STOCHASTIC MOMENTUM COOLING OF A LOW ENERGY BEAM AT TARN

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Stochastic momentum cooling is successfully applied to 7 MeV protons and 28 MeV α particles at the INS TARN. The cooling system, with a passband of 20 - 100 MHz, has a notch filter and travelling-wave couplers as a pickup and a kicker. Each coupler is installed with a helical inner conductor. Initial cooling time, final momentum spread, etc. were measured for various system gains and transmission time of the beam signal from the pickup to the kicker. The experimental results are compared with theoretical calculations according to the improved version of a Fokker-Planck equation approach to the stochastic cooling, and satisfactory agreements are obtained.

Introduction

TARN is a low energy, around ten MeV/nucleon, ion storage ring which accumulates the beam both in the longitudinal and transverse phase spaces from the injector sector focusing cyclotron. This storage ring has been mainly used to develop the accelerator technology related to the future project of high energy ion accelerator at INS. Details of the performance of beam accumulation are presented elsewhere.[1] In the meanwhile the stochastic momentum cooling has been also performed in this ring for the low energy 7 MeV/u RF stacked protons and α particles in view of future application of such a technique to the cooler ring for the study of precise nuclear and atomic physics experiments. The stochastic cooling of a low energy beam has been successfully tried at LEAR, CERN where they cool antiprotons, such as 50 or 175 MeV. However recently still lower energy beam, for example 5 MeV, is required for the nuclear experiments with nearly stopped antiprotons. In contrast with high energy beam cooling, two major experimental conditions should be attained. Firstly the beam life due to the residual gas scattering should be much longer than the cooling time. At the average pressure of around 10^{-11} Torr, the beam life of protons and α particles are several thousands seconds which is one order larger than the present cooling time of several hundreds seconds. Secondly the special attention should be paid to obtain the high coupling and high shunt impedance of the pickup and kicker as the particle's velocity is as low as 12 % of the light velocity. In the present experiment a travelling wave type coupler with a helical inner conductor has been used with a sufficient performance. The cooling system including these couplers and a notch filter operates successfully, and experiments have been carried out to measure cooling time, final momentum spread, etc. by varying system gain and signal's transmission time. For the analysis of experimental data, we improved the Sacherer's formulation of a Fokker-Planck equation.[2,3] In the improvement, coherent correction of particle energy is expressed explicitly in terms of the difference of particle-signal's transmission time from time-of-flight of the particle.

This paper describes the experimental results on cooling time, final momentum spread, etc., in compared with our theory. The hardware of the cooling system is presented elsewhere.[4,5]

Time Evolution of Schottky Signal

Figure 1 shows Schottky signals of protons at the 80th harmonic during a cooling process: at 0, 130, and 580 s. The passband is 20 - 95 MHz, and the system gain is 99 dB. The number of protons (N), 3×10^7 at the beginning of the cooling, decreases to 2×10^7 at 300 s. This beam loss is attributed to emittance blow-up caused by multiple Coulomb scattering of the beam from residual gas. The 1/e beam life time is 740 s.

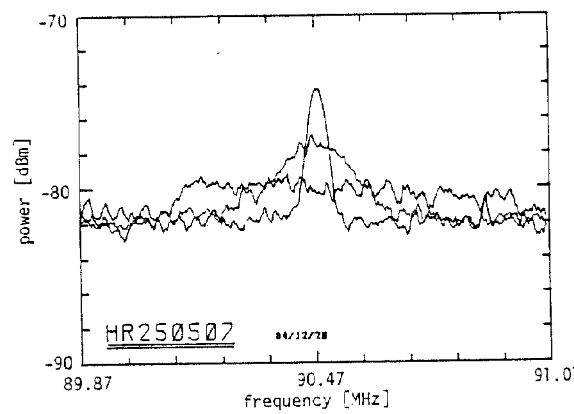

Fig.1 Time evolution of the Schottky signals of protons at the 80th harmonic: at 0, 130, and 580 s. Resolution bandwidth of the spectrum analyzer is 30 kHz.

The coupling impedance of the pickup (Z_p) is experimentally obtained from the electric power of the Schottky signal. The measured powers per Schottky band are 3.24×10^{-15} W before the cooling, and 1.45×10^{-15} W after the cooling. The width of a Schottky band equals to the particle's revolution frequency; f_0 = 1.13 MHz. The power per Schottky band is theoretically given by

$$P_b = 2\ e^2 f_0^2\ N \frac{|Z_p|^2}{Z_c}\ , \tag{1}$$

where Z_c, measured at 100 Ω, is the characteristic impedance of the pickup. From this equation, we have $|Z_p|$ = 406 Ω from the Schottky signal before the cooling, and 332 Ω from that after the cooling. The disagreement between the two values is due to inaccuracy of the measurement of the Schottky signal power. On the other hand, the amplifier noise power per band measures 6.75×10^{-15} W, whereas the theoretical value is 6.24×10^{-15} W, calculated from

$$P_a = 10^{NF/10}\ k\ T\ f_0\ , \tag{2}$$

with NF (noise figure of the preamplifier) = 1.4 dB, T = 290 K, and k = 1.381×10^{-23} J/K.

The time variation of the momentum spread in FWHM is shown in Fig.2 for two values of system gain: one is 99 dB, and the other is 109 dB. The momentum spread decreases exponentially with time at the beginning stage of the cooling, and stays at a final value. The

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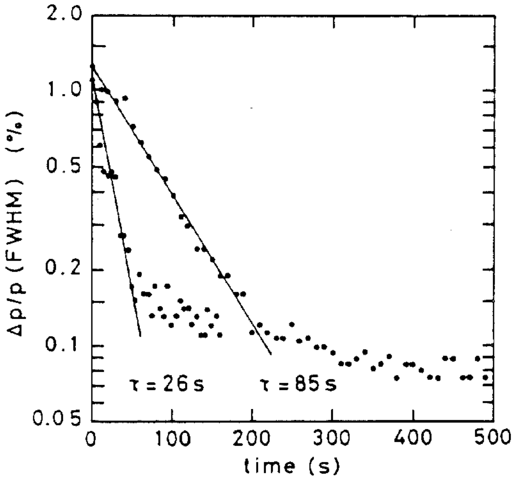

Fig.2 Decrease of momentum spread in FWHM with time for two cases of the system gain, 99 dB and 109 dB. A smaller final momentum spread is attained with a lower system gain at cost of the cooling time.

initial 1/e cooling time of 85 s at the 99 dB system gain is shortened to 26 s at the higher gain of 109 dB. On the other hand, the final momentum spread of 0.08 % at 99 dB increases to 0.125 % at 109 dB. These dependences on system gain are predicted by theory, and discussed in more detail in following paragraphs.

Cooling Time and System Gain

Initial cooling time is measured for various values of system gain. The result is shown in Fig.3 with dots, in comparison with theoretical predictions denoted by solid lines. The theory is described below in this paragraph. The measured initial cooling time is inversely proportional to system gain, except for the case of the 114 dB system gain. This exception is due to inaccuracy of the measurement because of significant beam loss at this high system gain.

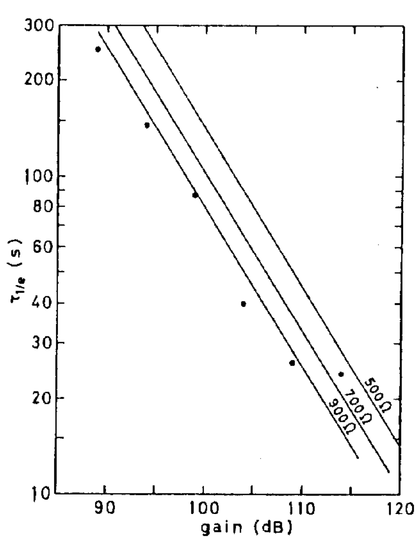

Fig.3 Dependence of the initial cooling time on the system gain. Measured values are denoted with dots, and the results of Eq.(14) with $|Z_p f_k|$ = 500, 700, and 900 Ω are with solid lines.

The theory is formulated with following assumptions for simplicity. First, the notch filter is assumed ideally to be linear:

$$|H(f)| = g_{pole}\left|\frac{f-nf_0}{f_0/2}\right| \qquad ((n-\tfrac{1}{2})f_0 \leq f \leq (n+\tfrac{1}{2})f_0) \quad (3)$$

$$\phi_{nf}(f) = \begin{cases} +\dfrac{\pi}{2} & (nf_0 \leq f \leq (n+\tfrac{1}{2})f_0) \\ -\dfrac{\pi}{2} & ((n-\tfrac{1}{2})f_0 \leq f \leq nf_0) \end{cases} , \quad (4)$$

where g_{pole} is the amplitude at the pole. Second the amplifier gain (G_{amp}), the coupling impedance of the pickup (Z_p), and the efficiency of the kicker (f_k) are constant in the passband, ($n_1 f_0$, $n_2 f_0$). Third, no Schottky band overlap takes place. For a particle with energy error (E) from the central energy (E_0), coherent energy correction per turn is

$$\Delta E_c = -4\left(\frac{qe}{A}\right)^2 f_0\, G\, |Z_p f_k|\, \kappa\, S(\alpha,\xi)\,\frac{E}{E_0} , \quad (5)$$

where

$$G \equiv G_{amp}\, g_{pole} , \quad (6)$$

$$\alpha \equiv \arg(Z_p f_k) , \quad (7)$$

$$\kappa \equiv \frac{(\Delta f_0/f_0)}{E/E_0} = 0.354 \quad \text{(at TARN)} , \quad (8)$$

$$S(\alpha,\xi) \equiv -\sum_{n=n_1}^{n_2} n \sin(n\xi - \alpha) , \quad (9)$$

$$\xi \equiv 2\pi f_0 T_F\left(\kappa\frac{E}{E_0} + \frac{\Delta T_D}{T_F}\right) , \quad (10)$$

$$\Delta T_D \equiv T_D - T_F , \quad (11)$$

T_F: time-of-flight of a particle, with energy E_0, from the pickup to the kicker,
T_D: signal's transmission time.

With adequate values of ΔT_D and the passband, $S(\alpha,\xi)$ stays almost constant,

$$S(\alpha,\xi) \simeq \frac{1}{2}(n_2{}^2 - n_1{}^2) , \quad (12)$$

in the range of ξ,

$$2\pi f_0 T_F\left(-\kappa\frac{\Delta E_0}{2E_0} + \frac{\Delta T_D}{T_F}\right) \leq \xi \leq 2\pi f_0 T_F\left(\frac{\Delta E_0}{2E_0} + \frac{\Delta T_D}{T_F}\right) , \quad (13)$$

where ΔE_0 is the full width of E; we have energy correction almost proportional to energy error. Then we obtain the single particle cooling time, i.e. cooling time in the case of no noise;

$$\tau_0 = \frac{E_0}{2(qe/A)^2(n_2{}^2 - n_1{}^2) f_0{}^2 |\kappa G Z_p f_k|} , \quad (14)$$

where G is system gain, $G_{amp} g_{pole}$. At our cooling, the gain of the amplifier is reduced to attain a small final momentum spread, and then the initial cooling time approximates the single particle cooling time. The cooling time is inversely porportional to the system gain, as in Eq.(14). The cooling time is calculated for various values of $|Z_p f_k|$, and the results are shown in Fig.3 as already mentioned. The best fit is obtained with $|Z_p f_k|$ = 900 Ω.

Final Momentum Spread and System Gain

Measured values of final momentum spread in FWHM are shown in Fig.4 as a function of system gain. The momentum spread is approximately proportional to square root of system gain.

In the case of the above mentioned reduced system

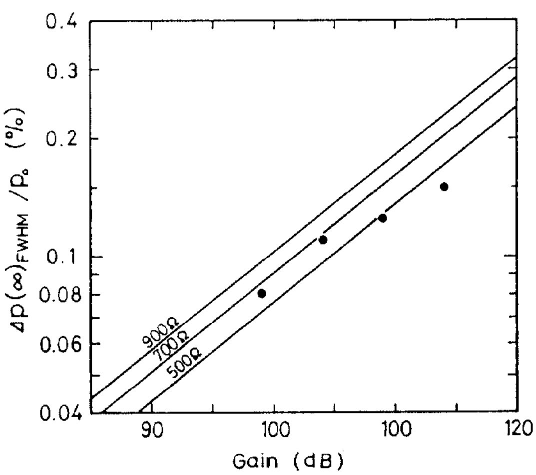

Fig.4 Dependence of the final momentum spread on the system gain. Experimental values are denoted with dots, and the results of Eq.(15) with $|Z_p f_k|$ = 500, 700, and 900 Ω are with solid lines.

gain, we have

$$\frac{\Delta p(\infty)}{p_0} = \frac{1}{(n_2+n_1)|\eta_f|}\sqrt{\frac{\ln 4}{3}\,\frac{N\,U_p}{(n_2-n_1)f_0\tau_0}}\,, \tag{15}$$

where η_f is frequency dispersion $((\Delta f/f_0)/(\Delta p/p_0)$ = 0.705), $U_p \equiv P_a/P_b$, and the final distribution function is assumed to be Gaussian. Equation (15) results in a final momentum spread proportional to $\sqrt{G}$, because $\tau_0 \propto G^{-1}$. In Fig.4, the result of calculation for Eq.(15) is shown with solid lines, for various values of $|Z_p f_k|$. In the calculation, $|Z_p|$ is set at 370 Ω, the average of the two values obtained above with Eq.(1). The calculation with $|Z_p f_k|$ = 500 ∿ 700 Ω agrees well with the experimental result.

Cooling of α Particles

28 MeV α particles are RF stacked in the initial momentum spread of ∿ 1 %. In the experiment, the beam life time was measured at around 2000 seconds at the average vacuum pressure of 3.4×10^{-11} Torr. Schottky signals of coasting α particles at the 80th harmonic are given in Fig.5 where the initial momentum spread

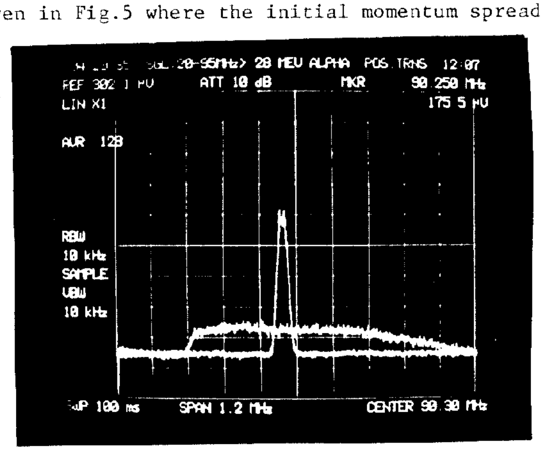

Fig.5 Schottky signals of coasting 28 MeV α particles ($N = 1.2 \times 10^7$) at the 80th harmonic. Before cooling, the momentum spread (FWHM) is 1.00×10^{-2} and it is reduced to 6.24×10^{-4} after 420 seconds cooling.

(FWHM) of 1.00×10^{-2} is reduced down to 6.24×10^{-4} after 420 seconds cooling. Theoretically the initial cooling time τ_0 depends upon the charge qe and mass number A of the particle in the form of $\tau_0 \propto (q/A)^{-2}$ and the final energy spread $\sigma_E(\infty) \propto A/q^2$. The analysis of the cooling data of α particles is now in progress from the point of view of q, A dependence.

Acceleration by Particle's Signal

Removing the notch filter from the cooling system, we can accelerate or decelerate the beam: a particle is kicked by its own signal provided that the signal's transmission time is set to be nearly equal to the time-of-flight of the particle. Measured acceleration rates are shown in Fig.6 with dots. The amplifier gain is set at 101 dB, and the system passband is 20 - 105 MHz. An error within ± 1 ns is allowable.

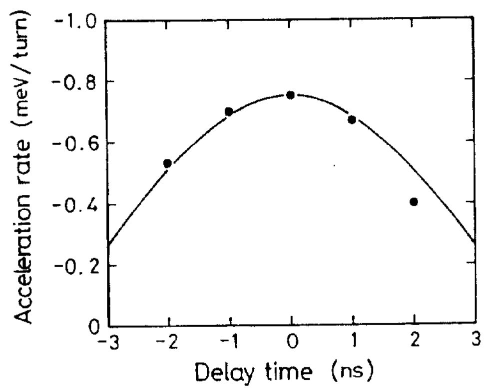

Fig.6 Rate of acceleration by particle's signal. Dots denote measured values, and the solid line the result of Eq.(16) with $|Z_p f_k|$ = 240 Ω. The delay time where the acceleration rate takes its maximum is defined to be zero.

The acceleration rate per turn is theoretically given by

$$\Delta E_\alpha \simeq 2\left(\frac{qe}{A}\right)^2 f_0 G_{amp}|Z_p f_k|\frac{\sin n_2\xi - \sin n_1\xi}{\xi}\,, \tag{16}$$

$$\xi \equiv 2\pi f_0 (T_D - T_F)\,. \tag{17}$$

In the derivation of Eq.(16), G_{amp}, Z_p, f_k, and the signal's delay time is assumed to be constant, being independent of frequency. The result of Eq.(16) with $|Z_p f_k|$ = 240 Ω is shown in Fig.6 with a solid line. The theoretical curve fits well the experimental data, though $|Z_p f_k|$ is smaller than those obtained from cooling time and final momentum spread.

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