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General Theory of the Diocotron Instability of a Relativistic Electron Beam

H. C. Chen

Naval Surface Weapons Center, White Oak
Silver Spring, MD 20903-5000

Abstract

The diocotron instability of a general relativistic electron beam is studied using a macroscopic, cold fluid model. In contrast with the previous treatments where the theoretical analyses are carried out for a teneous electron beam in a strong magnetic field, i.e., plasma frequency << cyclotron frequency, the restriction on the magnitude of the beam density and guiding magnetic field is removed in deriving the general eigenvalue equation. In the limit of long axial wavelengths, a dispersion relation is extracted for a special case of a sharp boundary density profile. The stability properties for various rotating beams are investigated for a broad range of beam parameters. The results show that the kink mode can be unstable as the plasma frequency approaches the cyclotron frequency.

Introduction

Considerable reserch effort has been focussed recently on the development of high-current powerful relativistic electron beams. The beams with high current become very attractive because it provides some nice features as far as the beam propagation on the plasma media. Historically high current beams generated from accelerators are annular and guided by a strong magnetic field. The applied magnetic field provides radial confinement of the electrons. The diocotron instability has been studied[1] previously in the parameter regime where the plasma frequency is much smaller than the cyclotron frequency ($\omega_{pb} << \omega_c$). Because the cost and physical limitation for the strength of the magnetic field, the criteria of $\omega_{pb} << \omega_c$ can be easily broken down if the beam density continues to increase. Consequently, a somewhat more general type of treatment of the instability is needed. In other words, this paper examines the general theory of the diocotron instability of a relativistic electron beam, especially in the regime where the plasma frequency is comparable or even larger than the cyclotron frequency.

Equilibrium

Let us consider a cylindrically symmetrical relativistic annular electron beam propagating parallel to a strong axial magnetic field in a conducting tube. The inner and outer radii of the electron beam are denoted by R_1 and R_2 respectively. R_c is the radius of the conducting wall. The analysis of dynamic properties is based on a maroscopic cold fluid model in which the electron flow is assumed to be laminar. The positive ions are assumed forming a stationary background ($m_i \rightarrow \infty$) which gives a partial fractional neutralization F. The balance forces in the radial direction give the angular velocity of an electron fluid element

$$\omega_b(r) = -\frac{\omega_c}{2}\{1 - [1 - \frac{2\omega_{pb}^2(1-\gamma^2 F)}{\omega_c^2\gamma^2}(1 - \frac{R_1^2}{r^2})]^{1/2}\}$$

Where the electron density profile is assumed to be a rectangle function in the radial direction and the angular velocity of an electron fluid element is in a slow rotational equilibrium. It is necessary for the confinement of the annular electron beam that

$$(\frac{\omega_c}{\omega_{pb}})^2 > \frac{2}{\gamma^2}(1 - \frac{R_1^2}{R_2^2})$$

Stability Analysis

In the linear stability analysis, the first-order perturbed quantities can be Fourier-decomposed according to

$$\delta\Phi(r,\theta,t) = \delta\Phi(r)\exp[i(\ell\theta - \omega t)]$$

where the oscillating angular frequency ω is assumed to be complex with $Im(\omega) > 0$, ℓ is the azimuthal harmonic number. All the perturbed quantities are axially independent. Upon linearizing the fluid-Maxwell equations, we obtain in the stability analysis the following eigenvalue eq.

$$\frac{1}{r}\frac{\partial}{\partial r}[r(1 - \frac{\omega_{pb}^2}{\gamma^2\nu^2})\frac{\partial\delta\psi}{\partial r}] - \frac{\ell^2}{r^2}(1 - \frac{\omega_{pb}^2}{\gamma^2\nu^2})\delta\psi$$

$$= -\frac{\ell}{r}\frac{\delta\psi}{(\omega - \ell\omega_b)}\frac{\partial}{\partial r}[\frac{\omega_{pb}^2(2\omega_b + \omega_c)}{\gamma^2\nu^2}]$$

where

$$\nu^2 = (\omega - \ell\omega_b)^2 - (2\omega_b + \omega_c)[\frac{1}{r}\frac{\partial}{\partial r}(r^2\omega_b) + \omega_c]$$

and the perturbed potential $\delta\psi = \delta\phi(r) - \beta\delta A_z$, ϕ and $\underset{\sim}{A}$ are the scalar and vector potential of the electromagnetic field. In order to solve the eigenvalue equation, the boundary conditions at the surface of the plasma column have to be enforced, continuity and jump condition of the eigenfunction at $r = R_1$ and R_2 connect the solution in different regions. Note that $\omega_{pb}^2(r) = 0$ outside the plasma column. Following the same treatment as in Reference 2, a dispersion relation is obtained

$$\left(\frac{R_1}{R_2}\right)^{2\ell} \frac{z-x}{\gamma^2} \left\{\frac{2Y+z+x-\ell Y}{\gamma^2} + \frac{2R_2^{2\ell}(x-\ell Y)}{(R_c^{2\ell}-R_2^{2\ell})} \left[(x-\ell Y)^2 - (2Y+z)^2\right]\right\}$$

$$+ \left[2x(x^2-z^2) - \frac{x+z}{\gamma^2}\right] \left\{\frac{2Y+z-x+\ell Y}{\gamma^2} + \frac{2R_c^{2\ell}(x-\ell Y)}{(R_c^{2\ell}-R_2^{2\ell})} \left[(x-\ell Y)^2 - (2Y+z)^2\right]\right\} = 0$$

where for the simplicity of notation, we use ω_{pb} to normalize $\omega, \omega_b, \omega_c$ and denote those quantities as

$$x = \omega/\omega_{pb}$$

$$Y = \omega_b/\omega_{pb}$$

$$z = \omega_c/\omega_{pb}$$

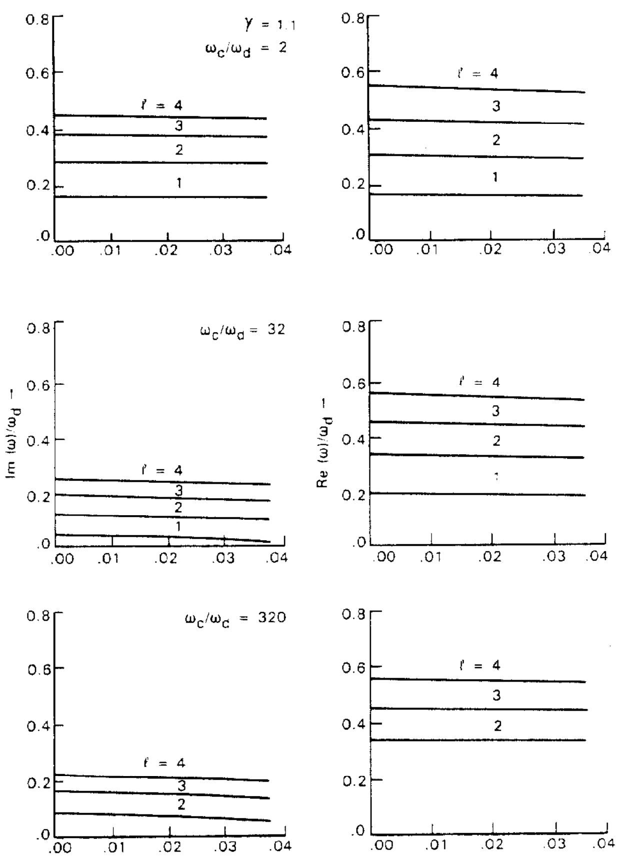

Figure 1 - Diocotron modes of $\ell = 1$ to $\ell = 4$ vs. fractional neutralization for the beam with $\gamma = 1.1$, $R_1 = 0.8\ R_c$, $R_2 = 0.9\ R_c$ and various values of ω_c/ω_d.

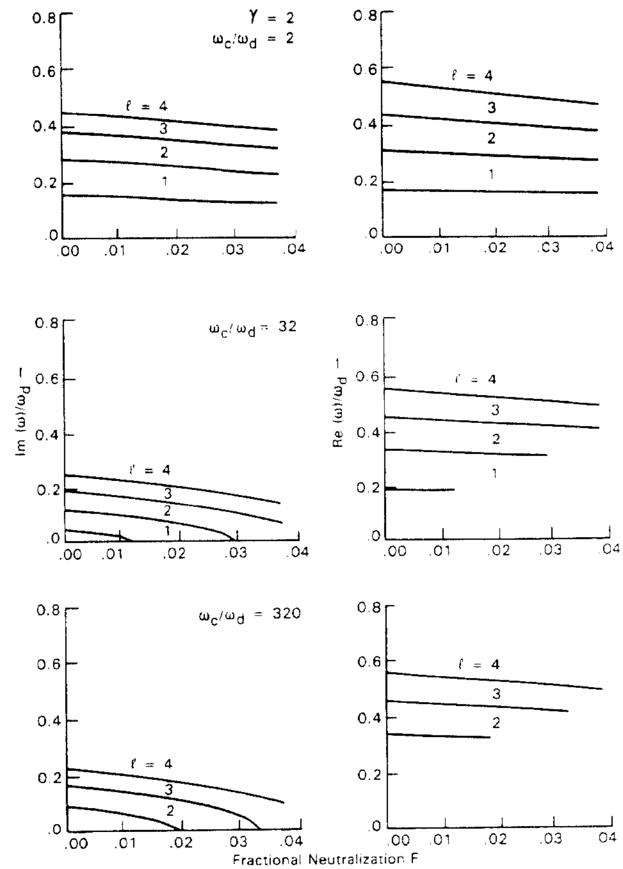

Figure 2 - Similar to Figure 1 except that $\gamma = 2$.

The result can be applied to systems in which the beam thickness is small compared with the radius of the conducting tube. When the beam thickness is sufficiently large, the approximation breaks down because it is no longer legitimate to neglect the variation of ω_b with r.

RESULTS

The dispersion relation is solved numerically for the complex eigenfrequency $\omega = \omega_r + i\omega_i$ with real oscillation frequency ω_r and growth rate ω_i for the unstable mode. Considering the annular beam with $R_1 = 0.8\ R_c$ and $R_2 = 0.9\ R_c$, the results normalized to the diocotron frequency ω_d are shown in Figures 1 and 2 for the non-relativistic ($\gamma = 1.1$) and relativistic case ($\gamma = 2$) respectively. The azimuthal mode numbers are taken from $\ell = 1$ to 4. In the case of a teneous beam in the strong field (e.g., $\omega_c = 320\ \omega_d$), the kink mode ($\ell = 1$) is stable which agrees with the previous results[3]. However, as ω_c/ω_d decreases, the $\ell = 1$ mode can be driven unstable even at larger fractional neutralization. It is important to notice that the kink mode can break the asymmetry of the beam.

Conclusions

We have formulated a fluid-Maxwell theory of the diocotron instability in an infinitely long relativistic electron beam propagating parallel to a uniform applied axial magnetic field. A somewhat more general type of treatment of the instability has been considered. For a general broad range of beam parameters, a closed algebraic dispersion relation is derived in the special case of a sharp boundary density profile, including the important influence of the fractional charge neutralization on stability behavior. It is found that the most dangerous kink mode can be unstable as the plasma frequency becomes comparable to the cyclotron frequency. It might have a strong impact on the stability of a high current relativistic electron beam.

Acknowledgement

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