COLLECTIVE EFFECTS AND THE DESIGN OF THE SSC*

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Evaluation of possible designs for the Superconducting Supercollider requires consideration of a variety of collective phenomena. Of particular interest is the scaling of current thresholds and growth rates with such parameters as machine circumference and aperture, magnetic field strength, momentum spread and acceptance, cell length, and phase advance. In this paper the interplay of collective effects and machine design is discussed with special attention to those issues which may be helpful in choosing a particular design.

Basic Issues

The self-interaction of a storage ring beam through its wakefield can lead to coherent instabilities which may cause beam loss. The electromagnetic environment is summarized by frequency-dependent longitudinal and transverse impedances which are determined by vacuum chamber smoothness and aperture, pick-up coupling and bandwidth, cold bore wall resistance, and so forth. Beam dynamics is sensitive to lattice functions and momentum spread, and the resulting threshold currents may be addressed with feedback systems of varying bandwidth, power, and complexity. The three elements of impedance environment, beam dynamics, and feedback utilization together determine whether a particular machine configuration will be successful in achieving performance goals.

For comparison of different approaches, we fix both luminosity and number of interactions per crossing. For the same RF frequency and bucket fill factor, this assumption yields constant bunch length σ_l, particles per bunch N_B and average total current I_0. Typical values for $10^{33}\mathrm{cm}^{-2}\mathrm{sec}^{-1}$ luminosity are σ_l = 0.07cm, $N_B = 1.5 10^{10}$. and I_0 = 85 ma [1].

Single-Bunch Instabilities

Both transverse and longitudinal single-bunch instabilities have limited storage ring current, with the transverse the major limitation in existing and proposed large electron storage rings. Because of its large beta-functions and small aperture, the transverse, single-bunch instability appears to be the primary limit in the SSC for rms momentum spread $\gtrsim 5\ 10^{-5}$. This is in contrast to the present generation of proton colliders, the Sp$\bar{\text{p}}$S and Tevatron, where longitudinal instability may be at least as important.

At low frequencies the threshold condition of mode-coupling due to coherent tune shifts is of the form [2,3,4]

$$N_B = \frac{8\pi^{3/2}E/e}{ec}\, R\, \frac{n}{\beta Z_{t,\ reac}}\, \frac{\sigma_E}{E}\, F_1 \qquad (1)$$

where $F_1 \simeq 1.5$ in the long bunch limit as determined by eigenmode analysis.

At high frequencies, the fast-blow-up condition yields

$$N_B = \frac{8\pi^{3/2}\ E/e}{ec}\, R\, \frac{n}{\beta Z_{t,\ res}}\, \frac{\sigma_E}{E}\, \frac{n_c\sigma_l}{\pi^{1/2}R}\, F_2 \qquad (2)$$

where $f_c = n_c f_0$ is the "resonant" frequency of the broad-band impedance. For a Q = 1 resonator the reactive part of Z_t at low frequency is equal in magnitude to the resistive transverse impedance at resonant frequency (typically at the cut-off of the beam pipe). For a 3 cm diameter beam pipe $f_c \sim$ 5GHZ, the factor $n_c\sigma_l/(\pi^{1/2}R)$ is greater than unity, and it would appear that the low frequency coupled-mode instability would dominate. However, discontinuities such as bellows and bellow shields can exhibit behavior typified by a higher Q [7]. Allowing for smearing of the $Z_{t,\ res}$ in equation (2) by finite bunch length effects, the fast blow-up threshold can approach the coupled-mode threshold current.

In either case the figure of merit in evaluating machine designs with respect to current limits is

$$\frac{n}{\beta}\quad \frac{R}{Z_t}\quad \frac{\sigma_E}{E} \qquad (3)$$

For beam pipes of similar smoothness, Z_t scales linearly with R. The value of $\sigma_{E/E}$ may be limited by the RF bucket height or the momentum acceptance (either physical or dynamic) of the lattice. The RF limit is soft in so far as it can be increased with additional voltage up to the point that the bucket height is comparable to the momentum acceptance. The lattice momentum acceptance (including random errors, etc.) provides the fundamental limitation on $\sigma_{E/E}$. For a FODO lattice

$$\frac{n}{\beta} \sim \frac{L}{R^2}\ f(\mu) \qquad (4)$$

where f(60°) ≃ 1., f(90°) ≃ .65 and L is the cell length. This suggests that low phase advance, long cell length, and small circumference (low tune) are to be preferred unless the momentum acceptance is severely affected.

Since the transverse, single-bunch instability has not been clearly observed in a proton storage ring, a few comments are in order. First, the SPS operates at top energy essentially at the threshold given by equation (1) [2]. At injection energy, this threshold is exceeded by a factor of five with space-charge induced tune spreads the probable cause of enhanced stability. Secondly, the gaussian bunch shape assumed in most analyses provides more severe mode-coupling than, for example, a rectangular current distribution [4]. The distribution typical of

* SSC-25
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0018-9499/85/1000-2392$01.00© 1985 IEEE

proton bunches is somewhere in between. Also, the success of the mode-coupling theory has been in electron machines where synchrotron frequency spread is unimportant. For SSC bunches synchrotron sideband overlap occurs at frequencies of a few Gigahertz. Also, the relatively long bunches allow for feedback on low modes. These issues raise the possibility that the form factor F_1 may be enhanced somewhat, but the basic scaling and figure of merit in relation (3) will remain unchanged.

Impedance and Threshold Current Estimates

First we will estimate the impedance of a "standard" 15000 meter radius ring with a full aperture of 33mm. Extrapolation to larger radius and smaller bore will follow together with a comparative study of four test lattices for the SSC.

Since the RF system of the SSC will be comparable to that of PEP and PETRA, rings some fifty times smaller, the machine impedance of the SSC will not be dominated by the cavities. Primary sources of broad-band impedance will be objects of great number such as bellows and pick-up electrodes. With fractional contraction of .004 and allowing bellows to shorten by only a third of their length, 1.2% of the ring will be bellows. Standard estimates indicate that such structures would exhibit a longitudinal $Z/n \simeq 1.2\ \Omega$ and $Z = 135M\ \Omega/m$. Since these values are far and away the largest impedance that has been recognized, shielding is recommended. To estimate the possible success of such a shielding program, we have taken the measured long-bunch tune shifts of PETRA [5], subtracted the known RF cavities and separator tanks, and have scaled the transverse impedance by the inverse radius cubed. It is assumed that the residual impedance is due to the bellow shields. For 5000 such shields in the SSC, a transverse impedance of $20M\ \Omega/m$ and a longitudinal Z/n of $.17\ \Omega$ are obtained. This rather circuitous method was chosen since some uncertainty continues to exist between calculation and observation of the PETRA impedance [5,6]. It has the advantage of scaling from an existing machine which was engineered with electrical smoothness as a goal and provides an alternative to the Reference Designs Study itemization [1].

A superconducting storage ring requires beam position monitoring at low currents during start-up to avoid magnet quenching. The pick-up electrodes will offer considerably more impedance than has been the case in electron storage rings. If both horizontal and vertical strip-lines of the type used in the Tevatron are placed in each quadrupole ($\simeq$ 4000 such objects), approximately $10\ M\ \Omega/m$ or transverse impedance and $Z/n = .08\ \Omega$ can be expected on averaging over the bunch spectrum [1,8]. Since overall pick-up response is required only to about 100 MHz, narrow bandwidth pick-ups can be considered [8] to reduce coupling to the $m = 0$ mode, which in turn can be handled fairly easily with feedback. We note that this pick-up configuration represents an increase in pick-up number from the Reference Designs Study [1].

The impedance introduced by the cold copper wall provides the major high-Q, transverse impedance ($300M\ \Omega/m$) which will drive coupled-bunch instabilities. A value of wall resistance of $5\ 10^{-10}\ \Omega$-m appears to be consistent with magneto-resistance [10] at 6 Tesla and the anomalous skin effect [11] out to several Gigahertz. Full evaluation of the anomalous skin effect in the presence of magneto-resistance awaits future measurements. The broad-band contribution appears to be small. Finally, allowances need to be made for the RF, warm bore abort sections, abort and injection kickers, and miscellaneous discontinuities. Values of $Z/n \simeq .10\ \Omega$ and $Z_t = 20M\ \Omega/m$ appear possible together with a peak, high-Q longitudinal parasitic mode impedance of $2.6M\Omega$ from higher order RF cavity modes and an additional high-Q $Z_t \simeq 100\ M\ \Omega/m$ from warm copper in high beta-function abort sections [1].

From these estimates total average broad-band impedances of approximately $Z/n \simeq .35\ \Omega$ and $Z_t \simeq 50\ M\ \Omega/m$ can be expected with shielded bellows. Without shielding values increase to $Z/n \simeq 1.4\ \Omega$ and $Z_t \simeq 165\ M\ \Omega/m$.

To investigate the scaling of the transverse coupled-mode instability with ring size, representative 6 Tesla and 3 Tesla designs (Table 1) with 60° and 90° phase advance per cell were evaluated. The 3T lattices were approximately a factor of 1.8 larger in circumference, and the cell length was a factor of 1.45 longer. For rings of similar longitudinal smoothness, the transverse impedance can be expected to scale linearly with circumference. This would be true, for example, for bellows which must account for roughly 1.2% of the machine to allow for contraction during cool down. On the other hand, pick-up electrodes, possibly in every quadrupole, would be spaced by the half cell length, and the total transverse impedance would scale as the ratio of circumference to cell length. Other impedance from kickers and warm bore area would remain constant.

Table I
Representative Lattices

	60° 6T	90° 6T	60° 3T	90° 3T
R	15661m	15845m	28032m	28586m
$\bar{\beta}$	212m	167m	307m	242m
η	$1.78\,10^{-4}$	$9.0\,10^{-5}$	$1.13\,10^{-4}$	$5.34\,10^{-5}$
Lcell	200m	200m	290m	290m

For various reasonable impedance mixes (e.g. Reference Designs study itemization or PETRA bellows estimate as above) scaling between 1.4 - 1.7 could be obtained. Since a larger ring may have longer magnets, if the inherently longer bellows could be as effectively shielded as the shorter bellows in the higher field design, the increase in transverse impedance with machine radius could be further minimized. Finally, a reduction of aperture from b = 33mm to b = 25mm yields an increase of transverse, broad-band impedance by a further factor between 1.7 - 2.0 when discontinuities are scaled as b^{-3}, pick-ups as b^{-2}, and warm bore as b^0.

If a value $Z_t = 50M\ \Omega/m$ is assumed for the 6 Tesla designs, the above scaling with $F_1 = 1$ yields threshold $\sigma E/E$ for $N_B = 1.5\ 10^{10}$ as summarized in Table II.

Table II
Minimum $\sigma E/E$

	60°	90°
6T, 33mm	6.1 10^{-5}	9.5 10^{-5}
3T, 33mm	1.1-1.3 10^{-4}	1.8-2.2 10^{-4}
3T, 25mm	1.8-2.6 10^{-4}	3.0-4.4 10^{-4}

The associated energy acceptance should be a factor of four larger than the entries of Table II to allow for a 3:1 bucket to bunch area ratio.

For a momentum acceptance at the $2 10^{-3}$ level, the 6T lattices would be a factor of 5-8 below threshold. Cost saving on RF voltage could be achieved if the safety margin is correspondingly reduced. On the other hand, the smaller aperture 3T design is only marginally stable. Given uncertainties in impedance estimates and form factors, it is difficult to draw absolute conclusions without detailed hardware designs, but the trend is clear.

Coupled-Bunch Oscillations

In scaling with storage ring dimension, the high-Q, longitudinal impedance remains unchanged if it is assumed that parasitic modes of RF cells are detuned to avoid coincidence in frequency. The low frequency resistive wall transverse impedance scales nearly linearly with R for the mix of impedance in the Reference Designs Study when allowance is made for reduced magneto-resistance at 3T. Aperture dependence is as $1/b^{2.5}$.

For fixed average current I_0, the growth rate for longitudinal dipole coupled-bunch oscillations is given by

$$\frac{1}{\tau_L} = \frac{I_o}{4\pi E/e}\,\omega Z_{par}(\omega) e^{-\omega^2\sigma_t^2}\;\frac{\sigma_l}{R}\;\frac{E}{\sigma_E} \qquad (5)$$

The real shift (which may destroy Landau damping) is

$$\frac{\Delta\omega_L}{\omega_s} = \frac{N_B ec\left|\frac{Z}{n}\right|}{2(2\pi)^{3/2}E/e\eta\sigma_l\left(\frac{\sigma_E}{E}\right)^2} \qquad (6)$$

The growth rate scales inversely with length whereas the real shift increases with length through η. Therefore, feedback damping requirements ease with larger machine radius, but Landau damping may be compromised. The transverse growth rate is given by

$$\frac{1}{\tau_T} = \frac{I_o Z_t^{rw}\,\beta}{4\pi(E/e)}\;\frac{c}{R}\cdot F_m$$

The product Z_t/R is approximately constant, with the growth rate scaling with β. Aperture reduction, however, would have strong impact on growth rates.

Conclusions

For evaluation of SSC lattice designs with respect to single-bunch instability, the suggested figure of merit is the product

$$\frac{R}{Z_t}\;\frac{\eta}{\beta}\;\frac{\Delta E}{E}_{acc}$$

where $(\Delta E/E)acc$ is the energy acceptance. Bellows should be shielded to obtain roughly a factor of three improvement in threshold currents and to ease requirements on RF voltage and energy acceptance. To compensate for lowered threshold values, the low field design should involve larger physical aperture to match peak current performance of the high field alternative, unless the larger ring can be made significantly smoother electrically or feedback proves effective. Coupled-Bunch instabilities show both advantages and disadvantages in larger ring size.

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