0.00425 | 8.5 | 0.017 | 13.3 | 0.010 |
| 0.00475 | 14.5 | 0.010 | 20.0 | 0.007 |
| 0.00525 | 19.5 | 0.007 | 24.2 | 0.006 |

The step size and inefficiency change with $P$ (i.e., the emittance of the beam remaining in the machine). The septum wires should be positioned at $x_{s}=65 m$ in order to maintain an extraction efficiency of greater than $99 \%$ over the entire range of $\mathrm{P}_{\mathrm{q}}$ If the septum $S$ length is 4.0 m and the horizontal electric field is $50 \mathrm{kV} / \mathrm{cm}$, then 45 GeV protons will receive an outward kick of 0.43 mr . They will arrive at the Lambertson displaced 40 mm radially and are vertically extracted. Before proceeding, we remark that the unstable trajectories can dictate the horizontal aperture in the bending arcs; excursions up to $\pm 40 \mathrm{~mm}$ occur in the arcs just before extraction.

## IV. Multiparticle Tracking

For a more realistic treatment we should consider particle behavior in the full six-dimensional phase space. Prior to tracking we specify the beam phase space at the septum $S$ at the onset of extraction with $P_{q}=0.00425 \mathrm{~m}^{-1}$. The transverse emittance area in both the $x-x^{\prime}$ and $y-y^{\prime}$ phase planes is taken to be $1.2 \pi \mathrm{~mm}-\mathrm{mr}$ and the longitudinal phase-space area is near $0.1 \mathrm{eVs}$. The matched betatron functions are $\beta_{x}=1040 \mathrm{~m}, \quad \alpha_{\mathrm{x}}=-0.17, \quad \beta_{\mathrm{y}}=14 \mathrm{~m}, \quad \alpha_{\mathrm{y}}=0$, and $\eta_{\mathrm{x}}=$ $\eta_{x}^{x}=0$. These Iunctions then set the limits upon the maximum values of the inftial coordinates $x_{m}, x_{m}^{\prime}, y_{m}$, and $y_{m}^{\prime}$. The initial beam was taken to be an ensemble of particles whose transverse coordinates $x, x^{\prime}, y$, and $y^{\prime}$ were generated according to a water bag distribution, i.e., uniformly in a 4 D hypervolume. The longitudinal distribution is a uniformly filled ellipse in $\Delta \ell-d p / p$ space; the half-bunch length $|\Delta l|$ is 0.5 m and maximum $|d p / p|=0.07 \%$.

The machine now consists of six major sections, each separated by a perturbing thin lens. The action of each section is represented by a matrix transformation accurate to second order in the particle coordinates $\psi_{f}=R_{f i} \psi_{i}+T_{f i k} \psi_{i} \psi_{k}$ where the column vector $\psi_{\mathrm{r}}$ contains $\mathrm{x}, \mathrm{x}^{\prime}, \mathrm{y}, \mathrm{y}^{\prime}, \Delta \ell$, and $\mathrm{d} p / \mathrm{p}, \mathrm{re}-$ spectively. The matrix elements are obtained from the program DTMAT ${ }^{3}$ by concatenating all of the transport elements within a section. The perturbing elements are, respectively, the quadrupole at $\theta=0.4 \pi$, the octupole at $\theta=0.668 \pi$, a single rf cavity at $\theta=\pi$, a quadrupole at $\theta=1.4 \pi$, and the octupole at $\theta=1.668 \pi$. The quadrupoles and octupoles have been described above; the momentum deviation is now included. The kicks from each quadrupole are $\Delta x^{\prime}=-( \pm) P_{q} x(1-\delta)$ and $\Delta y^{\prime}= \pm P_{q} y(1-\delta)$ where $\delta=d p / p$. The kicks fron each octupole are $\Delta x^{\prime}=-\{ \pm) P_{0} x\left(x^{2}-3 y^{2}\right)(1-\delta)$ and $\Delta y^{\prime}=$ $\pm P_{0} y\left(3 x^{2}-y^{2}\right)(1-\delta)$; the $\pm \stackrel{0}{\mathrm{~s}} \mathrm{igns}$ refer to the first and second elements of the pair, respectively. The rf cavity maintains the bunched beam-it is directly opposite the septum $S$ on the other side of the machine in the center of the LSS. We assume the voltage is $5 \mathrm{MV} / \mathrm{turn}$ and the action is only in the $\Delta \ell-\mathrm{d} p / \mathrm{p}$ phase space. The kick is $\Delta \delta=P_{r f} \Delta l$ where $P_{r f}=-e V 2 \pi / B^{3} \lambda E$ where $V$ is the $r f$ voltage, $\lambda$ is the $r f$ wavelength ( $\sim 5 \mathrm{~m}$ ) and $E$ is the total energy ( 45.93 GeV ). This
cavity represents the combined effect of 40 cavities in just the longitudinal phase space.

For tracking we generate 600 particles according to the above defined distribution. The number of turns used was 1000, much less than the actual 38000 but sufficient to study the processes. We fixed $P_{0}=3.0 \mathrm{~m}^{-3}$ as before, and set Prf $=-0.14 \times 10^{-3} \mathrm{~m}^{-1}$. The quadrupole power increased linearly with turn number from $0.00425 \mathrm{~m}^{-1}$ to $0.00525 \mathrm{~m}^{-1}$. The rms emittance of the starting beam was $0.195 \pi$ mm-mr in $x-x^{\prime}$ phase space. Particles were tracked until their $x$ value exceeded 65 mm ( $\equiv \mathrm{x}_{\mathrm{s}}$ ). Figure 4 shows the extracted particle phase space for a subset of the particles- $-95 \%$ of the particles were extracted durlag the 1000 turns. The final coordinates range from 65 to 95 mm , indicating step sizes up to 30 mm . The rms emittance of the extracted beam was $0.046 \pi$ mm-mr-mthis represents a reduction of $x 4.3$. The linear ramp of $P_{q}$ results in a smooth extraction from the machine.

## V. Conclusions

Half-integer resonant extraction from LAMPF II should be possible with an extraction efficiency of better than 99\%. The electrostatic septum needs to be 4.0 m in length with field capability of $50 \mathrm{kV} / \mathrm{cm}$ over a gap width of 3.5 cm .

## References

1. L. C. Teng, "Physics of High Energy Particle Accelerators," Fermilab Summer School, 1981, AIP Conference Proceedings, No. 87.
2. D. A. Edwards, "Comparison of Half Integer and Third Integer Extraction for the Energy Doubler," Fermilab Report TM-842, Dec. 1978 (unpublished).
3. K. L. Brown and R. L. Servranckx, $11^{\text {th }}$ Int. Conf. on High-Energy Accelerators, Geneva, Switzerland, July 1980 , p. 656.

## Figure Captions

Fig. 1. Site Plan for LAMPF If.
Fig. 2. Behavior of Betatron and dispersion functions through one main ring superperiod. Arrows indicate location of electrostatic septum $S$, Lambertson iron septum $L$, and perturbing quadrupole Q and octupole 0.

Fig. 3. $x-x^{\prime}$ phase spaces at the septum $S$ for the indicated values of the power of the perturbing quadrupoles $\pm P_{\text {. }}$. The trajectories $A, B$, and $C$ represent the motion of a single particle starcing at $x=0$, and $x^{\prime}=-\sqrt{\varepsilon / \beta_{s}}$. In (a) and (b) the emittances corresponding to $A, B$, and $C$ are $1.2,0.6$, and $0.3 \mathrm{rmm}-\mathrm{mr}$, respectively. In (c) the emittances are 0.5 , 0.25 , and $0.125 \mathrm{~mm}-\mathrm{mr}$, respectively; in (d) they are $0.05,0.025$, and $0.0125 \mathrm{~mm}-\mathrm{mr}$, respectively. The matched betatron functions $\beta_{s}$ are 925, 1038, 1439, and 3534 m , respectively, for (a)-(d).

Fig. 4. Phase-space plot of extracted beam as described in the text.



Fig. 3


