LONGITUDINAL COUPLING IMPEDANCE FOR A BEAM PIPE WITH A CAVITY*

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Summary

A method is presented for the computation of the longitudinal coupling impedance of an azimuthally symmetric obstacle of general shape in a long beam pipe. The method involves a numerical calculation of the fields at mesh points in the cavity alone, from which the additive contribution of the cavity to the coupling impedance of the beam pipe can be obtained as a surface integral confined to the cavity wall. Numerical work for obstacles of various geometries is in progress.

Introduction

The solution for the electromagnetic fields in a cavity - beam pipe combination driven by a periodic current source is complex, even for the simplest geometries. Keil and Zotter[1] have obtained the result for the longitudinal coupling impedance for a beam pipe of circular cross section and large circumference connected to a cylindrical cavity. They match field solutions within the beam, between the beam and the beam pipe walls, and in the cavity outside the beam pipe radius, and obtain the result for the coupling impedance as a slowly convergent infinite series.

In this paper we treat the field in the cavity region as a modification of that in the beam pipe. The solution for this modification of the field in the cavity can then be obtained numerically for an azimuthally symmetric obstacle of general shape by relatively simply changes in the numerical program SUPERFISH.[2]

Field Analysis

We consider a beam pipe of cross sectional radius b and circumferential length $2\pi R$ in which an azimuthally symmetric cavity-like obstacle with dimensions small compared to R is located, as shown in Figure 1. The longitudinal coupling impedance is defined as[3]

$$Z_L(\omega) = - \int \vec{J}^* \cdot \vec{E} dv / |I_o|^2 \tag{1}$$

where $\vec{E}$ is the electric field in the cavity beam pipe combination due to a driving current given by

$$J_z(r,z,t) = \begin{cases} (I_o/\pi a^2)\, e^{i\omega z/v} & , \ r < a \\ 0 & , \ r > a \end{cases} \tag{2}$$

The factor $\exp(-i\omega t)$ has been omitted from all fields and currents.

The solution for the fields for a lossless beam pipe without an obstacle (denoted by subscript 1) is well known.[4] Specifically, it is

$$E_{z1} = \frac{I_o}{\pi a^2} \frac{e^{i\omega z/v}}{i\omega\varepsilon} \begin{Bmatrix} 1 - \sigma a F_1(\sigma a)\, I_o(\sigma r) \\ \sigma a I_1(\sigma a)\, F_o(\sigma r) \end{Bmatrix} \tag{3}$$

$$E_{r1} = \frac{I_o}{\pi a^2} \frac{e^{i\omega z/v}}{\varepsilon\sigma v} \begin{Bmatrix} \sigma a F_1(\sigma a)\, I_1(\sigma r) \\ \sigma a I_1(\sigma a)\, F_1(\sigma r) \end{Bmatrix} \tag{4}$$

$$H_{\phi 1} = \varepsilon v\, E_{r1} \tag{5}$$

where

$$F_o(x) = K_o(x) - \frac{K_o(\sigma b)}{I_o(\sigma b)} I_o(x) \tag{6}$$

$$F_1(x) = K_1(x) + \frac{K_o(\sigma b)}{I_o(\sigma b)} I_1(x) \tag{7}$$

$$\sigma = \omega/v\gamma \quad , \quad \omega/v = n/R \tag{8}$$

and where the upper and lower entries in { } correspond to $r < a$ and $r > a$.

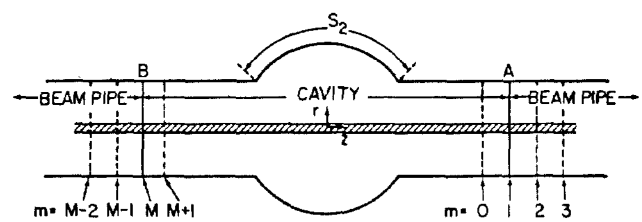

Figure 1. Typical cavity/beam pipe geometry.

The fields in the cavity/beam pipe combination (denoted by subscript 2) satisfy Maxwell's equations in the form

$$\nabla \times \vec{E}_2 = i\omega\mu\, \vec{H}_2 \ , \ \nabla \times \vec{H}_2 = -i\omega\varepsilon\, \vec{E}_2 + \vec{J} \ , \tag{9}$$

which is the same as that for the fields for the beam pipe without cavity:

$$\nabla \times \vec{E}_1 = i\omega\mu\, \vec{H}_1 \ , \ \nabla \times \vec{H}_1 = -i\omega\mu\, \vec{E}_1 + \vec{J} \ . \tag{10}$$

If we write $\vec{E}_2 \equiv \vec{E}_1 + \vec{e}$, $\vec{H}_2 \equiv \vec{H}_1 + \vec{h}$, then $\vec{e}$ and $\vec{h}$ satisfy the homogeneous Maxwell equations:

$$\nabla \times \vec{e} = i\omega\mu\, \vec{h} \ , \ \nabla \times \vec{h} = -i\omega\varepsilon\, \vec{e} \tag{11}$$

as well as the wall boundary condition

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$$\vec{n}_2 \times \vec{e} = - \vec{n}_2 \times \vec{E}_1 \text{ (on boundary surface } S_2). \quad (12)$$

Here $\vec{n}_2$ is a unit vector normal to the cavity/beam pipe wall surface. Clearly $\vec{n}_2 \times \vec{e} = 0$ on that portion of the boundary which coincides with the beam pipe. Equations (11) and (12) thus represent an equivalent SUPERFISH problem with specified frequency and boundary condition, and which can be solved on the usual triangular mesh in the cavity region. The only complication comes from the boundaries A and B which separate the cavity region from the beam pipe. These are discussed in the next section.

Boundary Equations at A and B

Let us denote the values of h_ϕ at mesh points corresponding to the same $z = z_m$ but for different r as h_m. For the beam pipe we assume a mesh structure in which the triangles connecting m-1 to m are a reflection of those connecting m to m+1. In this way the matrix equations in the pipe become

$$Bh_{m-1} + D_1 h_m + Bh_{m+1} = 0, \quad (13)$$

$$Fh_m + D_2 h_{m+1} + Fh_{m+2} = 0 \quad (14)$$

This choice insures that the eigenvalue of the phase advance for propagating modes in the pipe are exactly real, thus avoiding any spurious growth of field amplitude for a long beam pipe.

We can use Eqs. (13) and (14) to express h_m in terms of h_{m+2}, h_{m-2}. Repeated application of this reduction allows us to express h_m in terms of $h_{m\pm\ell}$, where $\ell = 2^p$. Further application of this reduction procedure and Eqs. (13) and (14) allows us ultimately to express h_o and h_{M+1} in terms of h_1 and h_M. These equations are then the boundary equations for SUPERFISH at m = 1, m = M (z = A, B) since h_o and h_{M+1} are now internal to the cavity.

With each matrix block corresponding to the dimensions of the "vector" h_m, the matrix equations are again in the form of tridiagonal blocks, as in SUPERFISH, but they now also contain non-vanishing entries in the upper right and lower left corner blocks. The matrix reduction process used in SUPERFISH can be readily modified to include these two non-vanishing blocks.

Longitudinal Impedance

The longitudinal coupling impedance corresponding to the field in Eq. (3) can be written, for $b \ll \gamma R/n$, as[4]

$$\frac{Z_{L1}}{nZ_o} = \frac{i}{\beta\gamma^2} \left(\frac{1}{4} + \ln \frac{b}{a} \right) \quad (15)$$

where $Z_o = \sqrt{\mu/\varepsilon} = 377$ ohms is the impedance of free space. Once the values of h_ϕ are obtained numerically for the cavity, it is necessary to recalculate Eq. (1) in order to obtain the modified coupling impedance.

The result can be put into a more convenient form by using the identity

$$\int_{V_2} dv\, \nabla \cdot (\vec{E}_1^* \times \vec{H}_2 + \vec{E}_2 \times \vec{H}_1^*)$$
$$= - \int dv\, (\vec{E}_2 \cdot \vec{J}^* + \vec{E}_1^* \cdot \vec{J}) \quad (16)$$

obtained from Eqs. (9) and (10), where the integral is taken over V_2, the cavity/beam pipe total volume. (For those portions of V_2 outside the beam pipe volume V_1, $\vec{E}_1$, $\vec{H}_1$ are defined by Eqs. (3)-(5).) The right side of Eq. (16) can be written, using Eq. (15), as

$$|I_o|^2 (Z_{L2} + Z_{L1}^*) = |I_o|^2 (Z_{L2} - Z_{L1}). \quad (17)$$

The left side can be written as an integral over the wall surface S_2. Since $\vec{n}_2 \times \vec{E}_2 = 0$ on S_2, only the first term survives, giving

$$\int_{S_2} dS\, (\vec{n}_2 \times \vec{E}_1^* \cdot \vec{H}_2) = - 2\pi \int_{s_2} r\, H_{\phi 2} (\vec{E}_1^* \cdot \vec{ds}) \quad (18)$$

where the surface integral over S_2 (or the line integral over s_2) now extends only over that portion of the surface different from S_1, since $\vec{n}_2 \times \vec{E}_1^* = 0$ everywhere S_2 and S_1 coincide. Thus we can express the contribution of the cavity to the coupling impedance as the line integral

$$\frac{\Delta Z_L}{nZ_o} \equiv \frac{Z_{L2} - Z_{L1}}{nZ_o}$$
$$= - \frac{2\pi}{|I_o|^2} \int_{s_2 \neq s_1} r\, H_{\phi 2} (\vec{E}_1^* \cdot \vec{ds}) , \quad (19)$$

a quantity readily obtained from the numerical results for $h_\phi = H_{\phi 2} - H_{\phi 1}$.

Numerical work for various cavity/beam pipe geometries is in progress.

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References

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2. K. Halbach and R.F. Holsinger, Particle Accelerators 7, 213 (1976).

3. See, for example, A.W. Chao, 1982 Summer School Lectures, SLAC, p. 396.

4. See, for example, Nielsen, Sessler and Symon, Proc. of the Int'l. Conf. on High Energy Accelerators, Geneva, 1959, p. 239.