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SIMULATION OF MULTITURN TRANSVERSE INSTABILITIES IN ELECTRON STORAGE RINGS *

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Abstract

Multiturn transverse beam instabilities place limitations on realizable beam currents in e^+e^- storage rings. Such instabilities become particularly important as one considers future accelerators employing superconducting RF cavities. Higher order cavity modes (HOM) can couple to betatron and synchrobetatron harmonics and, unless sufficiently damped, will lead to instability. A code has been developed for computer simulation of the beam-HOM interaction. Results obtained agree well with experience. Of particular interest is the result that a realistic spread in synchrotron frequencies provides adequate Landau damping of quadrupole oscillations while not significantly affecting dipole behavior. The simulation predicts a quadrupole instability threshold above 1 Ampere for the Cornell elliptical superconducting cavity design placed in CESR.

Introduction

Several theoretical treatments of storage ring transverse beam instabilities have been presented[1,2] and some detailed computer simulations of single beam stability have been reported.[3] Because superconducting RF cavities are much more susceptible to multiturn beam instabilities than are normal cavities, we have studied by simulation the beam-HOM interaction with the Cornell 1500MHz elliptically shaped five-cell superconducting cavity (LE5) design[4] placed in CESR. In general, an instability may occur whenever a cavity higher order deflecting mode has frequency $f_{HOM} = (k-\nu_\beta)f_o + mf_s$ (symbols used in this paper are defined in the Appendix). In practice, however, only the dipole mode (m=0) has been observed in circumstances without high chromaticity.[5]

Of particular interest in this simulation study has been the investigation of the contribution of Landau damping to the stability of these transverse oscillations. In actual operation dipole excitation can be fought with feedback, but quadrupole instabilities would present a more difficult problem, if encountered.[6] The questions therefore arise as to whether the natural spread in synchrotron frequencies due to the harmonic accelerating potential is sufficient to damp these oscillations and to what extent the behavior of transverse and quadrupole instabilities may differ.

The approach taken was to construct a starting bunch which, for the given machine parameters has a stationary ε-t phase space distribution. This bunch is then circulated around the ring and allowed to interact with a cavity dipole mode which is given an initial excitation. The resulting behavior depends strongly, of course, on cavity Q, shunt impedance, and tuning. In addition, the excitation growth rate is expected to be a nonlinear function of the beam current, since radiative and Landau damping, if present, tend to counteract the growth of coherent excitation. Quantum noise is not included in the simulation and zero chromaticity is the condition throughout.

Simulation studies have been performed using both linear and harmonic accelerating potentials. By doing so, the effects of the synchrotron frequency spread were made clearly evident.

Starting Bunch Generation

An initial distribution of particles was constructed for each set of beam energy and cavity fundamental accelerating voltage conditions considered. For the given beam energy spread, σ_ε, a set of 50 macroparticles with peak $\varepsilon=\varepsilon_{pk}$ distributed uniformly from $0-3\sigma_\varepsilon$ were weighted in charge according to

$$q_i = IT_oA_i \exp\left[\frac{A_i^2}{2\sigma_\varepsilon^2}\right] \frac{3}{50\sigma_\varepsilon} \; ; \; A_i = \frac{3\sigma_\varepsilon}{50}(i-1/2);\; i=1-50. \quad (1)$$

These macroparticles were circulated around the storage ring 100,000 revolutions with no transverse displacement and subjected to no HOM interaction. At each passage through the cavity the ε-t coordinates of each particle were recorded. The collection of coordinates for each ε_{pk} value thus collected were next sequenced around the ε-t contour, and every 2000'th point picked out and these points were composed into a new distribution.

This new distribution then consisted of 2500 macroparticles weighted in charge according to the spread in beam energy and distributed according to the empirically determined phase space density. By constructing the initial bunch in this manner, the distribution of synchrotron frequencies is determined by the machine parameters and the density of particles in ε-t phase space is stationary.

Single Particle Equations

The phase space coordinates of each macroparticle upon exiting the RF cavity on turn n are related to those on turn n-1 by

$$t(n)=t(n-1) + \alpha\varepsilon(n)T_o \quad (2)$$

$$\varepsilon(n)=\varepsilon(n-1) - \frac{U_o}{E_o} + \frac{eV_{pk}}{E_o}\sin(\omega t(n)+\phi) + q_i\frac{x(n)}{E_o}\left[\mathrm{Re}[V_{HOM}] - \frac{x(n)Z''\omega_{HOM}q_i}{4Q}\right] \quad (3)$$

$$x(n)=\left[x(n-1)\cos(2\pi\nu_\beta)+\beta x'(n-1)\sin(2\pi\nu_\beta)\right]\exp\left[-\frac{U_o}{2E_o}\right] \quad (4)$$

$$x'(n)=\left[-\frac{x(n-1)}{\beta}\sin(2\pi\nu_\beta)+x'(n-1)\cos(2\pi\nu_\beta)\right]\exp\left[-\frac{U_o}{2E_o}\right] + \frac{c\,\mathrm{Im}[V_{HOM}]}{E_o\omega_{HOM}} \quad (5)$$

$$V_{HOM}(n)=V_{HOM}(n-1)\exp\left[\left(-\frac{1}{2Q}+i\right)\omega_{HOM}T_o\right] + \sum_k^{2500} x_k(n)\frac{Z''\omega_{HOM}q_k}{2Q}\exp\left[i\omega_{HOM}t_k(n)\right] \quad (6)$$

The last term of (3) describes the effect of the dipole HOM on the particle energy. For x values consistent with actual beam pipe sizes, this term has no significant effect. It is important to note that there appears no significant direct coupling of the transverse and longitudinal motions of the particles with ξ=0. Any synchrobetatron excitations which are

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observed in simulation, then, arise from the appropriate tuning of the HOM onto a synchrobetatron line.

The last term of (5) expresses the transverse kick given each particle by the excited cavity deflecting mode.[7] The free decay of the HOM and the effect of beam transit on deflecting mode excitation is described by (6). The voltage induced by each particle is summed vectorially and added to the existing voltage.

Simulation Conditions

Machine parameters used in the simulations are given in Table 1. The four most threatening deflecting modes (highest shunt impedance) of the LE5 cavity design were examined in the 3.5GeV / 1.7MV condition. Simulations were done with each mode tuned first to a betatron line, then an associated synchrotron sideband. In each case, response to both harmonic and linear accelerating potentials were investigated.

In the case of the linear accelerating potential, the third term on the right hand side of (3) is replaced by:

$$\frac{eV_{pk}}{E_o}\,(t(n)\omega\cos\phi + \sin\phi)$$

The LE5 cavity design has an active length of 0.5 meters. Since the cryostat module tested in CESR [8] consisted of two such cavities, the worst case was used in the simulation, i.e. all HOMs of the two cavities were made identical in frequency so that each had an effective length of 1 meter.

Since transverse dipole oscillations in a storage ring can be fought with feedback, damping was added to the simulation code so as to allow no transverse displacement to the center of mass of each ring of macroparticles with common ε_{pk}. In this way the excitation of quadrupole oscillations could be examined independently.

TABLE 1
CESR MACHINE PARAMETERS USED IN SIMULATION

T_o	2.5632225 μsec	2.56319345 μsec
E_o	3.5 GeV	5.0 GeV
σ_ε	4.038×10^{-4}	5.8134×10^{-4}
ξ	0.0	0.0
ν_β	9.3899	9.3947
V_{pk}	1.7, 3.0, 5.0 MV	12.0 MV
U_o	0.1919 MeV	0.8219 MeV
α	0.014283	0.014915
β	18.00 m	13.70 m

Results

Under all conditions without the added damping, the dominant effect observed for beam currents I >10mA is growth of coherent transverse dipole oscillations. This is the case even when the HOM is tuned to a synchrobetatron line because the loaded cavity Q's are low enough to give a bandwidth which is comparable to the synchrotron frequency (>46kHz vs. 25kHz), so that the dipole resonance condition is still present.

The dipole growth rate determined by simulation may be compared with that calculated for the simple rigid bunch case for which there is the relation[7,1]:

$$\Gamma = \frac{I\,Z''c^2 e}{4\pi\nu_\beta\omega_{HOM}E_o}\,S\,D \qquad (7)$$

where S is the reduction factor $\exp(-\omega_{HOM}^2\sigma^2/c^2)$ resulting from the spread of the bunch, and D is the decay of the deflecting field between bunch passages, $\exp(-T_o\omega_{HOM}/2Q)$. For the LE5 cavity 1888MHz mode with 100mA single bunch beam current at 3.5GeV in CESR, (7) gives Γ=2540 sec^{-1}. The growth rate determined by simulation is 2100 sec^{-1}.

The instability growth rates obtained for the four worst LE5 modes are listed in Table 2.

TABLE 2
INSTABILITY GROWTH RATES FOR FOUR LE5 CAVITY MODES
3.5GeV / 1.7MV

Frequency(MHz)	1888.	1969.	2086.	2110.
Q (10^4)	3.2	0.4	1.0	1.3
Z"/Q (Ω/m²)(10^4)	6.95	16.4	5.0	10.0
Z" (Ω/m²)(10^9)	2.2	0.66	0.50	1.3
Γ/I Dipole (s^{-1}-A^{-1})				
	21,000	612	2,700	5840
Γ Quadrupole (s^{-1})				
I= 10.0 A	20,000		1,740	9000
1.0	<90		<25	<25
0.1	<20		<25	<25
Γ Quadrupole/Linear Potential (s^{-1})				
I= 10.0 A	20,000	798	2,990	9500
1.0	2,000		257	965
0.1	200		<30	<100

Figure 1 presents the dependence of Γ on beam current for both dipole and quadrupole instabilities with an accelerating potential of 5.0MV at 3.5GeV. Radiative damping alone provides dipole threshold currents of only about 0.5mA. In addition, the dipole excitation growth rates are the same with linear and harmonic fundamental accelerating potentials. The lack of significant direct coupling of the transverse and longitudinal motions prevents the spread in synchrotron frequencies from having any effect on the dipole growth rates, i.e. no Landau damping.

With the dipole excitations damped, however, the dependence on current of quadrupole growth rates proves more interesting. Note that in the case of Figure 1, the quadrupole threshold current with a harmonic accelerating potential is 1.7 Amperes.

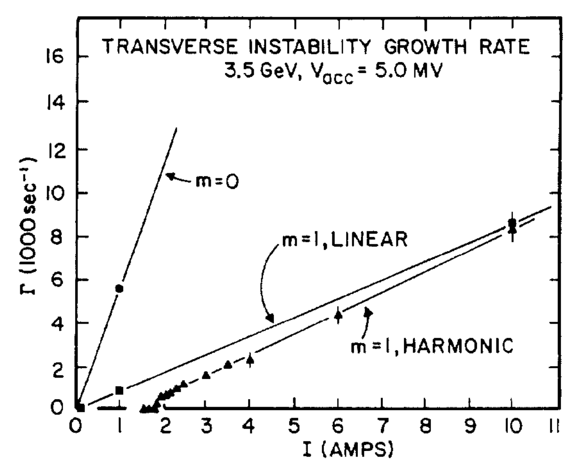

Figure 1 Transverse instability growth rate versus beam current with two 1500MHz superconducting cavities in CESR. f_{HOM}=1888MHz (3.5 GeV)

Normalized synchrotron frequency distributions generated for each of the operating conditions considered are presented in Figure 2 (full horizontal scale is 3600Hz). The associated distribution parameters are listed in Table 3 together with the threshold current and linear potential growth rate for the 1888MHz deflecting mode.

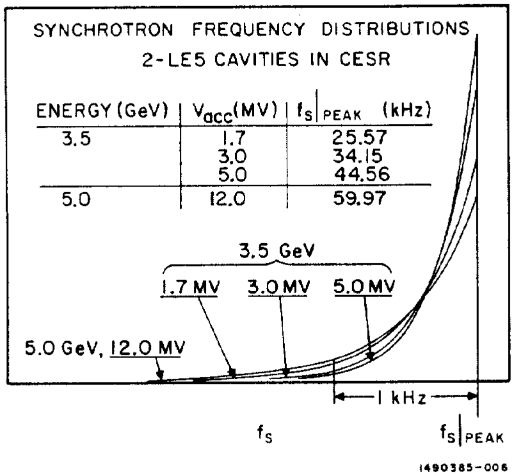

Figure 2 Synchrotron frequency distributions generated by simulation without quantum noise.

It should be noted that quantum noise which has not been included would contribute a smearing of the distributions.

Figure 3 compares the linear potential quadrupole growth rates at threshold with the variance, Δf_s, of the synchrotron frequency distributions. Essentially, the threshold occurs at that beam current which has a linear potential growth rate just matching the spread in synchrotron frequencies.

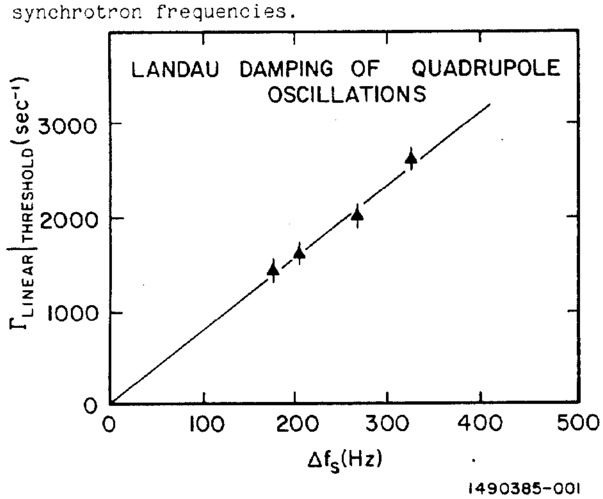

Figure 3 Landau damping of transverse quadrupole oscillations with two 1500MHz superconducting cavities in CESR.

Conclusion

Computer simulations of multiturn transverse instabilities in an electron storage ring have been performed. The results indicate that, for available beam currents, the natural spread in synchrotron frequencies is sufficient to damp transverse quadrupole excitations to the extent that quadrupole instabilities need not be a threat. In addition, it is noted that with zero chromaticity the spread in f_s does not contribute damping to transverse dipole oscillations.

TABLE 3
SYNCHROTRON FREQUENCY DISTRIBUTIONS GENERATED BY SIMULATION

E_o(GeV)	V(MV)	$f_s\vert_{mean}$(kHz)	Δf_s(Hz)	I_{th}(A)	Γ_{th}(sec^{-1})
3.5	1.7	25.313	267.3	1.0	2000
3.5	3.0	33.996	203.8	1.3	1600
3.5	5.0	44.427	174.7	1.7	1400
5.0	12.0	59.727	324.4	4.3	2600

Appendix

SYMBOLS

t	deviation from stable phase
ε	relative deviation from energy equilibrium
α	momentum compaction
T_o	beam revolution period
E_o	beam energy
U_o	synchrotron radiation loss per turn
V_{pk}	peak accelerating voltage
ω	angular frequency of accelerating RF
ω_{HOM}	angular frequency of HOM
ϕ	synchronous phase angle
σ_ε	relative beam energy spread
σ	bunch half-length
x	transverse displacement at cavity
x'	slope of transverse displacement at cavity
β	betatron oscillation amplitude at cavity
ν_β	betatron tune
Q	quality factor of cavity HOM
Z''	HOM shunt impedance (speed of light particle)
V_{HOM}	cavity HOM voltage transverse gradient
e	electron charge
k	revolution harmonic
ξ	chromaticity
Γ	instability growth rate
I	single bunch beam current

References

1. J.L. Laclare, "Bunched-Beam Instabilities," 11th Intnl. Conf. on High Energy Accelerators, Geneva, EXS 40, 1980, p. 526.
2. F. Sacherer, "Transverse Bunched Beam Instabilities - Theory," IEEE Trans. Nucl. Sci. NS-24, 347, (1977).
3. R.H. Siemann, "Computer Simulation Studies of Single Beam Stability," IEEE Trans. Nucl. Sci. NS-30, 2373, (1983).
4. P.Kneisel et al., "Performance of Superconducting Storage Ring Cavities at 1500 MHz." IEEE Trans. Mag. MAG-21, 1000, (1985).
5. E. Patterson, R. Kohaupt, and D.H. Rice, private communications.
6. T. Suzuki et al., "Collective Beam Instabilities Caused by RF Cavities in Tristan," IEEE Trans. Nucl. Sci. NS-30, 2563, (1983).
7. R. Sundelin, "Synchrobetatron Oscillation Driving Mechanism," IEEE Trans. Nucl. Sci. NS-26, 3604, (1979).
8. R. Sundelin, "High Gradient Superconducting Cavities for Storage Rings," contribution to this Conference.

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