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IMPASS - A COMPUTER PROGRAM FOR CALCULATIONS OF IMPEDANCES OF PERIODIC AXIALLY SYMMETRIC SMOOTH STRUCTURES (BELLOWS)

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Abstract

The computer program IMPASS approximately solves an infinite system of linear algebraic equations for the expansion coefficients of the general solution of the Maxwell equations derived in Ref.1. The coefficients are found by inverting the matrix, cut to a certain finite size. The values of longitudinal and transverse impedances, for a given frequency and for a given number (e.g. m=0 or m=1) of the azimuthal Fourier harmonic of the bunch density, are proportional to the zeroth coefficients of the solutions. Standard FFT routines are used to speed up the calculations of the matrix elements and terms in the right hand side of the equations. The stability of the solution, with respect to the matrix size and to the precision of the equation terms, is studied. The CPU time on a CYBER 875 computer, needed to find both the longitudinal and the transverse impedances of bellows for one value of the frequency, is of the order of a second.

1. Introduction

The experience obtained from large storage rings like PETRA and PEP shows that a substantial part of the transverse impedance of a ring comes from the elements of the vacuum pipe other than RF cavities. Most of the increase is produced by bellows. Typically, bellows contribute to the ring impedance approximately as much as all the RF cavities together. Hence, one needs reliable and quick methods for the evaluation of the impedances produced by bellows.

The computer code, described in the present paper, calculates both longitudinal and transverse impedances of periodic axially symmetric smooth structures (bellows). All the calculations are made for the limit $\gamma \to \infty$ (γ is the Lorentz factor of a particle).

The assumptions of periodicity and axial symmetry of the structure imply that the wall of the bellows may be described in a cylindrical coordinate system r,θ,z, by a curve :

$$r = b(z), \quad 0 < z < L \quad , \tag{1}$$

where L is the length of the boundary period and

$$b(z+L) = b(z) \quad . \tag{2}$$

The curve b(z) is assumed to be continuous and with a continuous finite derivative db/dz (Fig.1).

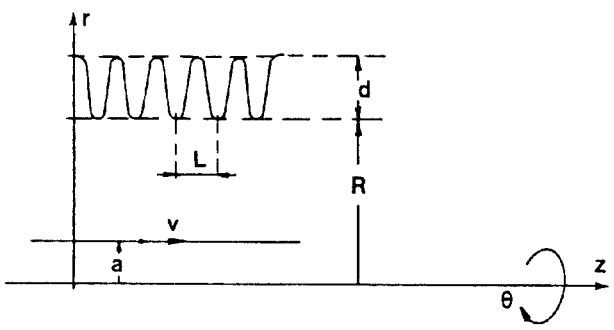

Fig.1 - Schematic layout of bellows geometry and coordinate system.

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2. Method

IMPASS approximately solves an infinite system of linear algebraic equations for the expansion coefficients of the general solution of the Maxwell equations derived in Ref.1. The coefficients are found by inverting the matrix, cut to a finite size. The values of the longitudinal and transverse impedances, Z_L and Z_T for a given frequency ω and a given mode number m (m=0 or m=1) of the azimuthal Fourier harmonic of the bunch density, are proportional to the zeroth coefficients

$$Z_{L0} = i \frac{Z_0}{n} B_0 \tag{3}$$

$$Z_{L1} = i \frac{Z_0}{n} \left(\frac{r}{R}\right) \left(\frac{a}{R}\right) D_0 \tag{4}$$

$$Z_{T1} = i \frac{Z_0}{Rn\kappa} D_0 \quad , \tag{5}$$

where $Z_0 = 377$ Ohm is the impedance of vacuum

$$n = 2\pi R/L \quad , \tag{6}$$

$$\kappa = \omega R/c \quad . \tag{7}$$

R is the reference radius and r and a in expression (4) are the radius of an observation point and the radial displacement of the charge from the axis of the bellows.

The choice of the reference radius R is more or less arbitrary, and it has been checked numerically that the answer does not depend on it. One should, however, bear in mind that the choice of R changes the function W(u) (see below) and might also influence the speed of convergence. One natural choice for R would be the average radius of the bellows, and another one the inner radius.

3. Input

The first line of input consists of up to 80 alphanumeric characters, which will appear as a heading on the output.

Then the following variables are read in via NAMELIST $DATA.

ETA	$n = 2\pi R/L$ (expression (6)).
KAPPA	$\kappa = \omega R/c$ (expression (7)).
EPS	$\varepsilon = d/2R$, where d is the depth of corrugations.
PRINT	a logical variable preset to FALSE. If PRINT=T then all the values of intermediate matrices will be printed out. This option is used mainly for checking.
IMAG	a logical variable preset to FALSE. If IMAG=T the program will find a solution for the imaginary part of the matrix (if any).
PLIM	the order of the matrix is 2×PLIM+1. The program allows for values of PLIM up to 20.
M	is the number of points for the Fast Fourier Transforms, used for calculating the matrix elements. The program allows for M up to 512 points. This, however, can be modified by a DIMENSION statement. The routine RFT requires a COMMON/FWORK/W(nnn), where nnn = $5*2^m$ if M>129. IMPASS has nnn = 2560 i.e. for m=9.

The boundary curve (1) is represented by dimensionless variables :

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$$u = 2\pi z/L \qquad (8)$$

$$W = b/R \quad . \qquad (9)$$

The user must provide his own definitions of W(u) and its derivative WP(u) by modifying the two FUNCTION subprograms FUNCTION W(U) and FUNCTION WP(U).

By default the program uses the functions :

$$W(u) = 1 + \varepsilon\cos(u) \qquad (10)$$

$$WP(u) = -\varepsilon\sin(u) \; . \qquad (11)$$

4. Output

The listing of the input parameters is followed by a table of the solutions for m=1 and m=0 for all values of N, -NLIM ≤ N ≤ NLIM (NLIM=PLIM). This is followed by a table of CHECKS, i.e. the solution is put back into the original equations and the resulting value is compared to the expected right hand side.

5. Accuracy and running time

The following table gives some idea of the accuracy and the time taken by the CYBER 875 to do the calculations. For W(U) = 1.0 + EPS*COS(U), ETA = 30.0, KAPPA = 0.1, EPS = 0.12 and M = 256 :

PLIM	D_0	TIME
5	0.01994	1.0 sec
6	0.01980	1.2 sec
7	0.01977	1.4 sec
8	0.01978	1.6 sec
9	0.01981	1.7 sec
10	0.01987	1.9 sec

Note that with the double precision test program D_0 = 0.01980 when PLIM=20.

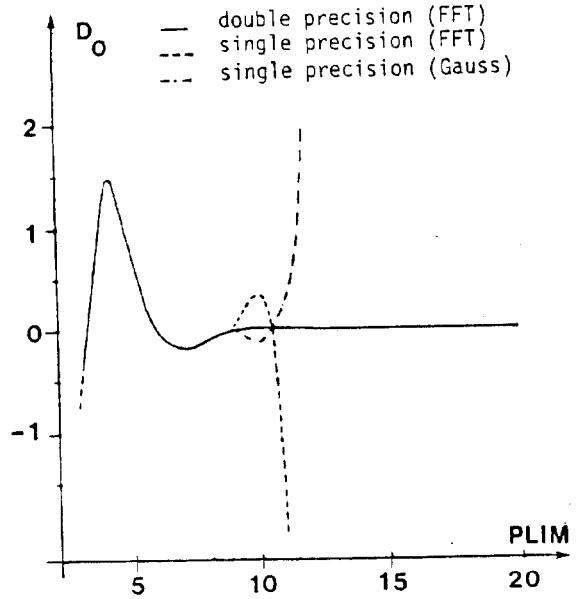

Fig.2 - Accuracy of D_0 in % versus PLIM for W(U) = 1+εcosu and η = 30, κ = 0.1, ε = 0.12 for single and double precision calculations.

6. Stability and accuracy tests

A number of tests have been made in order to check different parts of the program. Using the function (10) the calculations of the matrix elements and the right hand sides of the equations were compared to the analytical calculations. Also the numerical calculations were done twice, once with a Gaussian integration and once using the FFT.

The impedances should not depend on the initial z coordinate. It has also been checked that one gets the same results by substituting sine for cosine in the expression (10).

It has already been mentioned that the results do not depend on the choice of the reference radius R.

For the dipole mode (m=1) one out of the three boundary conditions is a consequence of the other two. This fact is used in IMPASS by checking that all three equations are satisfied by the found set of coefficients (although the third one is only approximate).

The results of the calculations should not depend on the size of the matrices, or on the parameter PLIM, within certain limits. If PLIM is too small the accuracy is poor, due to cutting off non-negligible terms. On the other hand, if PLIM is too large, the accuracy is lost due to the accumulation of machine errors (computational instability).

The computational stability of the results can be increased, for matrices of a larger order, by using double precision. The results of the stability dependance on the matrix size, for both single and double precision, can be seen in Fig.2 (note the suppressed zero of the abscissa).

7. Results

Figures 3-9, taken from Ref.1, illustrate the dependance of the coefficients B_0 and D_0 on the normalized frequency κ, for several different values of the parameters η and ε. The boundary curve in these calculations, is represented by the function:

$$W = 1 + \varepsilon + (4/\pi)\,\varepsilon\cos u.$$

The impedances, obtained from these results, agree quite well with those from calculations using the program TBCI [2].

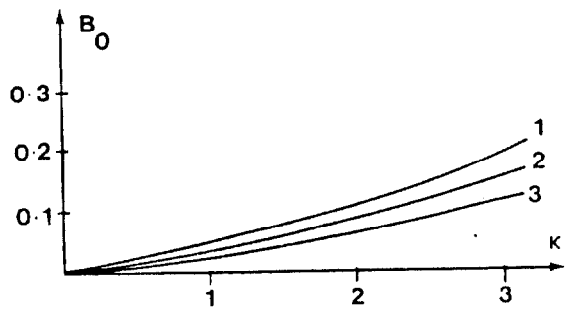

Fig.3 - Coefficient B_0 which is proportional to the m=0 longitudinal impedance, for bellows with the boundary w=1+ε+(4/π)εcosu as a function of the normalized frequency κ, for three values of the parameter η : (1) η=31.42, (2) η=20.94, (3) η=12.57. Corrugation parameter ε = 0.06.

REFERENCES

[1] S.Kheifets and B.Zotter, Longitudinal and transverse impedances of bellows in low frequency range (in preparation for publication).

[2] T. Weiland, Nucl. Instr. Meth. 212 (1983) 13.

[3] IMPASS uses the following subroutines from the CERN Program library :
CFT - Complex Fast Fourier Transform by R.C. Singleton (SLAC) RFT - Real Fast Fourier Transform by Ch. Iselin (CERN)
EBESI0, EBESI1 - Modified Bessel Functions by K. Kölbig (CERN).

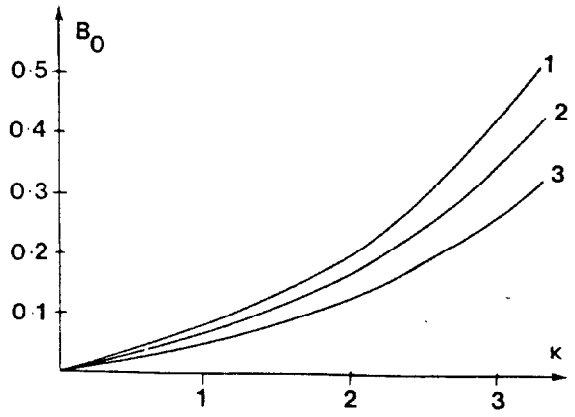

Fig.4 - The same as on Fig.3, but for $\varepsilon = 0.09$.

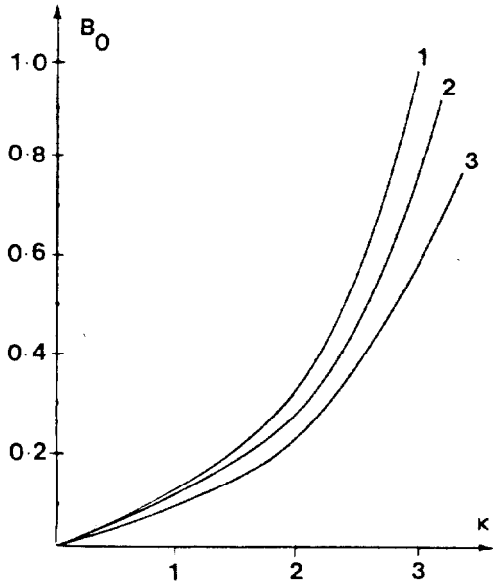

Fig.5 - The same as on Fig.3, but for $\varepsilon = 0.12$.

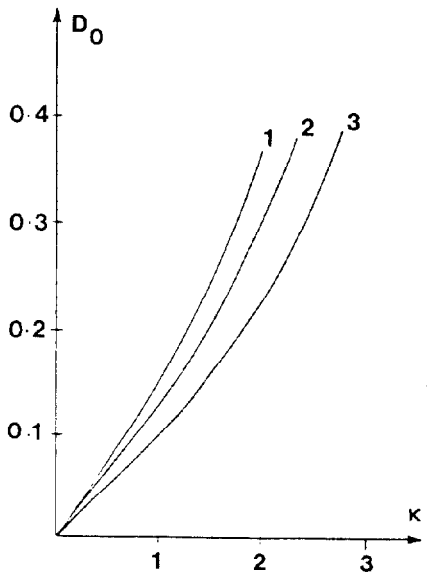

Fig.6 - Coefficient B_0^1 which is proportional to the m=1 longitudinal impedance and to the transverse impedance multiplied by κ, for bellows with the boundary $w=1+\varepsilon+(4/\pi)\varepsilon\cos u$ as function of the normalized frequency κ, for three values of the parameter η: (1) η=31.42, (2) η=20.94, (3) η=12.57. Corrugation parameter ε=0.09.

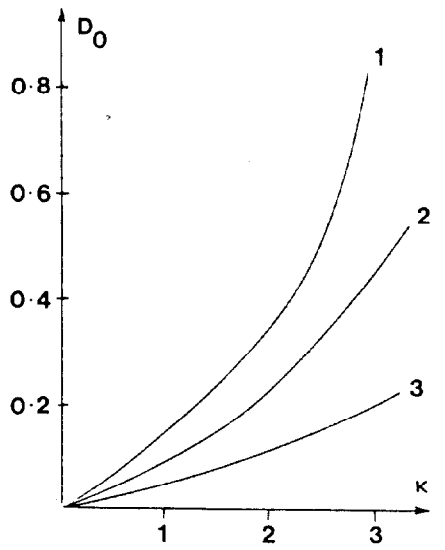

Fig.7 - The same as on Fig.6, but for η=12.57, and for three values of the corrugation parameter ε : (1) ε=0.12, (2) ε=0.09, (3) ε=0.06.

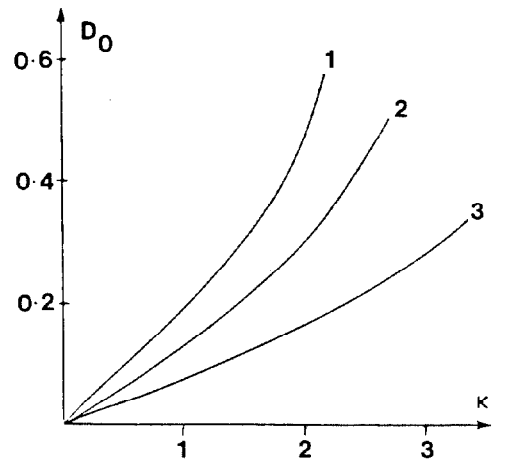

Fig.8 - The same as on Fig.7, but for $\eta = 20.94$.

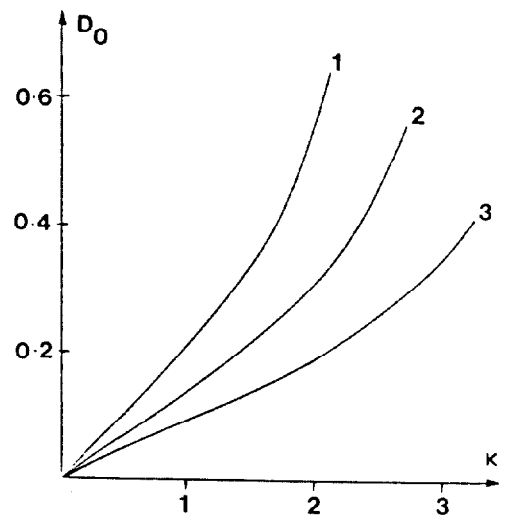

Fig.9 - The same as on Fig.7, but for $\eta = 31.42$.